Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 11, Number 4 (2015), pp. 2511-2516

http://www.ripublication.com

Solution Of Third Order Ordinary Differential Equations Using Differential Transform Method

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Abstract

In this study, a simple and Taylor series-based method known as differential transformation method (DTM) is used to solve initial-value problems involving third-order ordinary differential equations. We introduced briefly the concept of DTM and applied it to obtain the solution of three numerical examples for demonstration. The results are compared with the existing ones in literature and it is concluded that results yielded by DTM converge to the analytical solution more rapidly with few terms.

Keywords: Ordinary differential equations, Differential transformation method, Initial value problems (IVP)

1.0 INTRODUCTION

Most phenomena in sciences, economics, management, engineering etc. can be modelled by differential and integral theories. Interestingly, solutions to most of the differential equations arising from such models do not have analytic solutions, necessitating the development of numerical techniques.

However, most of the numerical methods that exist in literature require discretization, perturbation, linearization or complex computations. Some of the numerical methods used in literature include; Adomian Decomposition method in [1-3], modified Adomian decomposition method [4-5], variational Iteration method [6],homotopy perturbation method [7-8].Okuboye developed seven-step block method in [9].

The differential transform method proposed in this work is easy to apply; it requires no linearizationor discretization. It is an iterative technique for obtaining analytic

Taylor series solution of differential equations. The method was first introduced by Zhou [10] to solve linear and nonlinear initial value problems in electrical circuits. It has been widely applied to numerous problems, some of which are; Biazar et al for Riccati differential equations in [11], Opanuga et al in [12], Edeki et al in [13]. Gbadeyan and Agboola [14] used DTM to solve vibration problem. The three examples solved in this present work gave the exact solution and the results are presented in tables in comparison with the three-step block method.

2.0 FUNDAMENTALS OF DIFFERENTIAL TRANSFORM METHOD

Consider a function $g(x)$ whose differential transform can be defined as follows,

$$G(k) = \frac{1}{k!}\left[\frac{d^k g(x)}{dx^k}\right]_{x=0} \tag{1}$$

In equation (1), $g(x)$ is the original function and $G(x)$ is the transformed function. At about $x = 0$, the Taylor series is defined as,

$$g(x) = \sum_{k=0}^{x^k} \frac{x^k}{k!}\left[\frac{d^k g(x)}{dx^k}\right]_{x=0} \tag{2}$$

The inverse differential transform is expressed as,

$$g(x) = \sum_{k=0}^{\infty} x^k G(k) \tag{3}$$

Combining equations (1) and (2), we derive the following mathematical operations:

(i) If $g(x) = p(x) \pm r(x)$, then $G(k) = P(k) \pm R(k)$

(ii) If $g(x) = \alpha p(x)$, then $G(k) = \alpha P(k)$, α is a constant.

(iii) If $g(x) = \dfrac{d^n p(x)}{dx^n}$, then $G(k) = \dfrac{(k+n)!}{k!} P(k+n)$

(iv) If $g(x) = \sin(\beta x + \alpha)$, then $G(k) = \dfrac{\beta^k}{k!}\sin\left(\dfrac{k\pi}{2} + \alpha\right)$, α and β are constants

(v) If $g(x) = e^{\lambda x}$, then $G(k) = \dfrac{\lambda^k}{k!}$, where λ is a constant

3.0NUMERICAL EXAMPLES

We illustrate the method by the following problems

Example1:

Consider $u'''(t) = -u$,[7] (4)

The initial conditions are:

$$u(0) = 1,\ \ u'(0) = -1,\ \ u''(0) = 1 \tag{5}$$

While the theoretical solution of equation (4) is given as

$$u(t) = e^{-t} \tag{6}$$

Taking the differential transform of equation (4), we obtain

$$U(k+3) = -\frac{U(k)}{(k+3)!} \tag{7}$$

andthe transformation of the initial conditions yield

$$U(0)=1, \quad U(1)=-1, \quad U(2)=\frac{1}{2} \tag{8}$$

Substitutingequation (8) in (7), we obtain the following values

$$\begin{aligned} &U(3) = -\frac{1}{6}, \quad U(4) = \frac{1}{24}, \quad U(5) = -\frac{1}{120}, \\ &U(6) = \frac{1}{720}, \quad U(7) = -\frac{1}{5040}, \quad U(8) = \frac{1}{40320}, \end{aligned} \tag{9}$$

The series solution up to $0(t^8)$ is obtained as

$$u(t) = 1 - t + \frac{t^2}{2} - \frac{t^3}{6} + \frac{t^4}{24} - \frac{t^5}{120} + \frac{t^6}{720} - \frac{t^7}{5040} + 0(t^8) \tag{10}$$

Table 1: Numerical result for example 1

t	DTM	EXACT	DTM ERROR	ERROR IN KUBOYE [3]
0.1	0.904837418035960	0.904837418035960	0.000000000000000	2.138401E-12
0.2	0.818730753077982	0.818730753077982	0.000000000000000	6.055156E-13
0.3	0.740818220681718	0.740818220681718	0.000000000000000	7.395751E-12
0.4	0.670320046035639	0.670320046035639	0.000000000000000	2.158163E-12
0.5	0.606530659712633	0.606530659712633	0.000000000000000	1.484579E-11
0.6	0.548811636094026	0.548811636094026	0.000000000000000	1.098521E-11
0.7	0.496585303791410	0.496585303791410	0.000000000000000	3.142886E-11
0.8	0.449328964117222	0.449328964117222	0.000000000000000	2.309530E-11
0.9	0.406569659740599	0.406569659740599	0.000000000000000	5.154149E-11
1	0.367879441171442	0.367879441171442	0.000000000000000	8.200535E-11

Example 2:

Next, we solve $u'''(t) = e^t$, $0 \le t \le 1$, [7] (11)

Subject to the boundary conditions

$$u(0)=3, \quad u'(0)=1, \quad u''(0)=5 \tag{12}$$

The exact solution of (11) is

$$u(t) = 2 + 2t^2 + e^t \tag{13}$$

Transforming equation (11) we obtain

$$U(k+3) = \frac{1}{(k+3)!}.\left(\frac{1}{k!}\right) \tag{14}$$

and the differential transform of the initial conditions give

$$U(0) = 3, \quad U(1) = 1, \quad U(2) = \frac{5}{2} \tag{15}$$

Applying the transformed initial conditions (15) in equation (14), we obtain the following

$$U(3) = \frac{1}{6}, \quad U(4) = \frac{1}{24}, \quad U(5) = \frac{1}{120},$$

$$U(6) = \frac{1}{720}, \quad U(7) = \frac{1}{5040}, \quad U(8) = \frac{1}{40320} \tag{16}$$

We finally obtain the series solution up to $0(t^8)$ as

$$u(t) = 1 + t + \frac{t^2}{2} + \frac{t^3}{6} + \frac{t^4}{24} + \frac{t^5}{120} + \frac{t^6}{720} + \frac{t^7}{5040} + \cdots (t^8) \tag{17}$$

Table 2: Numerical result for example 2

t	DTM	EXACT	DTM ERROR	ERROR IN OKUBOYE[13]
0.1	3.125170918075650	3.125170918075650	0.000000000000000	2.531308E-14
0.2	3.301402758160170	3.301402758160170	0.000000000000000	1.612044E-13
0.3	3.529858807576000	3.529858807576000	0.000000000000000	4.023448E-13
0.4	3.811824697641270	3.811824697641270	0.000000000000000	7.536194E-13
0.5	4.148721270700130	4.148721270700130	0.000000000000000	1.212364E-12
0.6	4.542118800390510	4.542118800390510	0.000000000000000	1.780798E-12
0.7	4.993752707470480	4.993752707470480	0.000000000000000	2.456702E-12
0.8	5.505540928492470	5.505540928492470	0.000000000000000	2.212097E-11
0.9	6.079603111156950	6.079603111156950	0.000000000000000	5.231993E-11
1.0	6.718281828459050	6.718281828459040	0.000000000000000	8.860113E-11

Example3:

$$u'''(t) = 3\sin t, 0 \le t \le 1 \ [7] \tag{18}$$

With the initial conditions given as

$$u(0) = 2, \quad u'(0) = 0, \quad u''(0) = -2 \tag{19}$$

Theoretical solution is given as

$$u(t) = 3\cos t + \frac{t^2}{2} - 2 \tag{20}$$

Transforming equations (18) and (19) we have

$$U(k+3) = \frac{1}{(k+3)!}\left(\frac{3}{k!}\sin\frac{k\pi}{2}\right) \tag{21}$$

and

$$U(0) = 1, \quad U(1) = 0, \quad U(2) = -2 \tag{22}$$

Substituting equation (22) in (21) yields

Table 3: Numerical result for example3

t	DTM	EXACT	DTM ERROR	ERROR IN OKUBOYE[3]
0.1	0.990012495834077	0.990012495834077	0.000000000000000	1.743050E-14
0.2	0.960199733523725	0.960199733523725	0.000000000000000	1.082467E-13
0.3	0.911009467376818	0.911009467376818	0.000000000000000	2.711165E-13
0.4	0.843182982008654	0.843182982008655	0.000000000000001	5.079270E-13
0.5	0.757747685671099	0.757747685671118	0.000000000000019	8.164580E-13
0.6	0.656006844728792	0.656006844729035	0.000000000000243	1.199707E-12
0.7	0.539526561851364	0.539526561853465	0.000000000002101	1.654343E-12
0.8	0.410120128027871	0.410120128041497	0.000000000013626	1.674639E-10
0.9	0.269829904741115	0.269829904811993	0.000000000070878	3.336392E-10
1.0	0.120906917294566	0.120906917604419	0.000000000309854	5.001723E-10

$$\begin{aligned} &U(3)=0, \quad U(4)=\frac{1}{8}, \quad U(5)=0, \quad U(6)=-\frac{1}{240}, \\ &U(7)=0, \quad U(8)=\frac{1}{13440}, \quad U(9)=0, \quad U(10)=-\frac{1}{1209600}, \end{aligned} \tag{23}$$

The series solution up to $0(t^{14})$ is obtained as

$$u(t)=1-t^2+\frac{t^4}{8}-\frac{t^6}{240}+\frac{t^8}{13440}-\frac{t^{10}}{1209600}+\frac{t^{12}}{159667200}+\ldots(t^{14}) \tag{24}$$

4.0 CONCLUSION

In this work, we applied DTM for the solution of initial value problems in the third-order ordinary differential equation. Three examples are provided to illustrate the method. The obtained results agree with the conclusion made by several researchers that DTM is easy and simple to apply, it reduces the computational difficulties of other traditional methods. We then conclude that DTM is an excellent method for the solution initial value problem in the third-order.

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