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Dynamic Response of an Inclined Railway Bridge Supported by Winkler Foundation Under a Moving Railway Vehicle

[1]Michael C. Agarana, [2]Jacob A. Gbadeyan, [1]Olasunmbo O. Agboola, [1]Timothy A. Anake, [1]Olawale J. Adeleke

[1]*Department of Mathematics, College of Science & Technology, Covenant University, P.M.B. 1023, Ota, Nigeria*
[2]*Department of Mathematics, Faculty of Physical Science, University of Ilorin, P.M.B. 1515, Ilorin, Nigeria*

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ABSTRACT

An investigation into the dynamical behaviour of an inclined railway bridge traversed by uniform partially distributed moving railway vehicle, and supported by an elastic foundation is carried out. The effects of shear deformation and rotatory inertia are taken into consideration. The resulting coupled partially differential equations are solved using finite difference method. It was found that the foundation moduli and angle of inclination of the bridge have significant effect on the deflection of the bridge.

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INTRODUCTION

An inclined railway bridge is a railway bridge set at an angle, not perpendicular to a horizontal plane. However, the work done is the same: Work = Force × Distance, and the distance is increased, whereas the force is decreased [Molinear *et al* (2012), Gbadeyan and Agarana (2014)]. In Elementary Physics, an object placed on a tilted surface (inclined plane) will often slide down the surface. The greater the tilt of the surface (i.e. the angle of inclination), the faster the rate at which the object will slide down it (Sofi, 2013). According to Newton's laws of motion, a Railway vehicle on an inclined plane will continue to slide down the plane if there is no applied force to balance the forces acting on it, especially if the surface is frictionless or with minimal friction. There are always, at least, two forces namely: the force of gravity and the normal force, acting upon the railway vehicle positioned on an inclined bridge (Gerg and Dukkipati, 1984). The force of gravity acts in a downward direction, while the normal force acts in a direction perpendicular to the surface [Molinear *et al* (2012), Gbadeyan and Dada (2006)]. An inclined plane problem is in every way like any other net force problem with the sole exception that the surface has been tilted. An inclined plane therefore can be transformed into the form with which we are more comfortable, as illustrated in Figure 2. After this transformation, we can ignore the force of gravity since it has been replaced by its two components [Molinear (2012), Sofi (2013)]. We can now solve for the net force and the acceleration. For a railway vehicle mowing up the inclined bridge, the applied force must be greater than the component of its weight (F_{11}) moving down the inclined bridge, to avoid sliding down.

Problem Formulation:

A rectangular inclined railway bridge, modelled as rectangular inclined Mindlin plate, supported by Winkler foundation and traversed by a partially distributed moving railway vehicle is considered. M is the mass of the railway vehicle of rectangular dimension ε and μ, and , with one of its lines of symmetry moving along $y = y_1$ the plate is L_x by L_y in dimension and let $\xi = ut + \varepsilon/2$, where u is the velocity of the load. θ is the angle of inclination, F_{11} is the component of the weight of the railway vehicle parallel to the inclined plane and F_1 is the component perpendicular to the inclined plane.

Assumptions:

(i) The inclined bridge is of constant cross – section, (ii) the moving railway vehcile moves with a constant speed, (iii) The moving railway vehicle is guided in such a way that it keeps contact with the inclined bridge throughout the motion, (iv) The inclined bridge is continuously supported by a Winkler foundation, (v). The moving railway vehicle is uniformly partially distributed, (vi) The rectangular

Corresponding Author: M.C. Agarana, Department of Mathematics, College of Science & Technology, Covenant University, P.M.B. 1023, Ota, Nigeria;
Tel: +2348023214236; E-mail: michael.agarana@covenantuniversity.edu.ng

Mindlin railway bridge is elastic, (vii) No damping in the system, (viii) Uniform gravitational field; (ix) Constant mass (M_L) of the railway vehicle moving up the inclined plane. (x) Constant angle of inclination

Initial Conditions:

$$W(x,y,0) = 0 = \frac{\partial W}{\partial t}(x,y,0)$$

Boundary Conditions:

$$W(x,y,t) = M_x(x,y,t) = \psi_y(x,y,t) = 0, \quad \text{for } x = 0 \text{ and } x = L_x$$
$$W(x,y,t) = M_y(x,y,t) = \psi_x(x,y,t) = 0, \quad \text{for } y = 0 \text{ and } y = L_y$$

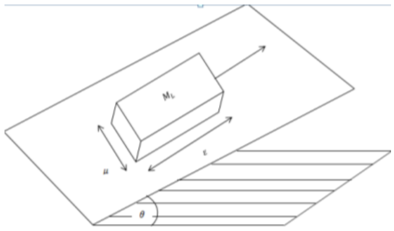

Fig. 1: A moving railway vehicle on an inclined plane supported by Winkler foundation.

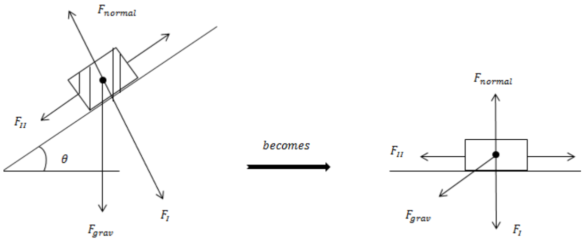

Fig. 2: Transformed inclined plane to a flat plane.

Problem Solution:

The set of dynamic equilibrium equations which govern the behaviour of rectangular inclined railway bridge supported by Winkler foundation and traversed by a partially distributed moving Railway vehicle can be written as [Gbadeyan and Dada (2006), Shadnam, (2001)];

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} - \rho h \frac{\partial^2 W}{\partial y^2} + KW + M_f \frac{\partial^2 W}{\partial f^2} = P(x,y,t) \tag{3}$$

$$Q_x - \frac{\partial M_x}{\partial x} - \frac{\partial M_{xy}}{\partial y} = \rho h^3 \frac{\partial^2 \psi_x}{\partial t^2} + \frac{\rho L h_1^3}{12} - \frac{d^2 \psi_x}{dt^2} B \tag{4}$$

$$Q_y - \frac{\partial M_{xy}}{\partial y} - \frac{\partial M_y}{\partial y} = \rho h^3 \frac{\partial^2 \psi_y}{\partial t^2} + \frac{\rho L h_1^3}{12} - \frac{d^2 \psi_y}{dt^2} B \tag{5}$$

where ψ_x and ψ_y are local rotations in the $x-$ and $y-$directions respectively, M_x and M_y are bending moments in the $x-$ and $y-$ directions respectively , M_{xy} is the twisting moment, Q_x and Q_y are the traversed shearing forces in $x-$ and $y-$ directions respectively, h and h_1 are thickness of the plate and load respectively, ρ and ρ_L are the densities of the plate and the load per unit volume respectively. $W(x,y,t)$ is the traverse displacement of the plate at time t, g is acceleration due to gravity, θ is the angle of inclination of the plate. The last terms in equations (4) and (5) account for inertia effects of the load in $x-$ and $y-$ directions respectively. Also, $B = B_x B_y$, where

$$B_x = \begin{cases} 1 - H(x - \xi - \frac{\varepsilon}{2}), \text{for } 0 \le t \le \frac{\varepsilon}{\mu} \\ H(x - \xi + \frac{\varepsilon}{2}) - H(x - \xi - \frac{\varepsilon}{2}), \text{ for } \frac{\varepsilon}{\mu} \le t \le \frac{L_x}{\mu} \\ H(x - \xi + \frac{\varepsilon}{2}), \text{for } \frac{L_x}{\mu} \le t \le \frac{(L_x + \varepsilon)}{\mu} \\ 0, \text{ for } \frac{(L_x + \varepsilon)}{\mu} \le t \end{cases} \tag{6}$$

$$B_y = H(y - y_1 + \tfrac{\mu}{2}) - H(y - y_1 - \tfrac{\mu}{2}) \tag{7}$$

$H(x)$ is the Heaviside function defined as:

$$H(x) = \begin{cases} 1, x > 0 \\ 0.5, \ x = 0 \\ 0, x < 0 \end{cases} \quad (8)$$

K is the foundation of stiffness and M_f is the mass of the foundation. D is the flexural rigidity of the plane. The bending moments, shearing forces and twisting moment can be written as (Gbadeyan and Dada, 2006)

$$M_x = -D\left(\frac{\partial \psi_x}{\partial x} + \upsilon \frac{\partial \psi_y}{\partial y}\right) \quad (9)$$

$$M_y = -D\left(\frac{\partial \psi_y}{\partial y} + \upsilon \frac{\partial \psi_x}{\partial x}\right) \quad (10)$$

$$M_{xy} = \frac{-D(1-\upsilon)}{2}\left(\frac{\partial \psi_x}{\partial y} + \upsilon \frac{\partial \psi_y}{\partial x}\right) \quad (11)$$

$$Q_x = -K^2 Gh\left(\psi_x - \frac{\partial W}{\partial x}\right) \quad (12)$$

$$Q_y = -K^2 Gh\left(\psi_y - \frac{\partial W}{\partial y}\right) \quad (13)$$

and

$$\frac{\partial W}{\partial x} = D_t \quad (14)$$

From equation (3), the moving load $P(x, y, t)$ can be expressed as follows [Gbadeyan and Dada (2006), Gbadeyan and Agarana (2014)]:

From equation (4), the straight derivative $\frac{d^2\psi_x}{dt^2}$ can be expressed as follows [Gbadeyan and Dada (2006), Gbadeyan and Agarana (2014)]:

By virtue of the inclined plane, the weight of the railway vehicle (M_L) has been resolved into its components. The component parallel to the plane is $M_L g \sin\theta$. Therefore, equation (12) becomes:

Application of the boundary conditions to the non-dimensional form of equations (9) – (14) and (22) – (24) yields nine equations with nine unknown variables: Q_x, Q_y, M_x, M_y, M_{xy}, $\psi_{x,t}$, $\psi_{y,t}$, D_t and W, from where the solutions are obtained.

A simply supported rectangular inclined plane (plate) has been taken as an illustrative example. If the edge $y = 0$ of the railway bridge (modelled as a plate) is simply supported, the deflection W along this edge must be zero. At the same time this edge rotate freely with respect to the $x -$ axis, that is, there are no bending moments (M_y) along this edge [Gbadeyan and Dada (2006), Gbadeyan and Agarana (2014)]. A numerical procedure, the finite difference method, can be used to solve the system of equations (7) – (12) and (20) – (22) (Gbadeyan and Agarana, 2014)

The resulting set of algebraic equations to be solved for the nine dependent variables may be written in matrix form as:

$$P(x, y, t) = \frac{1}{\mu\varepsilon}\left[-M_L g - M_L \frac{\partial^2 W}{\partial t^2}\right] B \quad (15)$$

$$\frac{d^2\psi_x}{dt^2} = \frac{\partial^2\psi_x}{\partial t^2} + u\frac{\partial^2\psi_x}{\partial x\partial t} + \frac{u}{\mathrm{D}(v^2-1)}\left\{\frac{\partial M_x}{\partial t} + u\frac{\partial M_x}{\partial t} + u\frac{\partial M_x}{\partial y}\right\} - \frac{uv}{\mathrm{D}(v^2-1)}\left\{\frac{\partial M_y}{\partial t} + u\frac{\partial M_y}{\partial x}\right\} \quad (16)$$

From equation (5), the straight derivative $\frac{d^2\psi_y}{dt^2}$ can be expressed as follows (Gbadeyan and Agarana, 2014):

$$\frac{d^2\psi_y}{dt^2} = \frac{\partial^2\psi_y}{\partial t^2} + u\frac{\partial^2\psi_y}{\partial x\partial t} + \frac{u}{\mathrm{D}(v^2-1)}\left\{\frac{\partial M_y}{\partial t} + u\frac{\partial M_y}{\partial t} + u\frac{\partial M_y}{\partial y}\right\} - \frac{uv}{\mathrm{D}(v^2-1)}\left\{\frac{\partial M_x}{\partial t} + u\frac{\partial M_x}{\partial y}\right\} \quad (17)$$

$$P(x, y, t) = \frac{1}{\mu\varepsilon}\left[-M_L g\cos\theta - M_L \frac{\partial^2 W}{\partial t^2}\right] B - M_L g \sin\theta \quad (18)$$

Substituting equations (16), (17) and (18) into equations (3), (4) and (5) respectively, we have

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + \rho h\frac{\partial W}{\partial y} + KW + M_f\frac{\partial W}{\partial t} = \frac{1}{\mu\varepsilon}\left[-M_L g\cos\theta - M_L\frac{\partial W}{\partial t} - \frac{u}{KGh}\left\{\frac{\partial Q_x}{\partial T} + M_L g\sin\theta\right\}\right] B \quad (19)$$

$$Q_x - \frac{\partial M_x}{\partial x} - \frac{\partial M_{xy}}{\partial y} = \frac{\rho h^3}{12}\frac{\partial\psi_x}{\partial t}\rho_L h_1^3\left[\frac{\partial^2\psi_x}{\partial t^2} + u\frac{\partial^2\psi_x}{\partial y\partial t} + \frac{u}{D(\upsilon^2-1)}\left\{\frac{\partial M_x}{\partial t} + u\frac{\partial M_x}{\partial x}\right\} - \frac{u\upsilon}{D(\upsilon^2-1)}\left\{\frac{\partial M_y}{\partial t} + u\frac{\partial M_y}{\partial x}\right\}\right] B \quad (20)$$

$$Q_y - \frac{\partial M_{xy}}{\partial x} - \frac{\partial M_y}{\partial y} = \frac{\rho h^3}{12}\frac{\partial^2\psi_x}{\partial t^2}\rho_L h_1^3\left[\frac{\partial^2\psi_x}{\partial t^2} + u\frac{\partial^2\psi_x}{\partial y^2} + \frac{u}{D(\upsilon^2-1)}\left\{\frac{\partial M_y}{\partial t} + u\frac{\partial M_y}{\partial x} + \frac{\partial M_y}{\partial y}\right\} - \frac{u\upsilon}{D(\upsilon^2-1)}\left\{\frac{\partial M_x}{\partial t} + u\frac{\partial M_x}{\partial y}\right\}\right] B \quad (21)$$

Equations (19), (20) and (21) can be written as first order partial differential equations as follows

$$\frac{dQ_x}{dx}+\frac{dQ_y}{dy}+KW+M_f\,\frac{\partial D_t}{\partial t}+\frac{M_L}{A}\left[g\cos\theta+\frac{\partial D_t}{\partial t}+u\frac{\partial D_t}{\partial t}+u\left\{\psi_{x,t}+\frac{u}{D(\upsilon^2-1)}M_x-\frac{u\upsilon M_y}{D(\upsilon^2-1)}\right\}-\frac{u}{\alpha Gh}\left\{\frac{\partial Q_x}{\partial t}+u\frac{\partial Q_x}{\partial x}\right\}\right]B=\rho h\frac{\partial D_T}{\partial T}-M_L g\sin\theta \tag{22}$$

$$Q_x-\frac{\partial M_x}{\partial x}-\frac{\partial M_{xy}}{\partial y}=\frac{\rho h^3}{12}\frac{\partial\psi_{x,t}}{\partial t}+\frac{\rho_L h_1^3}{12}\left[\frac{\partial\psi_{x,t}}{\partial y}+\frac{u}{D(\upsilon^2-1)}\left\{\frac{\partial M_x}{\partial t}+u\frac{\partial M_x}{\partial x}\right\}-\frac{u\upsilon}{D(\upsilon^2-1)}\left\{\frac{\partial M_x}{\partial t}+u\frac{\partial M_x}{\partial y}\right\}\right]B \tag{23}$$

$$Q_x-\frac{\partial M_x}{\partial x}-\frac{\partial M_{xy}}{\partial y}=\frac{\rho h^3}{12}\frac{\partial\psi_{x,t}}{\partial t}+\frac{\rho_L h_1^3}{12}\left[\frac{\partial\psi_{y,t}}{\partial y}+\frac{u}{D(\upsilon^2-1)}\left\{\frac{\partial M_x}{\partial t}+u\frac{\partial M_x}{\partial x}\right\}-\frac{u\upsilon}{D(\upsilon^2-1)}\left\{\frac{\partial M_x}{\partial t}+u\frac{\partial M_x}{\partial y}\right\}\right]B \tag{24}$$

where $\psi_{x,t}=\frac{\partial\psi_x}{\partial t}$ and $\psi_{y,t}=\frac{\partial\psi_y}{\partial t}$.

$$A_{i,j+1}S'_{i,j+1}+B_{i+1,j+1}S'_{i+1,j+1}=-C_{i,}S'_{i,j}-D_{i,+1j}S'_{i+1,j}+L_k,\quad i=1,2,3,\cdots,N-1;\ j=1,2,3,\cdots,M-1 \tag{25}$$

where N and M are the number of the nodal points along x and y axes respectively,

$$L_k=K_{i,j}S^0{}_{i,j}+L_{i,j+1,}S^0{}_{i,j+1}M_{i+1}S^0{}_{i+1,j}+N_{i+1,j+1}S^0{}_{i+1,j+1}+P_1 \tag{26}$$

Each term in equations (25) and (26) is a 9×9 matrix.

Effects Of The Angle Of Inclination On The Deflection Of The Bridge:

From equation (18), we have

$$P(x,y,t)=\frac{1}{\mu\varepsilon}\left[-M_L g\cos\theta-M_L\frac{\partial^2W}{\partial t^2}\right]B-M_L g\sin\theta$$

For free vibration, $P(x,y,t)=0$, which implies

$$\frac{1}{\mu\varepsilon}\left[-M_L g\cos\theta-M_L\frac{\partial^2W}{\partial t^2}\right]B-M_L g\sin\theta=0 \tag{27}$$

$$\frac{1}{\mu\varepsilon}\left[-M_L g\cos\theta-M_L\frac{\partial^2W}{\partial t^2}\right]B=M_L g\sin\theta \tag{28}$$

$$\left[-M_L g\cos\theta-M_L\frac{\partial^2W}{\partial t^2}\right]B-AM_L g\sin\theta=0 \tag{29}$$

$$\frac{\partial^2W}{\partial t^2}=-\left(\frac{A}{B}g\sin\theta+g\cos\theta\right) \tag{30}$$

For very small θ, $\sin\theta\to\theta$ and $\cos\theta\to 1$.

Considering small inclination, (i.e., $\theta\to 0$), equation (30) becomes:

$$-\frac{M_L}{A}\left[g+\frac{\partial^2W}{\partial t^2}\right]B=M_L g\theta \tag{31}$$

where $\mu\varepsilon=A$.

For numerical illustration purpose, let $M_L=10\,kg$, $A=2\,m^2$, $g=9.81$. If $B=1$, then

$$-9.81=\frac{\partial^2W}{\partial t^2} \tag{32}$$

That is, if θ tends to 0, the acceleration of the deflection is approximately the acceleration due to gravity in the opposite direction.

Integrating both sides of equation (32) twice, we have

$$-9.81t^2+ct+k=W \tag{33}$$

where c and k are constants.

Considering when θ is not small,

$$-\frac{M_L}{A}\left[g\cos\theta+\frac{\partial^2W}{\partial t^2}\right]B=M_L g\sin\theta \tag{34}$$

which can be written as

$$\frac{A}{B}g\sin\theta=-g\cos\theta-\frac{\partial^2W}{\partial t^2},\ B=1 \tag{35}$$

Specifically, for $\theta=90^\circ$, equation (35) becomes

$$\frac{A}{B}g=-\frac{\partial^2W}{\partial t^2} \tag{36}$$

So for $\theta=90^\circ$ and for free vibration and letting $B=1$, $\theta=90^\circ$, $A=2$, we obtain

$$-2gt^2 + c_1 t + k_1 = W \tag{37}$$

where c_1 and k_1 are constants.

Now, for $\theta \neq 90^\circ$, equation (35) can be written as

$$-g(2\sin\theta + \cos\theta) = \frac{\partial^2 W}{\partial t^2} \tag{38}$$

For forced vibration, we have

$$P = -\frac{M_L}{A}\left[g\cos\theta + \frac{\partial^2 W}{\partial t^2}\right]B - M_L g\sin\theta \tag{39}$$

where P is the applied force.

$$-\frac{A}{BM_L}(P + M_L g\sin\theta) - g\cos\theta = \frac{\partial^2 W}{\partial t^2} \tag{40}$$

For numerical illustration purpose, let $M_L = 10\,kg$, $A = 2\,m^2$, $g = 9.81\,ms^{-2}$, $B = 1$, $0 \le \theta \le 90^\circ$, equation (40) becomes

$$-\frac{1}{5}(P + 98.1\sin\theta) - 9.81\cos\theta = \alpha \tag{41}$$

where $\alpha = \dfrac{\partial^2 W}{\partial t^2}$ is the acceleration of the deflection.

From equation (27), $\alpha = k\tan\theta$, where $k = \left(M_L B^2\right)^{-1}$, is a constant.

RESULTS AND DISCUSSION

The numerical calculations were carried out for a simply supported rectangular inclined plate (inclined railway bridge) resting on a Winkler foundation and subjected to a moving railway vehicle (load.). Damping effect was neglected. For specific values of other parameters, deflection of the bridge is calculated and plotted (in Figure 3) as a function of time. It shows the deflection of the railway bridge for various values of velocity u. It can be seen that the response maximum amplitude of the bridge decreases as velocity decreases. In Figure 4, acceleration of the deflection of the bridge, α, without an applied force, is plotted against time. We can see that α increases gently, then later sharply with time, at a given value of angle of inclination, θ, as k_1 increases. Figure 5 shows that deflection of the railway bridge decreases with time if there is no applied load. In Figure 6 we plotted the deflection of the bridge under an applied load against time. It is clear that deflection increases as the applied load increases. Similarly, Figure 7 shows that the acceleration of the deflection of the bridge at different values of applied load decreases as with an increase in the applied load. Also, Figures 8 and 9 represent instantaneous dynamic response of the railway bridge, at any instant of time and at a given angle of inclination, for both forced and free vibration cases. The figures show, respectively, that deflection of the bridge decreases with an increase in time for forced vibration case, while the deflection increases with an increase in time for the free vibration case, at a given angle of inclination.

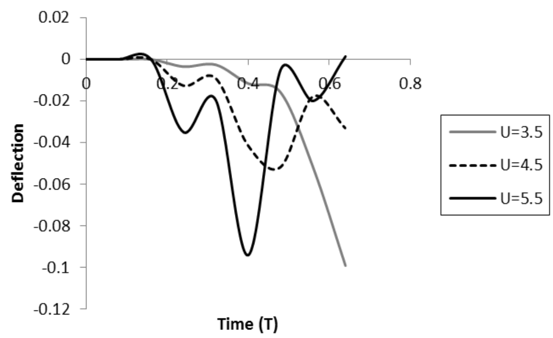

Fig. 3: Deflection of the bridge at different values of velocity and time.

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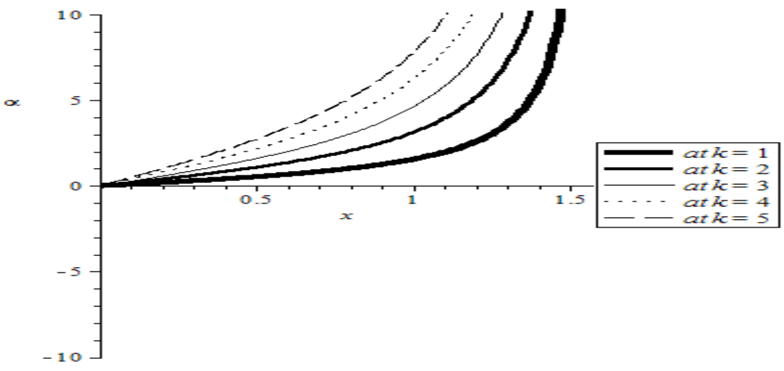

Fig. 4: Acceleration of deflection of inclined plate at various values of k.

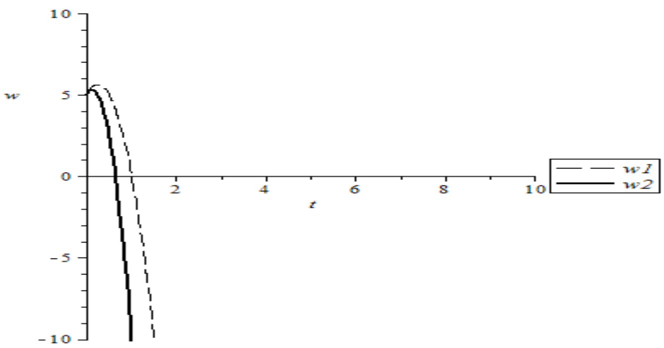

Fig. 5: Deflection of the bridge without applied load at given inclination angles.

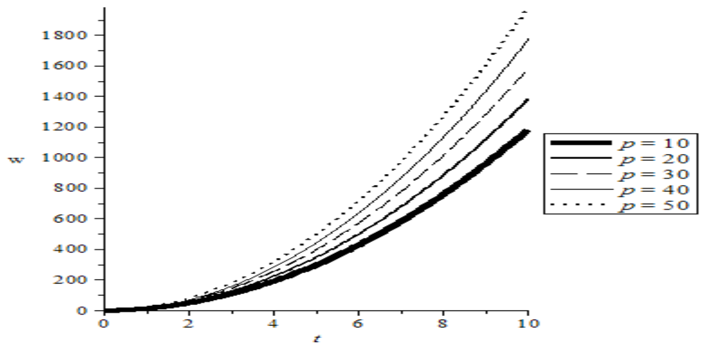

Fig. 6: Deflection of bridge at various values of applied load and different time.

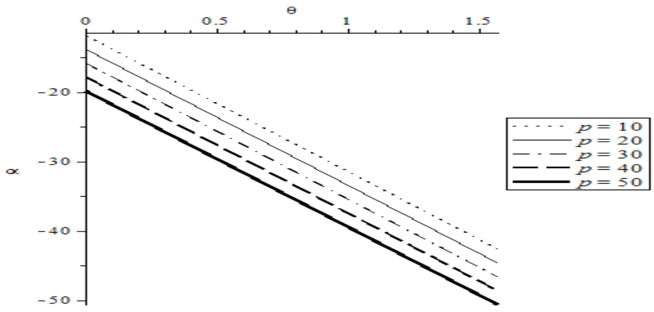

Fig. 7: Acceleration of deflection of the bridge at different values of applied load and different angles of inclination.

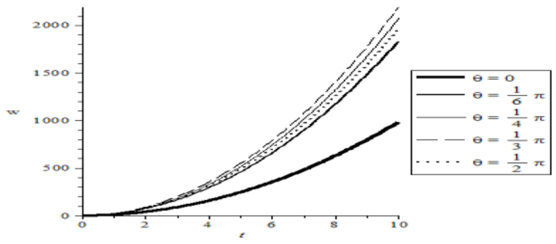

Fig. 8: Deflection of the railway bridge for forced vibration, at any instant of time and at a given angle of inclination.

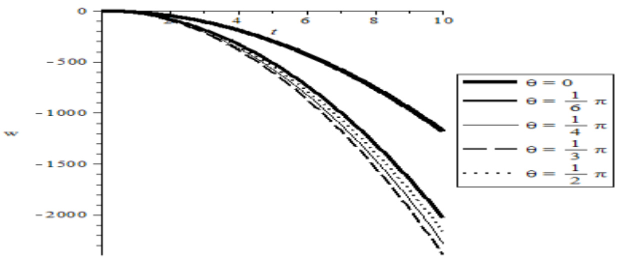

Fig. 9: Deflection of the railway bridge for free vibration, at any instant of time and at a given angle of inclination.

Conclusions:

The structure of interest was an inclined railway bridge on Winkler elastic foundation, under the influence of a uniform partially distributed moving railway vehicle). The problem was to determine the dynamic response of the whole system. Finite difference technique was adopted in solving the resulting first order coupled partial differential equations obtained from governing equations for the simply supported bridge. The study has contributed to scientific knowledge by showing that the angle of inclination of an incline railway bridge in addition to the elastic subgrade on which the bridge rests, have a significance effect on the dynamic response of the bridge to a partially distributed moving railway vehicle. Also, the influences of the moving railway vehicle speed and total mass of the moving railway vehicle on the dynamic response of the inclined bridge are significant in most cases.

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