International Journal of Applied Engineering Research ISSN 0973-4562 Volume 10, Number 18 (2015) pp 39168-39174
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Solution of Differential Equations by Three Semi-Analytical Techniques

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Abstract
In this work, we present some semi-analytical techniques namely Differential Transform Method (DTM), Adomian Decomposition Method (ADM) and Homotopy Perturbation Method (HPM) for the solution of differential equations. The equations considered include initial value problems and boundary value problems. The results indicated that DTM is easy to apply but requires transformation, while ADM does not need any transformation except the calculation of Adomian polynomials. In addition, it was demonstrated that HPM involves perturbation and more computations. The results obtained converged rapidly to the exact solution.

Key Words: Differential Equations, Adomian Decomposition Method, Differential Transform Method, Homotopy Perturbation Method, Series Solution.

Introduction

Several problems in sciences and engineering being studied by mathematical models contain differential equations ranging from initial value problems to boundary value problems. Some of the differential equations arising from these models do not have analytical solution. Hence the need for effective and efficient semi-analytical methods has led to the emergence of various numerical methods. The foremost of these methods include the Differential Transform Method (DTM), Adomian Decomposition Method (ADM) and Homotopy Perturbation Method (HPM).

The Adomian decomposition method was introduced by George Adomian in 1989 [1]. The method has been applied by several researchers to solve various problems [2-5] while the differential transform method was proposed by Zhou to solve electric circuit analysis problem [6]. It was afterwards applied to various functional equations [7-10]. The homotopy perturbation method was developed by He when he merged two techniques, the standard homotopy and the perturbation technique [11]. Since then it has gained tremendous application by researchers to solve differential equations ranging from linear to non-linear [12-16].

This paper which applies these three methods, DTM, ADM and HPM, was borne out of the quest to identify the most effective method for the solution of numerous mathematical models arising from different fields of applied sciences and engineering

Analysis of the Methods

Formulation of Adomian Decomposition Method

The ADM is applied by splitting the given equation into linear and non-linear parts, inverting the higher-order derivative operator in the linear operator on both sides. The initial/boundary conditions together with the source term are identified as the zeroth component while the nonlinear term is decomposed as Adomian polynomials and the successive terms as series solution by recurrent relation using Adomian polynomials.

Consider the generalized second order differential equation

$$y'' = f(t,u) \tag{1}$$

Let

$$Lu = f(t,u) \tag{2}$$

be the operator form of a differential equation (1). Since L is said to be a second order operator, we write

$$L = \frac{d^2}{dt^2}$$

with the inverse operator given as

$$L^{-1} = \int_0^t \int_0^t (\bullet) dt dt \tag{3}$$

Applying L^{-1} on both sides of equation (1), and using the initial/boundary conditions we have:

$$u(t) = u(0) + tu'(0) + L^{-1}(f(t,u)) \tag{4}$$

We can then represent u as

$$u(t) = \sum_{k=0}^{\infty} U_k \tag{5}$$

And the nonlinear function $f(u,t)$ can be determined by an infinite series of Adomian polynomials

$$f(t,u) = \sum_{k=0}^{\infty} A_k \tag{6}$$

The Adomian polynomials can be calculated using

$$A_k = \frac{1}{k!} \frac{d^k}{d\alpha^k} \left[N\left(\sum_{k=0}^{\infty} \alpha U_k \right) \right]_{\alpha=0} \tag{7}$$

substituting (5) and (6) in (4), we obtain

$$\sum_{k=0}^{\infty} U_k = u(0) + tu'(0) + L^{-1}\left[\sum_{k=0}^{\infty} A_k \right] \tag{8}$$

We identify the zeroth component $u_0(t)$ by all the terms arising from the initial/boundary conditions together with source term (if present).

$$u_0(t) = u(0) + tu'(0) \tag{9}$$

The remaining components are recursively determined using the recurrence relations stated below.

$$u_{k+1}(t) = L^{-1}(A_k), k \geq 0 \tag{10}$$

We can then write other terms as

$$u_1(t) = L^{-1}(A_0)$$
$$u_2(t) = L^{-1}(A_1).$$
$$u_3(t) = L^{-1}(A_2)$$
$$\dots\dots\dots\dots$$

2.2. Formulation of Differential Transform Method

Let the differential transform of an arbitrary function $u = f(t)$ in Taylor series about a point $t = 0$ be defined as

$$U(k) = \frac{1}{k!}\left[\frac{d^k u(t)}{dt^k}\right]_{t=0} \tag{11}$$

where $u(t)$ is the original function and $U(k)$ is the transformed function. We can write the inverse differential transform of $U(k)$ as

$$u(t) = \sum_{k=0}^{\infty} U(k)(t-t_0)^k \tag{12}$$

Function $u(t)$ can then be written as a finite series with equation (12) stated as

$$u(t) = \sum_{k=0}^{\infty} U(k)t^k \tag{13}$$

The following theorems can be derived from equations (11), (12) and (13)

(1) If $u(t) = v(t) \pm w(t)$, then $U(k) = V(k) \pm W(k)$

(2) If $u(t) = \alpha v(t)$, then $U(k) = \alpha V(k)$

(3) If $u(t) = \frac{dv(t)}{dt}$, then $U(k) = (k+1)V(k+1)$

(4)If $u(t) = \frac{d^r v(t)}{dt^r}$,then

$$U(k) = (k+1)\cdots(k+r)V(k+r)$$

(5) If $u(t) = x^r$, then $U(k) = \delta(k-r) = \begin{cases} 1 & k = r \\ 0 & otherwise \end{cases}$

(6) If $u(t) = v(t)w(t)$, then $U(k) = \sum_{n=0}^{k} V(k)W(k-n)$

2.3. Formulation of Homotopy Perturbation Method

We consider the following systems of integral equation

$$Q(t) = R(t) + \alpha\int_0^t K(t,s)Q(s)ds \tag{14}$$

$$Q(t) = (q_1(t), q_2(t), \dots, q_n(t))^T$$
$$R(t) = (r_1(t), r_2(t), \dots, r_n(t))^T$$
$$K(t,s) = \left[K_{ij}(t,s)\right]$$
$$i = 1,2,3,\dots,n;\ j = 1,2,3,\dots,n$$

let

$$L(u) = 0 \tag{15}$$

where L is an integral/differential operator. Defining a convex homotopy $H(u, p)$

$$H(u, p) = (1-p)F(u) + pL(u) \tag{16}$$

Then

$$H(u,0) = F(u), \text{ and } H(u,1) = L(u). \tag{17}$$

It then means that $H(u, p)$ traces an implicitly defined curve continuously from a starting point $H(v_0, 0)$ to a solution $H(f,1)$. The embedding parameter increases monotonically from zero to unit as the trivial problem $F(u) = 0$ deforms continuously, the original problem becomes $L(u) = 0$. We then consider the embedding parameter $p \in (0,1]$ as an expanding one.

In the homotopy perturbation method the parameter p is used as expanding parameter to obtain:

$$u = \sum_{i=0}^{\infty} p^i u_i = u_0 + pu_1 + p^2u_2 + p^3u_3 + \dots \tag{18}$$

at $p \to 1$ equation(18) corresponds to (16) and then yield the approximate solution of the form.

$$f = \lim_{p\to 1} u \sum_{i=0}^{\infty} u_i = u_0 + u_1 + u_2 + u_3 + \dots \tag{19}$$

series (19) converges for most of the cases and its convergence rate is dependent on $L(u)$

The solution of various order are obtained by comparing equal power of p.

Test Examples

Example 1: Consider a first order differential equation

$$\frac{dy}{dx} = 2y, y(0) = 1 \tag{20}$$

The exact solution of (20) is

$$y = \exp(2x) \tag{21}$$

Solution by Adomian decomposition method

In operator form, equation (20) becomes

$$Ly = 2y \tag{22}$$

Applying the inverse operator L^{-1} and imposing the initial condition, we obtain

$$y(x) = y(0) + L^{-1}(2y) \tag{23}$$

The zeroth component is

$$y_0 = y(0) = 1 \tag{24}$$

Other components are determined using the recursive relation

$$y_{n+1}(x) = 2L^{-1}(y_n) \tag{25}$$

$$y_1(x) = 2L^{-1}(y_0) = 2\int_0^x y_0 dx = 2x$$

$$y_2(x) = 2L^{-1}(y_1) = 2\int_0^x y_1 dx = 2x^2$$

$$y_3(x) = 2L^{-1}(y_2) = 2\int_0^x y_2 dx = \frac{4x^3}{3}$$

The series solution is given as

$$y(x) = 1 + 2x + 2x^2 + \frac{4x^3}{3} + \cdots \qquad (26)$$

Solution by Differential Transform method

Transformation of equation (20) with the initial condition gives

$$Y(k+1) = \frac{2Y(k)}{(k+1)} \qquad (27)$$

$$Y(0) = 1 \qquad (28)$$

Using (28) in (27), we get at

$$k = 0, \quad Y(1) = \frac{2Y(0)}{1} = 2 \cdot 1 = 2$$

$$k = 1, \quad Y(2) = \frac{2Y(1)}{2} = 2 = 2$$

$$k = 2, \quad Y(3) = \frac{2Y(2)}{3} = \frac{4}{3}$$

$$k = 3, \quad Y(4) = \frac{2Y(3)}{2} = \frac{2}{3}$$

The series solution is obtained as

$$y(x) = 1 + 2x + 2x^2 + \frac{4x^3}{3} + \cdots \qquad (29)$$

Solution by Homotopy Perturbation method

To use homotopy perturbation method, equation (20) is written as

$$y_0 + py_1 + p^2 y_2 + p^3 y_3 + \cdots = 1 + 2p\int_0^x \left(y_0 + py_1 + p^2 y_2 + p^3 y_3 + \cdots\right) dr \qquad (30)$$

Comparing coefficient of like powers of p, we obtain

$$p^0 : y_0 = 1$$

$$p^1 : y_1 = 2\int_0^x (y_0) dr = 2x$$

$$p^2 : y_2 = 2\int_0^x (y_1) dr = 2x^2$$

$$p^3 : y_3 = 2\int_0^x (y_2) dr = \frac{4x^3}{3}$$

Collecting the terms together, we obtain

$$y(x) = 1 + 2x + 2x^2 + \frac{4x^3}{3} + \cdots \qquad (31)$$

Example 2: We now consider a second order boundary value problem

$$\frac{d^2 y}{dx^2} = 4y \qquad (32)$$

The boundary conditions are stated as

$$y(0) = 0, \; y(1) = 1 \qquad (33)$$

The exact solution takes the form

$$y(x) = \frac{\exp(-2x) - \exp(2x)}{\exp(-2) - \exp(2)} \qquad (34)$$

Solution by Adomian decomposition method

Writing equation (32) in operator form (L is a second order operator) gives

$$Ly = 4y \qquad (35)$$

Applying the inverse operator L^{-1} on both sides of (35) and imposing the boundary conditions at $x = 0$ yields

$$y(x) = y(0) + Ax + L^{-1}(4y) \qquad (36)$$

and constant $A = y'(0)$ will be determined later. We then write the zeroth component as

$$y_0 = y(0) + Ax \qquad (37)$$

The recurrent relation y_{n+1} is given as

$$y_{n+1}(x) = L^{-1}(4y_n) \qquad (38)$$

at $n = 0, 1, 2, 3, \ldots$ we obtain the following

$$y_1 = 4L^{-1}(y_0) = \frac{2Ax^3}{3},$$

$$y_2 = 4L^{-1}(y_1) = \frac{2Ax^5}{15},$$

$$y_3 = 4L^{-1}(y_2) = \frac{4Ax^7}{315} \ldots$$

The series form of $y(x)$ is given as

$$y(x) = Ax + \frac{2}{3}Ax^3 + \frac{2}{15}Ax^5 + \frac{4}{315}Ax^7 + \ldots \qquad (39)$$

To determine the constant A, we impose the boundary conditions in (33) at $x = 1$, we have

$$\frac{4481309325763513A}{959490350280000} = 1 \qquad (40)$$

which gives

$$A = 0.5514411295$$

The series solution is then written as

$$y(x) = 0.5514411295x + 0.3676274197x^3 + 0.07352548393x^5 + 0.007002427041x^7 + 0.0003890237245x^9 + 0.00001414631726x^{11} \qquad (41)$$

Solution by Differential Transform method

The differential transformation of equation (32) leads to the recurrent relation

$$Y(k+2)=\frac{4}{(k+2)!}\left[Y(k)\right] \tag{42}$$

and the transformation of the boundary conditions (33) gives

$$Y(0)=0, Y(1)=A \tag{43}$$

Substituting (43) in (42), we obtain the following

$$Y(2)=Y(4)=Y(6)=Y(8)=0,$$

$$Y(3)=\frac{2A}{3}, Y(5)=\frac{2A}{15},$$

$$Y(7)=\frac{4A}{315}, Y(9)=\frac{2A}{2835},$$

which form the series

$$y(x)=Ax+\frac{2}{3}Ax^3+\frac{2}{15}Ax^5+\frac{4}{315}Ax^7+\frac{2}{2835}Ax^9+\ldots \tag{44}$$

The constant A can be determined using equation (11), to obtain $A=y'(0)$. Imposing the condition (33) at $x=1$, we have following equation

$$\frac{124521522151944238607085592915628353511A}{135221758735957163816824250022832031 25}=1 \tag{45}$$

Then

$$A=0.5514411295$$

and equation (44) can be written as

$$y(x)=0.5514411295x+0.3676274197x^3+0.07352548393x^5+0.007002427041x^7+0.0003890237245x^9+\ldots \tag{46}$$

Solution by Homotopy Perturbation method

To solve equation (32) by homotopy perturbation method, we write it as a system of two differential equations:

$$\frac{dy}{dx}=h(x),\quad \frac{dh}{dx}=4y$$
$$y(0)=0,\quad h(0)=A \tag{47}$$

writing (47) as a system of integral equation, we obtain

$$y(x)=0+\int_0^x h(r)dr,\; h(x)=A+\int_0^x 4y(r)dr \tag{48}$$

Applying (17) and (19) in (48), we have

$$y_0+py_1+p^2y_2+p^3y_3+...=0+p\int_0^x\left(y_0+py_1+p^2y_2+p^3y_3+...\right)dr$$

$$h_0+ph_1+p^2h_2+p^3h_3+...=A+p\int_0^x 4\left(h_0+ph_1+p^2h_2+p^3h_3+...\right)dr \tag{49}$$

by comparing the coefficient of like powers, we obtain the following

$$p^0:\begin{cases}y_0=0\\ h_0=A\end{cases},\; p^1:\begin{cases}y_1=Ax\\ h_1=0\end{cases},\; p^2:\begin{cases}y_2=0\\ h_2=2Ax^2\end{cases},$$

$$p^3:\begin{cases}y_3=\dfrac{2Ax^3}{3}\\ h_3=0\end{cases},\; p^4:\begin{cases}y_4=0\\ h_4=\dfrac{2Ax^4}{3}\end{cases},$$

$$p^5:5\begin{cases}y_5=\dfrac{2Ax^5}{15}\\ h_5=0\end{cases}$$

Collecting the terms together, we have the series

$$y(x)=Ax+\frac{2}{3}Ax^3+\frac{2}{15}Ax^5+\frac{4}{315}Ax^7+\cdots \tag{50}$$

imposing the boundary conditions at $x=1$ we obtain

$$\frac{1052633503A}{58046625}=1 \tag{51}$$

Solving (51) yields

$$A=0.5514411296$$

and equation (50) can be written as

$$y(x)=0.5514411296x+0.3676274197x^3+0.07352548394x^5+0.007002427042x^7+0.0003890237246x^9+0.00001414631726x^{11} \tag{52}$$

Example 3: We finally consider the equation below

$$y''-3y'+2y=0 \tag{53}$$

with the boundary conditions

$$y(0)=0\quad y(1)=1 \tag{54}$$

The theoretical solution of equation (53) is

$$y(x)=\frac{\exp(2x)-\exp(x)}{\exp(2)-\exp(1)} \tag{55}$$

Solution by Adomian decomposition method

In operator form, equation (53) becomes

$$Ly=3y'-2y \tag{56}$$

Using the inverse operator on both sides of (53) and imposing the boundary conditions, we have

$$y(x)=y(0)+Ax+L^{-1}(3y')-L^{-1}(2y) \tag{57}$$

The zeroth component is

$$y_0(x)=Ax \tag{58}$$

The recursive relation is

$$y_{n+1}(x)=L^{-1}(3y_n')-L^{-1}(2y_n) \tag{59}$$

at $n=0,1,2,3\ldots$

$$y_1(x)=3L^{-1}\left(\frac{dy_0}{dx}\right)-2L^{-1}\left(y_0\right)=\frac{3A}{2}x^2-\frac{A}{3}x^3$$

$$y_2(x)=3L^{-1}\left(\frac{dy_1}{dx}\right)-2L^{-1}\left(y_1\right)=$$
$$-\frac{A}{2}x^4+\frac{3A}{2}x^3+\frac{A}{30}x^5$$

$$y_3(x)=3L^{-1}\left(\frac{dy_2}{dx}\right)-2L^{-1}\left(y_2\right)=$$
$$-\frac{9A}{20}x^5-\frac{9A}{8}x^4+\frac{A}{20}x^6-\frac{A}{630}x^7$$

$$y(x)=y_0+y_1+y_2+y_3+y_4+\cdots= \quad (60)$$

$$y(x)=Ax+\frac{3A}{2}x^2+\frac{7A}{6}x^3+\frac{5A}{8}x^4+\frac{31A}{120}x^5+\cdots \quad (61)$$

To calculate constant A, we apply the boundary conditions (54) at $x=1$ which yields

$$\frac{451548028965692419114791158151841A}{96675198332942058413447935085414}=1 \quad (62)$$

Solving the equation gives
A = 0.2140972657
Then we can write the series solution as

$$\begin{aligned}y(x)&=0.2140972657x+0.3211458986x^2+\\&0.2497801433x^3+0.1338107911x^4+\\&0.05530846031x^5+0.01873351075x^6+\\&0.005394911259x^7+\cdots\end{aligned} \quad (63)$$

Solution by differential transform method
We begin by transforming the differential equation and the boundary conditions as

$$Y(k+2)=\frac{1}{(k+2)!}\left(3(k+1)Y(k+1)-2Y(k)\right) \quad (64)$$

$$Y(0)=0, Y(1)=A \quad (65)$$

Substituiting (65) in (64) gives the following iterates

$$Y(2)=\frac{3A}{2}, Y(3)=\frac{7A}{6}, Y(4)=\frac{5A}{8},$$
$$Y(5)=\frac{31A}{120}, Y(6)=\frac{7A}{80}, Y(7)=\frac{127A}{5040},$$
$$Y(8)=\frac{17A}{2688}, Y(9)=\frac{73A}{51840}, Y(10)=\frac{341A}{21209600}$$

The equation is written as

$$\begin{aligned}y(x)&=Ay+\frac{3A}{2}y^2+\frac{7A}{6}y^3+\frac{5A}{8}y^4+\\&\frac{31A}{120}y^5+\frac{7A}{80}y^6+\cdots\end{aligned} \quad (66)$$

To calculate the constant A we apply the boundary conditions at $x=1$ to give

$$\frac{20833226744937617610336078867056970792227A}{44603368817551742810893416071254335535}=1 \quad (67)$$

Solving the equation above yields
$A=0.2140972657$
Substituting for A in (66) gives the series solution as

$$\begin{aligned}y(x)&=0.2140972657y+0.3211458986y^2+\\&0.2497801433y^3+0.1338107911y^4+\\&0.05530846031y^5+0.01873351075y^6+\cdots\end{aligned} \quad (68)$$

Solution by Homotopy perturbation method
Using homotopy perturbation method, we write (53) as a system of two differential equations:

$$\frac{dy}{dx}=h(x), \quad \frac{dh}{dx}=3h(x)-2y$$
$$y(0)=0, \quad h(0)=A \quad (69)$$

writing (69) as a system of integral equation, we obtain

$$y(x)=0+\int_0^x h(x)dr,$$
$$h(x)=A+\int_0^x\left(3h(x)-2x\right)dr \quad (70)$$

Applying (17) and (19) in (70), we have

$$y_0+py_1+p^2y_2+p^3y_3+...=$$
$$0+p\int_0^x\left(y_0+py_1+p^2y_2+p^3y_3+...\right)dr$$
$$h_0+ph_1+p^2h_2+p^3h_3+...=$$
$$A+p\int_0^x\left(3\left(y_0+py_1+p^2y_2+p^3y_3+...\right)-2x\right)dr \quad (71)$$

by comparing the coefficient of like powers, we obtain the following

$$p^0:\begin{cases}y_0=0\\h_0=A\end{cases} \qquad p^1:\begin{cases}y_1=\int_0^x h_0=Ax\\h_1=3\int_0^x h_0-\int_0^x y_0=3Ax\end{cases}$$

$$p^2:\begin{cases}y_2=\int_0^x h_1=\int_0^x 3Axdr=\frac{3Ax^2}{2}\\h_2=3\int_0^x h_1-2\int_0^x y_1=\frac{7A}{2}x^2\end{cases}$$

$$p^3:\begin{cases} y_3 = \int_0^x h_2 = \frac{7A}{6}x^3 \\ h_3 = \int_0^x h_2 = \frac{5A}{2}x^3 \end{cases}$$

$$p^4:\begin{cases} y_4 = \int_0^x h_3 = \frac{5A}{8}x^4 \\ h_4 = \int_0^x h_3 - \int_0^x y_3 = \frac{31A}{24}x^4 \end{cases}$$

$$p^5:\begin{cases} y_5 = \int_0^x h_4 = \frac{31A}{120}x^5 \\ h_5 = \int_0^x h_4 - \int_0^x y_4 = \frac{21A}{40}x^5 \end{cases}$$

$$y(x) = y_0 + y_1 + y_2 + y_3 + y_4 + y_5 + \cdots = \quad (72)$$

$$y(x) = Ax + \frac{3A}{2}x^2 + \frac{7A}{6}x^3 + \frac{5A}{8}x^4 + \frac{31A}{120}x^5 + \frac{7A}{80}x^6 + \cdots \quad (73)$$

Using the boundary conditions at x = 1, we obtain

$$\frac{13960804095241A}{2988969984000} = 1 \quad (74)$$

which gives

$$A = 0.2140972657 \quad (75)$$

The series solution is

$$y(x) = 0.2140972657x + 0.3211458986x^2 + 0.2497801433x^3 + 0.1338107911x^4 + 0.05530846031x^5 + 0.01873351075x^6 + 0.005394911259x^7 + 0.001354037767x^8 + \ldots \quad (76)$$

TABLE1: NUMERICAL RESULT FOR EXAMPLE 1

x	EXACT	ADM	DTM	HPM
0	1.000000000	1.000000000	1.000000000	1.000000000
0.1	1.221402758	1.221402758	1.221402758	1.221402758
0.2	1.491824698	1.491824698	1.491824698	1.491824698
0.3	1.8221188	1.8221188	1.8221188	1.8221188
0.4	2.225540928	2.225540928	2.225540928	2.225540928
0.5	2.718281828	2.718281828	2.718281828	2.718281828
0.6	3.320116923	3.320116923	3.320116923	3.320116923
0.7	4.055199967	4.055199967	4.055199967	4.055199967
0.8	4.953032424	4.953032424	4.953032424	4.953032424
0.9	6.049647464	6.049647464	6.049647464	6.049647464
1	7.389056099	7.389056099	7.389056099	7.389056099

TABLE2: NUMERICAL RESULT FOR EXAMPLE 2

x	EXACT	ADM	DTM	HPM	ADM ERROR	DTM ERROR	HPM ERROR
0.1	0.055512476	0.055512476	0.055512476	0.055512476	4.352E-12	4.35E-12	5.65E-12
0.2	0.113252863	0.113252863	0.113252863	0.113252863	8.682E-12	8.68E-12	1.13E-11
0.3	0.175538485	0.175538485	0.175538485	0.175538485	1.298E-11	1.3E-11	1.7E-11
0.4	0.244869083	0.244869083	0.244869083	0.244869083	1.725E-11	1.72E-11	2.29E-11
0.5	0.324027137	0.324027137	0.324027137	0.324027137	2.154E-11	2.15E-11	2.88E-11
0.6	0.416189537	0.416189537	0.416189537	0.416189537	2.595E-11	2.59E-11	3.48E-11
0.7	0.525055085	0.525055085	0.525055085	0.525055086	3.063E-11	3.06E-11	4.09E-11
0.8	0.654992938	0.654992938	0.654992938	0.654992938	3.583E-11	3.58E-11	4.54E-11
0.9	0.811217956	0.811217956	0.811217956	0.811217956	4.189E-11	4.19E-11	3.74E-11
1	1	1	1	1	4.928E-11	4.93E-11	4.1E-11

TABLE 3: NUMERICAL RESULT FOR EXAMPLE 3

x	EXACT	ADM	DTM	HPM	ADM ERROR	DTM ERROR	HPM ERROR
0.1	0.024884919	0.024884919	0.024884919	0.024884919	7.331E-13	7.33E-13	7.33E-13
0.2	0.057896598	0.057896598	0.057896598	0.057896598	2.5E-12	2.5E-12	2.5E-12
0.3	0.101109573	0.101109573	0.101109573	0.101109573	5.364E-12	5.36E-12	5.36E-12
0.4	0.157086639	0.157086639	0.157086639	0.157086639	9.494E-12	9.49E-12	9.49E-12
0.5	0.228989991	0.228989991	0.228989991	0.228989991	1.518E-11	1.52E-11	1.52E-11
0.6	0.320717302	0.320717302	0.320717302	0.320717302	2.282E-11	2.28E-11	2.28E-11
0.7	0.437068276	0.437068276	0.437068276	0.437068276	3.299E-11	3.3E-11	3.28E-11
0.8	0.583948471	0.583948472	0.583948472	0.583948472	4.638E-11	4.64E-11	4.44E-11
0.9	0.76861868	0.76861868	0.76861868	0.76861868	6.379E-11	6.39E-11	4.93E-11
1	1	1	1	1	8.543E-11	8.67E-11	2.03E-12

Concluding Remarks

In this work, we have applied three numerical methods namely: Adomian decomposition method (ADM), differential transform method (DTM) and homotopy perturbation method (HPM) to solve some differential equations (IVP and BVP). We observed that DTM is the simplest methods out of the three for solving differential equations. It converges to the exact solution rapidly with few terms but it requires transformation. The ADM is also a powerful method for solving differential equations. It does not require any form of transformation, perturbation, or linearization. However, rigorous calculation of Adomian polynomials is a requirement. It can result to intensive computations for the nonlinear equations. The homotopy perturbation method is equally simple in application but it results to large computations when compared to DTM.

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