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SOME FIXED AND COINCIDENCE POINT RESULTS FOR EXPANSIVE MAPPINGS ON G-PARTIAL METRIC SPACES

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Abstract. In this paper, we introduced a class of expansive mappings on G-partial metric spaces and proved fixed point and common fixed point theorems for a pair of those maps on G-partial metric spaces. We also establish a coincidence point theorem for two expansive maps on G-partial metric spaces. The results generalize and extend some results in literature.

Keywords: Fixed point; Expanding map; Reciprocally continuous map; Weakly compatible map; Coincidence point.

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1. Introduction

In 1992, Matthew [1] introduced the concept of partial metric spaces which generalized the notion of metric spaces in the sense that the distance from a point to itself need not be zero. Partial metrics are useful in modelling partially defined information which often appears in computer science. Mustafa and Sims [2] also generalized the concept of metric spaces to G-metric spaces by assigning real numbers to each triplet of an arbitary set. In 2013, Eke and

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Olaleru [3] generalized the notion of G-metric spaces to the context of partial metric spaces and called it G-partial metric spaces. Olaleru *et al.* [4] proved the existence of fixed points for generalized Ciric-type contractive mappings in ordered G-partial metric spaces. In this paper, we prove some fixed point theorems in G-partial metric spaces.

2. Preliminaries

We now recall the following definition, as analogue of partial metric space, introduced in [3].

Definition 1.1. [3] Let X be a nonempty set, and let $G_p : X \times X \times X \to R^+$ be a function satisfying the following:

(G_p1) $G_p(x,y,z) \geq G_p(x,x,x) \geq 0$ for all $x,y,z \in X$(small self distance),

(G_p2) $G_p(x,y,z) = G_p(x,x,y) = G_p(y,y,z) = G_p(z,z,x)$ iff $x = y = z$, (equality),

(G_p3) $G_p(x,y,z) = G_p(z,x,y) = G_p(y,z,x)$ (symmetry in all three variables),

(G_p4) $G_p(x,y,z) \leq G_p(x,a,a) + G_p(a,y,z) - G_p(a,a,a)$ (rectangle inequality).

Then the function G_p is called a G- partial metric and the pair (X, G_p) is called a G-partial metric space

Definition 1.2. [3] A G-partial metric space is said to be symmetric if $G_p(x,y,y) = G_p(y,x,x)$ for all $x,y \in X$.

Let (X, G_p) be a G-partial metric space, define d_{Gp} on X by

$$d_{Gp}(x,y) = G_p(x,y,y) + G_p(y,x,x) - G_p(y,y,y) - G_p(x,x,x).$$

Then (X, d_{Gp}) is a metric space.

Example 1.3. [3] *Let $X = R^+$ and let $G_p : X \times X \times X \to R^+$ be the map defined by $G_p(x,y,z) = max\{x,y,z\}$, then (X, G_p) is a G-partial metric space.*

We state the following definitions.

Definition 1.4. [3] A sequence $\{x_n\}$ of points in a G-partial metric space (X, G_p) converges to a point $a \in X$ if

$$\lim_{n\to\infty} G_p(x_n,x_n,a) = \lim_{n\to\infty} G_p(x_n,x_n,x_n) = G_p(a,a,a).$$

Definition 1.5. [3] A sequence $\{x_n\}$ of points in a G-partial metric spaces (X,G_p) is said to be Cauchy if, for each $\varepsilon > 0$ there exists a positive integer N such that $G_p(x_n,x_m,x_l) < \varepsilon$ for $n,m,l > N$; i.e $G_p(x_n,x_m,x_l) \to 0$ as $n,m,l \to \infty$.

The proof of the following proposition easily follows from the definitions.

Proposition 1.6. [3] *Let (X,G_p) be a G-partial metric space. Then the following are equivalent:*
$G_p(x_n,x_m,x_l) \to G_p(x,x,x)$ *as* $n,m,l \to \infty$
$G_p(x_n,x_m,x_m) \to G_p(x,x,x)$ *as* $n,m \to \infty$.

Definition 1.7. [3] A G-partial metric space (X,G_p) is said to be complete if every Cauchy sequence in (X,G_p) converges to an element in (X,G_p). That is, $G_p(x,x,x) = lim_{n\to\infty} G_p(x_n,x,x) = lim_{n,m\to\infty} G_p(x_n,x_m,x_m)$.

Definition 1.8. [5] Let f and g be self-mappings on a set X. If $w = fx = gx$ for some $x \in X$, then the point x is called a coincidence point of f and g and w is called a point of coincidence of f and g.

Definition 1.9. [5] Let f and g be self-mappings on a set X. Then f and g are said to be weakly compatible if they commute at each of their coincidence points.

The contraction mapping principle introduced by Banach in 1922 has wide range of applications in fixed point theory. Different authors generalized this mapping. In 1981, Gillespie and Williams [6] introduced a new class of maps where the existing constant is greater than one.

Suppose (X,d) is a metric space, $T: X \to X$ and there exists a constant $h > 1$ such that $d(Tx,Ty) \geq hd(x,y)$, for all $x,y \in X$. Then T is called an expanding map.

Several authors have recently proved some fixed points and common fixed points for expanding maps on abstract spaces; see [7-11] and the references therein. In this work, we introduced the class of expanding maps in G-partial metric spaces and proved some fixed point theorems in the new setting. In 1999, Pant [12] introduced a new continuity condition known as reciprocal continuity and proved a common fixed point theorem by using the compatibility in metric spaces. The notion of reciprocal continuity is weaker than the continuity of one of the mappings.

Definition 1.10. [29] Two self-mappings T and S are called reciprocally continuous if $\lim_{n\to\infty} T Sx_n = Tz$ and $\lim_{n\to\infty} STx_n = Sz$, whenever $\{x_n\}$ is a sequence such that $\lim_{n\to\infty} Tx_n = \lim_{n\to\infty} Sx_n = z$ for some z in X.

Han and Xu [7] proved the existence of common fixed point for a pair of expanding mappings in cone metric spaces by assuming the surjectivity of the maps. Esakkiappan [13] later proved a common fixed point theorem using compatible and reciprocal continuous map in a cone metric space. Manro and Kumar [10] proved common fixed point theorems for expansion mapping using the concept of compatible maps and weakly reciprocal continuity in both metric and G-metric spaces. Huang *et al.* [8] proved the fixed point and common fixed point theorems for expansion mappings and pairs of weakly compatible expansion maps respectively in partial metric spaces. In this work, the existence of the fixed point of an expanding map and common fixed point for a pair of expanding mappings on G-partial metric spaces using the concept of compatible maps and reciprocal continuity are proved. Shatanawi and Awawdeh [14] prove some results for fixed and coincidence points for some expansive mappings in cone metric spaces in which the surjectivity of the two maps is not assumed in proving the coincidence point theorem. Also we prove the coincidence point theorem for expanding maps without assuming the surjectivity of the maps therein in G-partial metric spaces. Our results generalize the recent results of Huang *et al.* [8], Manro and Kumar [10] and an analogue results to the results of Han and Xu [7], Esakkiappan [13] and Shatanawi and Awawdeh [14] in the cone metric spaces.

3. Main results

Theorem 3.1. *Let (X, G_p) be a complete G-partial metric space and $T : X \to X$ be a surjection. Suppose that there exist a_1, a_2, a_3, a_4, $a_5 \geq 0$ with $a_1 + a_2 + a_3 > 1$, $a_2 \leq 1 + a_5$, such that*

$$\begin{aligned} G_p(Tx, Ty, Ty) \quad &\geq \quad a_1 G_p(x, y, y) + a_2 G_p(x, Tx, Tx) + a_3 G_p(y, Ty, Ty) \\ &\quad + a_4 G_p(x, Ty, Ty) + a_5 G_p(y, Tx, Tx), \qquad (3.1) \end{aligned}$$

for all $x, y \in X$, $x \neq y$. Then T has a fixed point in X.

Proof. Let $x_0 \in X$ be chosen. Since T is surjective, choose $x_1 \in X$ such that $Tx_1 = x_0$. Continuing the process, we can define a sequence $\{x_n\} \in X$ such that $x_{n-1} = Tx_n$, $n = 1, 2, \cdots$. Without loss of generality, we suppose that $x_{n-1} \neq x_n$ for $n \geq 1$. From (3.1) we have

$$\begin{aligned}
G_p(x_{n-1}, x_n, x_n) &\geq a_1 G_p(x_n, x_{n+1}, x_{n+1}) + a_2 G_p(x_n, Tx_n, Tx_n) \\
&\quad + a_3 G_p(x_{n+1}, Tx_{n+1}, Tx_{n+1}) + a_4 G_p(x_n, Tx_{n+1}, Tx_{n+1}) + a_5 G_p(x_{n+1}, Tx_n, Tx_n) \\
&= a_1 G_p(x_n, x_{n+1}, x_{n+1}) + a_2 G_p(x_n, x_{n-1}, x_{n-1}) + a_3 G_p(x_{n+1}, x_n, x_n) \\
&\quad + a_4 G_p(x_n, x_n, x_n) + a_5 G_p(x_{n+1}, x_{n-1}, x_{n-1}),
\end{aligned}$$

From

$$G_p(x_{n+1}, x_{n-1}, x_{n-1}) \geq G_p(x_{n+1}, x_n, x_n) - G_p(x_{n-1}, x_n, x_n) + G_p(x_{n-1}, x_{n-1} x_{n-1}).$$

We have

$$\begin{aligned}
G_p(x_{n-1}, x_n, x_n) &\geq a_1 G_p(x_n, x_{n+1}, x_{n+1}) + a_2 G_p(x_n, x_{n-1}, x_{n-1}) + a_3 G_p(x_{n+1}, x_n, x_n) \\
&\quad + a_5 [G_p(x_{n+1}, x_n, x_n) - G_p(x_{n-1}, x_n, x_n)] \\
&\geq (a_1 + a_3 + a_5) G_p(x_{n+1}, x_n, x_n) + (a_2 - a_5) G_p(x_n, x_{n-1}, x_{n-1}).
\end{aligned}$$

It follows that

$$(1 - a_2 + a_5) G_p(x_{n-1}, x_n, x_n) \geq (a_1 + a_3 + a_5) G_p(x_n, x_{n+1}, x_{n+1}).$$

Hence

$$G_p(x_n, x_{n+1}, x_{n+1}) \leq \frac{1 - a_2 + a_5}{a_1 + a_3 + a_5} G_p(x_{n-1}, x_n, x_n).$$

Let $k = \frac{1-a_2+a_5}{a_1+a_3+a_5}$. By $a_1 + a_2 + a_3 > 1$, $a_2 \leq 1 + a_5$, we have $a_1 + a_3 + a_5 > 1 - a_2 + a_5 \geq 0$. Thus $k \in [0, 1)$. It follows that $G_p(x_n, x_{n+1}, x_{n+1}) \leq k G_p(x_{n-1}, x_n, x_n)$, and consequently

$G_p(x_n,x_{n+1},x_{n+1}) \leq k^n G_p(x_0,x_1,x_1)$. For $n > m$, we get

$$\begin{aligned}
G_p(x_m,x_n,x_n) &\leq G_p(x_m,x_{m+1},x_{m+1}) + G_p(x_{m+1},x_{m+2},x_{m+2}) \\
&\quad + \cdots + G_p(x_{n-1},x_n,x_n) - G_p(x_{m+1},x_{m+1},x_{m+1}) \\
&\quad - G_p(x_{m+2},x_{m+2},x_{m+2}) - \ldots - G_p(x_{n-1},x_{n-1},x_{n-1}) \\
&\leq (k^m + k^{m+1} + \ldots + K^{n-1}) G_p(x_0,x_1,x_1) \\
&\leq (k^m + k^{m+1} + \cdots) G_p(x_0,x_1,x_1) \\
&\leq k^m (1 + k + \cdots) G_p(x_0,x_1,x_1) \\
&\leq \frac{k^m}{1-k} G_p(x_0,x_1,x_1).
\end{aligned}$$

Therefore $\{x_n\}$ is a Cauchy sequence. Since X is complete, there exists $p \in X$ such that $Tx_{n+1} = x_n \to p$ as $n \to \infty$. Therefore $\lim_{n\to\infty} G_p(x_n,p,p) = \lim_{n\to\infty} G_p(x_n,x_n,x_n) = \lim_{n,m\to\infty} G_p(x_m,x_n,x_n) = G_p(p,p,p)$. Since T is a surjection, we find $q \in X$ such that $p = Tq$. Now we prove that $p = q$ is the fixed point of T. Using (3.1) we obtain

$$\begin{aligned}
G_p(p,x_n,x_n) &= G_p(Tq,Tx_{n+1},Tx_{n+1}) \\
&\geq a_1 G_p(q,x_{n+1},x_{n+1}) + a_2 G_p(q,Tq,Tq) + a_3 G_p(x_{n+1},Tx_{n+1},Tx_{n+1}) \\
&\quad + a_4 G_p(q,Tx_{n+1},Tx_{n+1}) + a_5 G_p(x_{n+1},Tq,Tq),
\end{aligned} \tag{3.2}$$

$$G_p(q,p,p) \geq G_p(q,x_{n+1},x_{n+1}) - G_p(p,x_{n+1},x_{n+1}) + G_p(p,p,p), \tag{3.3}$$

$$G_p(q,x_n,x_n) \geq G_p(q,x_{n+1},x_{n+1}) - G_p(x_n,x_{n+1},x_{n+1}) + G_p(x_n,x_n,x_n) \tag{3.4}$$

and

$$G_p(p,x_n,x_n) \leq G_p(p,x_{n+1},x_{n+1}) + G_p(x_{n+1},x_n,x_n) - G_p(x_{n+1},x_{n+1},x_{n+1}) \tag{3.5}$$

Using (3.3), (3.4) and (3.5) in (3.2), we find that

$$
\begin{aligned}
& G_p(p,x_{n+1},x_{n+1})+G_p(x_{n+1},x_n,x_n)-G_p(x_{n+1},x_{n+1},x_{n+1}) \\
& \geq a_1G_p(q,x_{n+1},x_{n+1})+a_2G_p(q,x_{n+1},x_{n+1})-a_2G_p(p,x_{n+1},x_{n+1}) \\
& \quad +a_2G_p(p,p,p)+a_3G_p(x_{n+1},x_n,x_n)+a_4G_p(q,x_{n+1},x_{n+1}) \\
& \quad -a_4G_p(x_n,x_{n+1},x_{n+1})+a_4G_p(x_n,x_n,x_n)+a_5G_p(x_{n+1},p,p) \\
& \geq (a_1+a_2+a_4)G_p(q,x_{n+1},x_{n+1})+(a_5-a_2)G_p(x_{n+1},p,p) \\
& \quad +(a_3-a_4)G_p(x_{n+1},x_n,x_n).
\end{aligned}
$$

Taking the limit as $n\to\infty$ yields $(a_1+a_2+a_4)G_p(q,p,p)\leq 0$. Since $a_1+a_2+a_4\geq 0$, we have $G_p(q,p,p)\leq 0$. But $G_p(q,p,p)\geq 0$. Hence $q=p$. That is $q=p=Tq$. This gives that p is the fixed point of T. This completes the proof.

Corollary 3.2. *Let (X,G_p) be a complete G-partial metric space and $T:X\to X$ be a surjection. Suppose that there exists a_1, a_2, $a_3\geq 0$ with $a_1+a_2+a_3>1\geq a_2$ such that*

$$G_p(Tx,Ty,Ty)\geq a_1G_p(x,y,y)+a_2G_p(x,Tx,Tx)+a_3G_p(y,Ty,Ty), x,y\in X, x\neq y. \tag{3.6}$$

Then T has a fixed point in X.

Remarks 3.3. Corollary 2.2 is an analogue of the results of [8] which they proved in the context of partial metric spaces. In [7], Han and Xu proved a parallel result of Theorem 2.1 in the context of cone metric spaces.

3. Common fixed point theorems

We prove a theorem on the coincidence point of two expansive type mappings in the G-partial metric spaces in which the surjectivity condition of the maps is not assumed.

Theorem 3.4. *Let (X,G_p) be a G-partial metric space. Let $T,S:X\to X$ be mappings satisfying:*

$$
\begin{aligned}
G_p(Tx,Ty,Ty)\geq\ & a_1G_p(Sx,Sy,Sy)+a_2G_p(Sx,Tx,Tx)+a_3G_p(Sy,Ty,Ty) \\
& +a_4G_p(Sx,Ty,Ty)+a_5G_p(Sy,Tx,Tx).
\end{aligned}
\tag{3.7}
$$

for all $x, y \in X$ where a_1, a_2, a_3, a_4, $a_5 \geq 0$ with $a_1+a_2+a_3+a_4+a_5 > 1$. Suppose the following hypotheses are also satisfy: (1) $a_2 - a_5 < 1$ or $a_3 - a_4 < 1$; (2) $S(X) \subseteq T(X)$ and (3) $T(X)$ is a complete subspace of X. Then T and S have a coincidence point.

Proof. Let $x_0 \in X$ be chosen. We chose $x_1 = Sx_0$ and $x_2 = Tx_1$. Since $S(X) \subseteq T(X)$ then there exists a sequence $\{x_n\}$ such that $Sx_n = Tx_{n+1}$. Without loss of generality, we claim that $x_{n-1} \neq x_n$ for $n \geq 1$. From (3.7) with $x = x_n$ and $y = x_{n+1}$ we have the following. Case (i)

$$\begin{aligned}
&G_p(Sx_{n-1}, Sx_n, Sx_n)\\
&= G_p(Tx_n, Tx_{n+1}, Tx_{n+1})\\
&\geq a_1 G_p(Sx_n, Sx_{n+1}, Sx_{n+1}) + a_2 G_p(Sx_n, Sx_{n-1}, Sx_{n-1}) + a_3 G_p(Sx_{n+1}, Sx_n, Sx_n)\\
&\quad + a_4 G_p(Sx_n, Sx_n, Sx_n) + a_5 G_p(Sx_{n+1}, Sx_{n-1}, Sx_{n-1})\\
&\geq a_1 G_p(Sx_n, Sx_{n+1}, Sx_{n+1}) + a_2 G_p(Sx_n, Sx_{n-1}, Sx_{n-1}) + a_3 G_p(Sx_{n+1}, Sx_n, Sx_n)\\
&\quad + a_4 G_p(Sx_n, Sx_n, Sx_n) + a_5 G_p(Sx_{n+1}, Sx_n, Sx_n) - a_5 G_p(Sx_{n-1}, Sx_n, Sx_n)\\
&\quad + a_5 G_p(Sx_{n-1}, Sx_{n-1}, Sx_{n-1})\\
&\geq a_1 G_p(Sx_n, Sx_{n+1}, Sx_{n+1}) + a_2 G_p(Sx_n, Sx_{n-1}, Sx_{n-1})\\
&\quad + a_3 G_p(Sx_{n+1}, Sx_n, Sx_n) + a_5 G_p(Sx_{n+1}, Sx_n, Sx_n) - a_5 G_p(Sx_{n-1}, Sx_n, Sx_n)\\
&\geq (a_1 + a_3 + a_5) G_p(Sx_n, Sx_{n+1}, Sx_{n+1}) + (a_2 - a_5) G_p(Sx_n, Sx_{n-1}, Sx_{n-1}).
\end{aligned} \tag{3.8}$$

If $a_2 - a_5 < 1$, then (3.8) becomes

$$(1 - a_2 + a_5) G_p(Sx_n, Sx_{n-1}, Sx_{n-1}) \geq (a_1 + a_3 + a_5) G_p(Sx_n, Sx_{n+1}, Sx_{n+1}),$$

$$G_p(Sx_n, Sx_{n+1}, Sx_{n+1}) \leq \frac{1 - a_2 + a_5}{a_1 + a_3 + a_5} G_p(Sx_n, Sx_{n-1}, Sx_{n-1}). \tag{3.9}$$

Case (ii)

$$\begin{aligned}
&G_p(Sx_n, Sx_{n-1}, Sx_{n-1})\\
&= G_p(Tx_{n+1}, Tx_n, Tx_n)\\
&\geq a_1 G_p(Sx_{n+1}, Sx_n, Sx_n) + a_2 G_p(Sx_{n+1}, Tx_{n+1}, Tx_{n+1}) + a_3 G_p(Sx_n, Tx_n, Tx_n)\\
&+ a_4 G_p(Sx_{n+1}, Tx_n, Tx_n) + a_5 G_p(Sx_n, Tx_{n+1}, Tx_{n+1})\\
&= a_1 G_p(Sx_{n+1}, Sx_n, Sx_n) + a_2 G_p(Sx_{n+1}, Sx_n, Sx_n) + a_3 G_p(Sx_n, Sx_{n-1}, Sx_{n-1})\\
&+ a_4 G_p(Sx_{n+1}, Sx_{n-1}, Sx_{n-1}) + a_5 G_p(Sx_n, Sx_n, Sx_n)\\
&\geq a_1 G_p(Sx_{n+1}, Sx_n, Sx_n) + a_2 G_p(Sx_{n+1}, Sx_n, Sx_n) + a_3 G_p(Sx_n, Sx_{n-1}, Sx_{n-1})\\
&+ a_4 G_p(Sx_{n+1}, Sx_n, Sx_n) - a_4 G_p(Sx_{n-1}, Sx_n, Sx_n) + a_4 G_p(Sx_{n-1}, Sx_{n-1}, Sx_{n-1})\\
&+ a_5 G_p(Sx_n, Sx_n, Sx_n)\\
&\geq a_1 G_p(Sx_{n+1}, Sx_n, Sx_n) + a_2 G_p(Sx_{n+1}, Sx_n, Sx_n) + a_3 G_p(Sx_n, Sx_{n-1}, Sx_{n-1})\\
&+ a_4 G_p(Sx_{n+1}, Sx_n, Sx_n) - a_4 G_p(Sx_{n-1}, Sx_n, Sx_n)\\
&= (a_1 + a_2 + a_4) G_p(Sx_{n+1}, Sx_n, Sx_n) + (a_3 - a_4) G_p(Sx_{n-1}, Sx_n, Sx_n).
\end{aligned}$$

If $a_3 - a_4 < 1$, then the above inequality becomes

$$(1 - a_3 + a_4) G_p(Sx_{n-1}, Sx_n, Sx_n) \geq (a_1 + a_2 + a_4) G_p(Sx_{n+1}, Sx_n, Sx_n),$$

$$G_p(Sx_{n+1}, Sx_n, Sx_n) \leq \frac{1 - a_3 + a_4}{a_1 + a_2 + a_4} G_p(Sx_{n-1}, Sx_n, Sx_n) \tag{3.10}$$

Put $k = \frac{1-a_2+a_5}{a_1+a_3+a_5}$ in (3.9) and $k = \frac{1-a_3+a_4}{a_1+a_2+a_4}$ in (3.10) Thus in (3.9) and (3.10), we have $k < 1$. Hence $G_p(Sx_n, Sx_{n+1}, Sx_{n+1}) \leq k G_p(Sx_n, Sx_{n-1}, Sx_{n-1})$. Consequently, we have

$$G_p(Sx_n, Sx_{n+1}, Sx_{n+1}) \leq k^n G_p(Sx_n, Sx_{n-1}, Sx_{n-1}).$$

For $n > m$, we obtain

$$\begin{aligned}
G_p(Sx_m, Sx_n, Sx_n) &\leq G_p(Sx_m, Sx_{m+1}, Sx_{m+1}) + G_p(Sx_{m+1}, Sx_{m+2}, Sx_{m+2}) \\
&\quad + \cdots + G_p(Sx_{n-1}, Sx_n, Sx_n) - G_p(Sx_{m+1}, Sx_{m+1}, Sx_{m+1}) \\
&\quad - G_p(Sx_{m+2}, Sx_{m+2}, Sx_{m+2}) - \cdots - G_p(Sx_{n-1}, Sx_n, Sx_n) \\
&\leq k^m G_p(Sx_0, Sx_1, Sx_1) + k^{m+1} G_p(Sx_0, Sx_1, Sx_1) \\
&\quad + \cdots + k^{n-1} G_p(Sx_0, Sx_1, Sx_1) \\
&\leq (k^m + k^{m+1} + \cdots + k^{n-1}) G_p(Sx_0, Sx_1, Sx_1) \\
&\leq (k^m + k^{m+1} + \cdots) G_p(Sx_0, Sx_1, Sx_1) \\
&\leq \frac{k^m}{1-k} G_p(Sx_0, Sx_1, Sx_1).
\end{aligned}$$

Thus $\{Tx_n\}$ is a Cauchy sequence. Since $T(X)$ is a complete subspace of X, there exists a point $z \in X$ such that $Tx_n \to Tz$ as $n \to \infty$. Likewise, $Sx_n \to Tz$ as $n \to \infty$. Also $G_p(Tz, Tz, Tz) = lim_{n\to\infty} G_p(Tx_n, Tz, Tz) = lim_{n,m\to\infty} G_p(Tx_m, Tx_n, Tx_n) = 0$. Since $a_1 + a_2 + a_3 + a_4 + a_5 > 1$ we have a_1, a_2, a_3, a_4, a_5 are not all zero. So we obtain the following cases.

Case (a) If $a_1 \neq 0$, then we have

$$\begin{aligned}
G_p(Tx_n, Tz, Tz) &\geq a_1 G_p(Sx_n, Sz, Sz) + a_2 G_p(Sx_n, Tx_n, Tx_n) \\
&\quad + a_3 G_p(Sz, Tz, Tz) + a_4 G_p(Sx_n, Tz, Tz) + a_5 G_p(Sz, Tx_n, Tx_n).
\end{aligned}$$

Hence, $G_p(Tx_n, Tz, Tz) \geq a_1 G_p(Sx_n, Sz, Sz)$. As $n \to \infty$, we have $G_p(Tz, Tz, Tz) \geq a_1 G_p(Tz, Sz, Sz)$. Thus $G_p(Tz, Sz, Sz) \leq 0$. But $G_p(Tz, Sz, Sz) \geq 0$. Therefore $G_p(Tz, Sz, Sz) = 0$, which implies that $Tz = Sz$.

Case (b) If $a_2 \neq 0$, then we have

$$\begin{aligned}
G_p(Tz, Tx_n, Tx_n) &\geq a_1 G_p(Sz, Sx_n, Sx_n) + a_2 G_p(Sz, Tz, Tz) \\
&\quad + a_3 G_p(Sx_n, Tx_n, Tx_n) + a_4 G_p(Sz, Tx_n, Tx_n) + a_5 G_p(Sx_n, Tz, Tz)
\end{aligned}$$

Hence, $G_p(Tz, Tx_n, Tx_n) \geq a_2 G_p(Sz, Tz, Tz)$. Similar to Case (a) we have $Sz = Tz$.

Case(c) If $a_3 \neq 0$, then we have

$$\begin{aligned}
G_p(Tx_n, Tz, Tz) &\geq a_1 G_p(Sx_n, Sz, Sz) + a_2 G_p(Sx_n, Tx_n, Tx_n) \\
&\quad + a_3 G_p(Sz, Tz, Tz) + a_4 G_p(Sx_n, Tz, Tz) + a_5 G_p(Sz, Tx_n, Tx_n).
\end{aligned}$$

Hence, $G_p(Tx_n,Tz,Tz) \geq a_3G_p(Sz,Tz,Tz)$. Similar to Case (a) we have $Sz = Tz$.

Case (d) If $a_4 \neq 0$, then we have

$$G_p(Tz,Tx_n,Tx_n) \geq a_1G_p(Sz,Sx_n,Sx_n)+a_2G_p(Sz,Tz,Tz)$$
$$+a_3G_p(Sx_n,Tx_n,Tx_n)+a_4G_p(Sz,Tx_n,Tx_n)+a_5G_p(Sx_n,Tz,Tz).$$

Hence, $G_p(Tz,Tx_n,Tx_n) \geq a_4G_p(Sz,Tx_n,Tx_n)$. Similar to Case (a) we have $Sz = Tz$.

Case (e) If $a_5 \neq 0$, then we have

$$G_p(Tx_n,Tz,Tz) \geq a_1G_p(Sx_n,Sz,Sz)+a_2G_p(Sx_n,Tx_n,Tx_n)$$
$$+a_3G_p(Sz,Tz,Tz)+a_4G_p(Sx_n,Tz,Tz)+a_5G_p(Sz,Tx_n,Tx_n).$$

Hence, $G_p(Tx_n,Tz,Tz) \geq a_5G_p(Sz,Tx_n,Tx_n)$. Similar to Case (a) we have $Sz = Tz$. Thus S and T have coincidence point which is z.

Corollary 3.5. *Let (X,G_p) be a G-partial metric space. Let $T,S : X \to X$ be mappings satisfying:*

$$G_p(Tx,Ty,Ty) \geq a_1G_p(Sx,Sy,Sy)+a_2G_p(Sx,Tx,Tx)+a_3G_p(Sy,Ty,Ty) \tag{3.11}$$

for all $x,y \in X$ where a_1, a_2, $a_3 \geq 0$ with $a_1+a_2+a_3 > 1$. Suppose the following hypotheses are also satisfy: (1) $a_2 < 1$ or $a_3 < 1$, (2) $S(X) \subseteq T(X)$ and (3) $T(X)$ is a complete subspace of X. Then T and S have a coincidence point.

Remark 3.6. *Corollary 2.5 is an analogue of the result proved by Shatanawi and Awaedeh [14] in the setting of cone metric spaces.*

Example 3.7. *Let $X = [0,1]$ and $G_p(x,y,y) = max\{x,y,y\}$. Then (X,G_p) is a complete G-partial metric space. Define $T,S : X \to X$ by $Tx = \frac{x}{3}$ and $Sx = \frac{x}{9}$ for all $x \in X$. Then for every $x,y \in X$ we have $G_p(Tx,Ty,Ty) \geq 3G_p(Sx,Sy,Sy)$ i.e. the condition (2.7) holds for $a_1 = 3, a_2 = a_3 = a_4 = a_5 = 0$. Therefore we have all the hypothese of Theorem 2.4 satisfied and o is the coincidence point of T and S.*

In the next theorem, we prove the existence of the common fixed point for a pair of weakly compatible maps satisfying certain conditions in G-partial metric spaces in which the surjectivity of the two maps is assumed.

Theorem 3.8. *Let T and S be two weakly compatible and surjective selfmappings of a complete G-partial metric space (X,G_p) satisfying the following conditions: for any $x,y\in X$ and $a_1+a_2+a_3>1$, $a_2\le 1+a_5$, $a_3\le 1+a_4$ we have that*

$$\begin{aligned}G_p(Tx,Sy,Sy)\geq a_1G_p(x,y,y)+a_2G_p(x,Tx,Tx)+a_3G_p(y,Sy,Sy)\\+a_4G_p(x,Sy,Sy)+a_5G_p(y,Tx,Tx).\end{aligned}\tag{3.12}$$

If S and T are compatible pair of reciprocal continuous maps, then S and T have a unique common fixed point in X.

Proof. Let $x_0\in X$ be chosen. Since T and S are surjective then there exist $x_1,x_2\in X$ such that $x_0=Tx_1$ and $x_1=Sx_2$. Continuing the process, we can define a sequence $\{x_n\}\in X$ such that $x_{2n}=Tx_{2n+1}$, $x_{2n+1}=Sx_{2n+2}$. Using (3.12), we have

$$\begin{aligned}G_p(x_{2n},x_{2n+1},x_{2n+1})&=G_p(Tx_{2n+1},Sx_{2n+2},Sx_{2n+2})\\&\geq a_1G_p(x_{2n+1},x_{2n+2},x_{2n+2})+a_2G_p(x_{2n+1},Tx_{2n+1},Tx_{2n+1})\\&\quad+a_3G_p(x_{2n+2},Sx_{2n+2},Sx_{2n+2})+a_4G_p(x_{2n+1},Sx_{2n+2},Sx_{2n+2})\\&\quad+a_5G_p(x_{2n+2},Tx_{2n+1},Tx_{2n+1})\\&=a_1G_p(x_{2n+1},x_{2n+2},x_{2n+2})+a_2G_p(x_{2n+1},x_{2n},x_{2n})\\&\quad+a_3G_p(x_{2n+2},x_{2n+1},x_{2n+1})\\&+a_4G_p(x_{2n+1},x_{2n+1},x_{2n+1})+a_5G_p(x_{2n+2},x_{2n},x_{2n}).\end{aligned}$$

By rectangle inequality, the above inequality becomes

$$\begin{aligned}&G_p(x_{2n},x_{2n+1},x_{2n+1})\\&\geq a_1G_p(x_{2n+1},x_{2n+2},x_{2n+2})+a_2G_p(x_{2n+1},x_{2n},x_{2n})+a_3G_p(x_{2n+2},x_{2n+1},x_{2n+1})\\&\quad+a_4G_p(x_{2n+1},x_{2n+1},x_{2n+1})+a_5G_p(x_{2n+2},x_{2n+1},x_{2n+1})-a_5G_p(x_{2n},x_{2n+1},x_{2n+1})\\&+a_5G_p(x_{2n},x_{2n},x_{2n})\\&\geq a_1G_p(x_{2n+1},x_{2n+2},x_{2n+2})+a_2G_p(x_{2n+1},x_{2n},x_{2n})+a_3G_p(x_{2n+2},x_{2n+1},x_{2n+1})\\&+a_5G_p(x_{2n+2},x_{2n+1},x_{2n+1})-a_5G_p(x_{2n},x_{2n+1},x_{2n+1})\\&\geq(a_1+a_3+a_5)G_p(x_{2n+1},x_{2n+2},x_{2n+2})+(a_2-a_5)G_p(x_{2n+1},x_{2n},x_{2n}).\end{aligned}$$

Therefore, we have $(1-a_2+a_5)G_p(x_{2n+1},x_{2n},x_{2n}) \geq (a_1+a_3+a_5)G_p(x_{2n+1},x_{2n+2},x_{2n+2})$ and

$$G_p(x_{2n+1},x_{2n+2},x_{2n+2}) \leq \frac{1-a_2+a_5}{a_1+a_3+a_5} G_p(x_{2n},x_{2n+1},x_{2n+1}).$$

Let $M = \frac{1-a_2+a_5}{a_1+a_3+a_5}$. Since $a_2 \leq 1+a_5$ and $a_1+a_2+a_3 > 1$, we have $a_1+a_3+a_5 > 1-a_2+a_5 \geq 0$. Thus $M \in [0,1)$, and

$$G_p(x_{2n+1},x_{2n+2},x_{2n+2}) \leq MG_p(x_{2n},x_{2n+1},x_{2n+1}) \tag{3.13}$$

Similarly, we have

$$\begin{aligned}
G_p(x_{2n-1},x_{2n},x_{2n}) &= G_p(Tx_{2n+1},Sx_{2n},Sx_{2n}) \\
&\geq a_1G_p(x_{2n+1},x_{2n},x_{2n}) + a_2G_p(x_{2n+1},Tx_{2n+1},Tx_{2n+1}) \\
&\quad + a_3G_p(x_{2n},Sx_{2n},Sx_{2n}) + a_4G_p(x_{2n+1},Sx_{2n},Sx_{2n}) \\
&\quad + a_5G_p(x_{2n},Tx_{2n+1},Tx_{2n+1}) \\
&= a_1G_p(x_{2n+1},x_{2n},x_{2n}) + a_2G_p(x_{2n+1},x_{2n},x_{2n}) \\
&\quad + a_3G_p(x_{2n},x_{2n-1},x_{2n-1}) + a_4G_p(x_{2n+1},x_{2n-1},x_{2n-1}) \\
&\quad + a_5G_p(x_{2n},x_{2n},x_{2n}).
\end{aligned}$$

By rectangle inequality, the above inequality becomes

$$\begin{aligned}
G_p(x_{2n-1},x_{2n},x_{2n}) &\geq a_1G_p(x_{2n+1},x_{2n},x_{2n}) + a_2G_p(x_{2n+1},x_{2n},x_{2n}) \\
&\quad + a_3G_p(x_{2n},x_{2n-1},x_{2n-1}) \\
&\quad + a_4G_p(x_{2n+1},x_{2n},x_{2n}) - a_4G_p(x_{2n-1},x_{2n},x_{2n}) \\
&\quad + a_4G_p(x_{2n-1},x_{2n-1},x_{2n-1}) + a_5G_p(x_{2n},x_{2n},x_{2n}) \\
&\geq a_1G_p(x_{2n+1},x_{2n},x_{2n}) + a_2G_p(x_{2n+1},x_{2n},x_{2n}) \\
&\quad + a_3G_p(x_{2n},x_{2n-1},x_{2n-1}) + a_4G_p(x_{2n+1},x_{2n},x_{2n}) \\
&\quad - a_4G_p(x_{2n-1},x_{2n},x_{2n}) \\
&\geq (a_1+a_2+a_4)G_p(x_{2n+1},x_{2n},x_{2n}) + (a_3-a_4)G_p(x_{2n},x_{2n-1},x_{2n-1}).
\end{aligned}$$

Therefore, $(1-a_3+a_4)G_p(x_{2n},x_{2n-1},x_{2n-1}) \geq (a_1+a_2+a_4)G_p(x_{2n+1},x_{2n},x_{2n})$ and

$$G_p(x_{2n+1},x_{2n},x_{2n}) \leq \frac{1-a_3+a_4}{a_1+a_2+a_4}G_p(x_{2n},x_{2n-1},x_{2n-1}).$$

By symmetry, we have $G_p(x_{2n},x_{2n+1},x_{2n+1}) \leq \frac{1-a_3+a_4}{a_1+a_2+a_4}G_p(x_{2n-1},x_{2n},x_{2n})$

Let $N = \frac{1-a_3+a_4}{a_1+a_2+a_4}$. Since $a_3 \leq 1+a_4$ and $a_1+a_2+a_3 > 1$, we have that $a_1+a_2+a_4 > 1-a_3+a_4 \geq 0$. Thus $N \in [0,1)$ and

$$G_p(x_{2n},x_{2n+1},x_{2n+1}) \leq NG_p(x_{2n-1},x_{2n},x_{2n}). \tag{3.14}$$

Let $h = MN \in [0,1)$. Then by induction we have

$$\begin{aligned}
G_p(x_{2n+1},x_{2n+2},x_{2n+2}) &\leq MG_p(x_{2n},x_{2n+1},x_{2n+1}) \\
&\leq MNG_p(x_{2n-1},x_{2n},x_{2n}) \\
&\leq M^2NG_p(x_{2n-2},x_{2n-1},x_{2n-1}) \\
&\vdots \\
&\leq Mh^nG_p(x_0,x_1,x_1)
\end{aligned}$$

and

$$\begin{aligned}
G_p(x_{2n},x_{2n+1},x_{2n+1}) &\leq NG_p(x_{2n-1},x_{2n},x_{2n}) \\
&\leq kNG_p(x_{2n-2},x_{2n-1},x_{2n-1}) \\
&\vdots \\
&\leq h^nG_p(x_0,x_1,x_1).
\end{aligned}$$

For $n > m$, we get

$$\begin{aligned}
G_p(x_{2m+1}, x_{2n+1}, x_{2n+1}) &\leq G_p(x_{2m+1}, x_{2m+2}, x_{2m+2}) + G_p(x_{2m+2}, x_{2m+3}, x_{2m+3}) \\
&+ \ldots + G_p(x_{2n}, x_{2n+1}, x_{2n+1}) - G_p(x_{2m+2}, x_{2m+2}, x_{2m+2}) \\
&- G_p(x_{m+3}, x_{m+3}, x_{m+3}) - \ldots - G_p(x_{2n}, x_{2n}, x_{2n}) \\
&\leq G_p(x_{2m+1}, x_{2m+2}, x_{2m+2}) + G_p(x_{2m+2}, x_{2m+3}, x_{2m+3}) \\
&+ \ldots + G_p(x_{2n}, x_{2n+1}, x_{2n+1}) \\
&\leq (\sum_{i=m+1}^{n} h^i + M \sum_{i=m}^{n-1} h^i) G_p(x_0, x_1, x_1) \\
&\leq (N+1) \frac{Mh^m}{1-h}) G_p(x_0, x_1, x_1).
\end{aligned}$$

Similarly, we have

$$\begin{aligned}
G_p(x_{2m}, x_{2n+1}, x_{2n+1}) &\leq G_p(x_{2m}, x_{2m+1}, x_{2m+1}) + G_p(x_{2m+1}, x_{2m+2}, x_{2m+2}) \\
&+ \ldots + G_p(x_{2n}, x_{2n+1}, x_{2n+1}) - G_p(x_{2m+1}, x_{2m+1}, x_{2m+1}) \\
&- G_p(x_{m+2}, x_{m+2}, x_{m+2}) - \ldots - G_p(x_{2n}, x_{2n}, x_{2n}) \\
&\leq G_p(x_{2m}, x_{2m+1}, x_{2m+1}) + G_p(x_{2m+1}, x_{2m+2}, x_{2m+2}) \\
&+ \ldots + G_p(x_{2n}, x_{2n+1}, x_{2n+1}) \\
&\leq (M \sum_{i=m}^{n} h^i + \sum_{i=m+1}^{n+1} h^i) G_p(x_0, x_1, x_1) \\
&\leq (\frac{Mh^m}{1-h} + \frac{h^{m+1}}{1-h}) G_p(x_0, x_1, x_1) \\
&\leq (N+1) \frac{Mh^m}{1-h}) G_p(x_0, x_1, x_1).
\end{aligned}$$

Therefore $\{x_n\}$ is a Cauchy sequence. Since X is complete, there exists a point $z \in X$ such that $x_n \to z$ as $n \to \infty$. It is equivalent to $x_{2n} = Tx_{2n+1} \to z$, $x_{2n+1} = Sx_{2n+2} \to z$ as $n \to \infty$. Also $G_p(z,z,z) = \lim_{n\to\infty} G_p(x_n, z, z) = \lim_{n,m\to\infty} G_p(x_n, x_m, x_m) = 0$. Suppose T and S are compatible and reciprocal continuous. By reciprocal continuity of T and S, $\lim_{n\to\infty} TSx_n = Tz$ and $\lim_{n\to\infty} STx_n = Sz$. By compatibility of T and S, $Tz = Sz$. Since T and S are weakly compatible, $Tz = Sz$ implies $TTz = TSz = STz = SSz$.

Next we show that z is a common fixed point of S and T. From (3.12) we have

$$\begin{aligned}G_p(Tz,Sx_{2n+2},Sx_{2n+2}) \geq{}& a_1G_p(z,x_{2n+2},x_{2n+2})+a_2G_p(z,Tz,Tz)\\ &+a_3G_p(x_{n+2},Sx_{2n+2},Sx_{2n+2})+a_4G_p(z,Sx_{2n+2},Sx_{2n+2})\\ &+a_5G_p(x_{n+2},Tz,Tz).\end{aligned}$$

From the following three inequalities, we have

$$G_p(z,Tz,Tz) \geq G_p(z,x_{2n+2},x_{2n+2})-G_p(Tz,x_{2n+2},x_{2n+2})+G_p(Tz,Tz,Tz),$$

$$G_p(z,x_{2n+1},x_{2n+1}) \geq G_p(z,x_{2n+2},x_{2n+2})-G_p(x_{2n+1},x_{2n+2},x_{2n+2})+G_p(x_{2n+1},x_{2n+1},x_{2n+1})$$

and

$$G_p(Tz,x_{2n+1},x_{2n+1}) \leq G_p(Tz,x_{2n+2},x_{2n+2})+G_p(x_{n+2},x_{2n+1},x_{2n+1})-G_p(x_{n+2},x_{2n+2},x_{2n+2}),$$

we obtain

$$\begin{aligned}&G_p(Tz,x_{2n+2},x_{2n+2})+G_p(x_{n+2},x_{2n+1},x_{2n+1})-G_p(x_{n+2},x_{2n+2},x_{2n+2})\\ &\geq a_1G_p(z,x_{2n+2},x_{2n+2})+a_2G_p(z,x_{2n+2},x_{2n+2})\\ &-a_2G_p(Tz,x_{2n+2},x_{2n+2})+a_2G_p(Tz,Tz,Tz)\\ &+a_3G_p(x_{n+2},x_{2n+1},x_{2n+1})+a_4G_p(z,x_{2n+2},x_{2n+2})\\ &-a_4G_p(x_{2n+1},x_{2n+2},x_{2n+2})+a_4G_p(x_{2n+1},x_{2n+1},x_{2n+1})+a_5G_p(x_{2n+2},Tz,Tz)\\ &\geq (a_1+a_2+a_4)G_p(z,x_{n+2},x_{2n+2})\\ &+(a_3-a_4)G_p(x_{n+2},x_{2n+1},x_{2n+1})+(a_5-a_2)G_p(Tz,x_{2n+2},x_{2n+2}).\end{aligned}$$

Therefore, we have

$$\begin{aligned}&(a_5-a_2-1)G_p(Tz,x_{2n+2},x_{2n+2})\\ &\leq (1-a_3+a_4)G_p(x_{n+2},x_{2n+1},x_{2n+1})\\ &\quad -(a_1+a_2+a_4)G_p(z,x_{2n+2},x_{2n+2})G_p(Tz,x_{2n+2},x_{2n+2})\\ &\leq \frac{1-a_3+a_4}{a_5-a_2-1}G_p(x_{n+2},x_{2n+1},x_{2n+1})-\frac{a_1+a_2+a_4}{a_5-a_2-1}G_p(z,x_{2n+2},x_{2n+2}).\end{aligned}$$

As $n \to \infty$, we get $G_p(Tz,z,z) \leq 0$. $G_p(Tz,z,z) \geq 0$ implies that $G_p(Tz,z,z) = 0$. Therefore $Tz = Sz = z$. Suppose there exists $u \in X$ such that u is another common fixed point of T and S then we show that $u = z$. On the contrary, letting $u \neq z$ and using (3.12) we have

$$\begin{aligned} G_p(u,z,z) &= G_p(Tu,Sz,Sz) \\ &\geq a_1 G_p(u,z,z) + a_2 G_p(u,Tu,Tu) + a_3 G_p(z,Sz,Sz) + a_4 G_p(u,Sz,Sz) a_5 G_p(z,Tu,Tu) \\ &= a_1 G_p(u,z,z) + a_4 G_p(u,z,z) + a_5 G_p(z,u,u) \\ &= (a_1 + a_4 + a_5) G_p(u,z,z). \end{aligned}$$

Since $a_1 + a_4 + a_5 > 1$, then we have $u = z$. The uniqueness proved.

Corollary 3.9. *Let (X, G_p) be a complete G-partial metric space. Suppose mappings $T, S : X \to X$ are onto, compatible, reciprocally continuous, and satisfy*

$$G_p(Tx,Sy,Sy) \geq a_1 G_p(x,y,y) + a_2 G_p(x,Tx,Tx) + a_3 G_p(y,Sy,Sy) \tag{3.15}$$

for all $x, y \in X$, with $a_1 + a_2 + a_3 > 1$. Then S and T have a common fixed point.

Remark 3.10. *Theorem 2.4 is an analogue result of [13 Theorems 3.1 and 3.2] from cone metric spaces to the setting of G-partial metric spaces.*

Example 3.11. *Let $X = R^+$ and $G_p(x,y,y) = max\{x,y,y\}$; then (X, G_p) is a complete G-partial metric space. Let $T, S : R \to R$ be defined by $Tx = 2x$ and $Sx = 3x$. T and S are surjective, reciprocally continuous, compatible, and satisfy the inequality of Theorem 2.4 with $a_1 = 2$ and $a_2 = a_3 = a_4 = a_5 = 0$. Then T and S have a unique common fixed point 0 in X.*

Conflict of Interests

The author declares that there is no conflict of interests.

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