MecSol 2015
International Symposium on Solid Mechanics
May 25-27th, 2015
Belo Horizonte, MG, Brazil

DYNAMIC BEHAVIOR OF AN INCLINED RECTANGULAR MINDLIN PLATE RESTING ON PASTERNAK FOUNDATION UNDER UNIFORM PARTIALLY DISTRIBUTED MOVING LOAD

Agarana M.C.[1]
Department of Mathematics
Covenant University, Ota, Ogun State, Nigeria
michael.agarana@covenantuniversity.edu.ng

Gbadeyan J.A.[2]
Department of Mathematics
University of Ilorin, Ilorin, Kwara State, Nigeria
j.agbadeyan@yahoo.com

Agboola O.O.[3]
Department of Mathematics
Covenant University, Ota, Ogun State, Nigeria
olasunmbo.agboola@cu.edu.ng

Abstract. *In this article, the dynamic behavior of inclined rectangular Mindlin plate under the influence of moving load along the mid-plate on the plate surface is considered. Finite difference method is used to solve the non-dimensionalised form of the resulting coupled partial differential equations. It was found that the response amplitude of the plate is affected significantly by the foundation moduli. Also, the effects of the shear deformation, rotatory inertia and angle of inclination of the plate are noticeable.*

Keywords: *Pasternak foundation, Inclined Mindlin plate, Shear deformation, Rotatory inertia, Moving load.*

1. INTRODUCTION

An inclined rectangular Mindlin plate is a plate set at an angle, not perpendicular to a horizontal plane. However, the work done is the same: Work = Force × Distance, and the distance is increased, whereas the force is decreased. (Civalek, 2005; Zhang and Zheng, 2010) In Elementary Physics, an object placed on a tilted surface (inclined plane) will often slide down the surface. The greater the tilt of the surface (i.e. the angle of inclination), the faster the rate at which the object will slide down it. (Gbadeyan and Dada, 2006) According to Newton's laws of motion, a moving load on an inclined plane will continue to slide down the plane if there is no applied force to balance the forces acting on it, especially if the surface is frictionless or with minimal friction. There are always, at least, two forces namely: the force of gravity and the normal force, acting upon the moving load positioned on an inclined plate (Khan Academy, 2014). The force of gravity acts in a downward direction, while the normal force acts in a direction perpendicular to the surface. (Civalek, 2005; Nguyan-Thoi *et al*, 2013) An inclined plane problem is in every way like any other net force problem with the sole exception that the surface has been tilted. An inclined plane therefore can be transformed into the form with which we are more comfortable, as illustrated in figure 2. After this transformation, we can ignore the force of gravity since it has been replaced by its two components. (Civalek, 2005; Gbadeyan and Dada, 2006). We can now solve for the net force and the acceleration. For a load mowing up the inclined plate, the applied force must be greater than the component of its weight (F_{11}) moving down the inclined plate, to avoid sliding down.

2. THE GOVERNING EQUATION

The set of dynamic equilibrium equations which governs the behavior of inclined Mindlin plate supported by Pasternak foundation, and traversed by a partially distributed moving load can be written as follows [Gbadeyan and Agarana, 2014]:

$$\begin{aligned} &Q_x - \frac{\partial M_x}{\partial x} - \frac{\partial M_{xy}}{\partial y} = \frac{\rho h^3}{12}\frac{\partial^2 \psi_x}{\partial T^2} \\ &\quad + \frac{\rho_L h_1^3}{12}\left[\frac{\partial^2 \psi_x}{\partial T^2} + u\frac{\partial^2 \psi_x}{\partial x \partial T} + \frac{u}{D(\upsilon^2 - 1)}\left\{\frac{\partial M_x}{\partial T} + u\frac{\partial M_x}{\partial x}\right\} - \frac{u\upsilon}{D(\upsilon^2 - 1)}\left\{\frac{\partial M_y}{\partial T} + u\frac{\partial M_y}{\partial y}\right\}\right]B \end{aligned} \tag{1}$$

$$Q_y - \frac{\partial M_{xy}}{\partial y} - \frac{\partial M_y}{\partial y} = \frac{\rho h^3}{12}\frac{\partial^2 \psi_x}{\partial T^2} + \frac{\rho_L h_1^3}{12}\left[\frac{\partial^2 \psi_y}{\partial T^2} + u\frac{\partial^2 \psi_x}{\partial x \partial T} + \frac{u}{D(\upsilon^2-1)}\left\{\frac{\partial M_x}{\partial T} + u\frac{\partial M_x}{\partial x}\right\} - \frac{u\upsilon}{D(\upsilon^2-1)}\left\{\frac{\partial M_x}{\partial T} + u\frac{\partial M_x}{\partial x}\right\}\right]B \tag{2}$$

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + kW + (M_f - \rho h)\frac{\partial \Delta_t}{\partial t} + \frac{M_L}{A}[g\cos\theta + \frac{\partial \Delta_t}{\partial t} + u\frac{\partial \Delta_t}{\partial t} + G_1\left(\frac{\partial \Delta_x}{\partial x} + \frac{\partial \Delta_y}{\partial y}\right) + u\left\{\frac{\partial \psi_x}{\partial t} + \frac{u}{D(\upsilon^2-1)}M_x - \frac{u\upsilon}{D(\upsilon^2-1)}M_y\right\} - \frac{u}{2Gh}\left\{\frac{\partial Q_x}{\partial t} + u\frac{\partial Q_x}{\partial x}\right\}B] = \rho h\frac{\partial^2 W}{\partial t^2} - M_L g\sin\theta \tag{3}$$

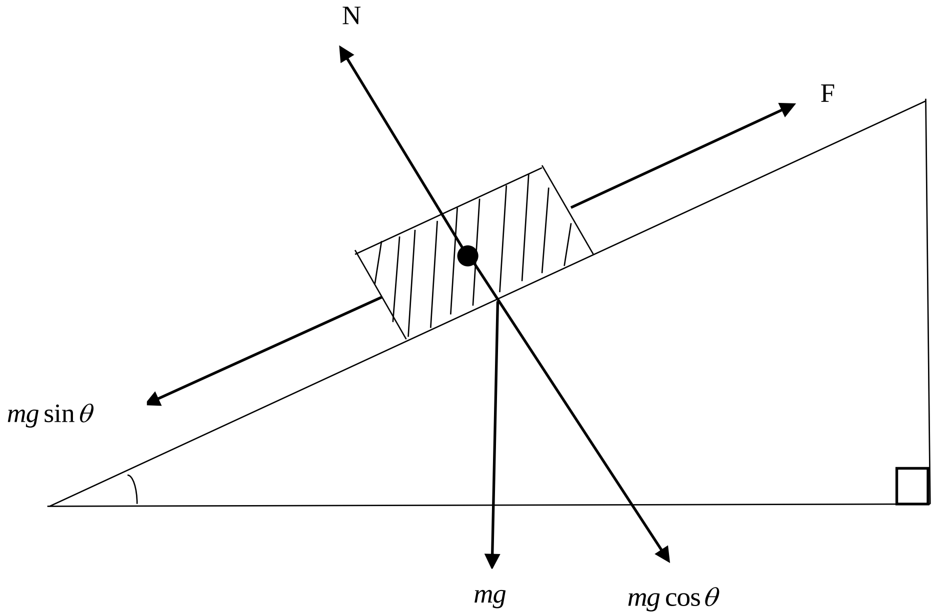

Figure 1. Diagram of moving load on an inclined plane

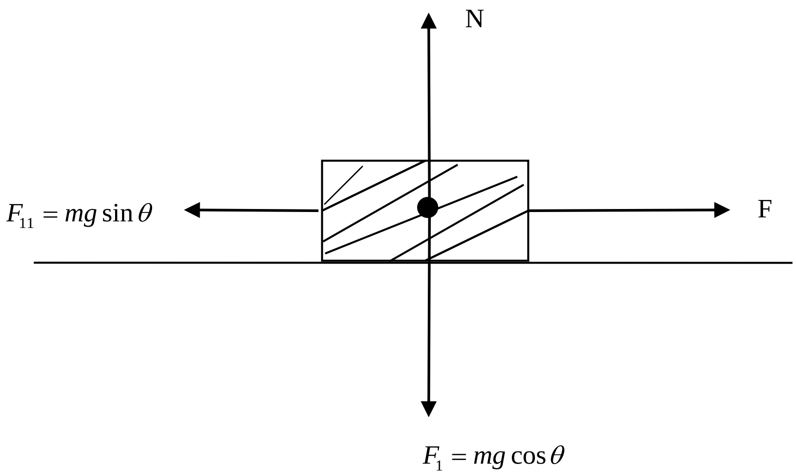

Figure 2. Diagram of a transformed inclined plane to a flat plane

$$M_x = -D\left(\frac{\partial \psi_x}{\partial x} + \upsilon \frac{\partial \psi_y}{\partial y}\right) \tag{4}$$

$$M_y = -D\left(\frac{\partial \psi_y}{\partial y} + \upsilon \frac{\partial \psi_x}{\partial x}\right) \tag{5}$$

$$M_{xy} = \frac{-D(1-\upsilon)}{2}\left(\frac{\partial \psi_x}{\partial y} + \upsilon \frac{\partial \psi_y}{\partial x}\right) \tag{6}$$

$$Q_x = -\kappa^2 Gh\left(\psi_x - \frac{\partial W}{\partial x}\right) \tag{7}$$

$$Q_y = -\kappa^2 Gh\left(\psi_y - \frac{\partial W}{\partial y}\right) \tag{8}$$

and

$$\frac{\partial W}{\partial t} = \Delta_t \tag{9}$$

$$\frac{\partial W}{\partial x} = \Delta_x \tag{10}$$

$$\frac{\partial W}{\partial y} = \Delta_y \tag{11}$$

where Eqs. (4 – 8) are the equations for bending moments, twisting moments and shear force, ψ_x and ψ_y are local rotations in the $x-$ and $y-$ directions respectively. h and h_1 are the thickness of the plate and load respectively, ρ and ρ_L are the densities of the plate and the load per unit volume respectively. $W(x,y,t)$ is the traverse displacement of the plate at time t, g is the acceleration due to gravity, θ is the angle of inclination of the plate, u is the velocity of the load (M_L) of rectangular dimension ξ by μ with one of its lines of symmetry moving along $Y = Y_1$, the plate is I_x by I_y in dimensions and $\xi = ut + \frac{\varepsilon}{2}$, $B = B_x B_y$ where

$$B_x = \begin{cases} 1 - H\left(x - \xi - \frac{\varepsilon}{2}\right), & 0 \le t \le \varepsilon/u \\ H\left(x - \xi + \frac{\varepsilon}{2}\right) - H\left(x - \xi - \frac{\varepsilon}{2}\right), & \frac{\varepsilon}{u} \le t \le \frac{L_x}{u} \\ H\left(x - \xi + \frac{\varepsilon}{2}\right), & \frac{L_x}{u} \le t \le \frac{(L_x + \varepsilon)}{u} \\ 0, & (L_x + \varepsilon)/u \le t \end{cases} \tag{12}$$

$$B_y = \left\{H\left(y - y_1 + \mu/2\right) - H\left(y - y_1 - \mu/2\right)\right\} \tag{13}$$

$$H(x) = \begin{cases} 1, & x > 0 \\ 0.5, & x = 0 \\ 0, & x < 0 \end{cases} \tag{14}$$

$H(x)$ is called Heaviside function.

G is the modulus of rigidity of the plate, D is the flexural rigidity of the plate defined by $D=\frac{1}{12}Eh^3(1-\upsilon^2)^{-1}=\frac{Gh^3}{6(1-\upsilon)}$ for isotropic plate, κ^2 is the shear correction factor and υ is the Poisson's ration of the plate.

Since the inertia effect of the load is considered, the uniform partially distributed applied load takes on the form [Gbadeyan and Dada, 2006]:

$$P(x,y,t)=\frac{-M_L}{A}\left[g\sin\theta+\frac{d^2W}{dt^2}\right]B-M_L g\sin\theta \tag{15}$$

the acceleration $\frac{d^2W}{dt^2}$ is defined as

$$\frac{d^2W}{dt^2}=\frac{\partial^2W}{\partial t^2}+2u\frac{\partial^2W}{\partial x\partial t}+u^2\frac{\partial^2W}{\partial x^2} \tag{16}$$

Similarly,

$$\frac{d^2\psi_x}{dt^2}=\frac{\partial^2\psi_x}{\partial t^2}+2u\frac{\partial^2\psi_x}{\partial x\partial t}+u^2\frac{\partial^2\psi_x}{\partial x^2} \tag{17}$$

and

$$\frac{d^2\psi_y}{dt^2}=\frac{\partial^2\psi_y}{\partial t^2}+2u\frac{\partial^2\psi_y}{\partial x\partial t}+u^2\frac{\partial^2\psi_y}{\partial x^2} \tag{18}$$

2.2. Initial Conditions

$$W(x,y,0)=\frac{\partial W}{\partial T}(x,y,0) \tag{19}$$

2.3. Boundary Conditions

$$\begin{aligned} &W(x,y,t)=M_x(x,y,t)=\psi_y(x,y,t)=0, \text{ for } x=0 \text{ and } x=a\\ &W(x,y,t)=M_x(x,y,t)=\psi_x(x,y,t)=0, \text{ for } y=0 \text{ and } y=b \end{aligned} \tag{20}$$

3. PROBLEM SOLUTION

The set of partial differential Eqs. (1) - (11)**,** are the partial differential equations to be solved for the following eleven dependent variables M_x, M_y, M_{xy}, Q_x, Q_y, ψ_{xt}, W, Δ_t, Δ_x and Δ_y.

A numerical procedure, finite difference method, can be used to solve the system of Eqs. (1) - (11). Rearranging them in matrix form results in

$$R_{i,j+1}S'_{i,j+1}+P_{i+1,j+1}=-T_{i,j}S'_{i,j}-Y_{i+1,j}S_{i+1,j}+Z_k \tag{21}$$

$$i=1,\ 2,\ 3\ldots N-1; \quad j\ =\ 1,\ 2,\ 3\ldots M-1$$

Where N and M are the number of the nodal points along x and y axes respectively, Z_k is a matrix representing the right hand side of Eqs. (12) – (22) defined by

$$Z_k = A_{i,j} S_{i,j} + P_{i,j+1} S^0_{i,j+1} + G_{i+1,j} S^0_{i+1,j} + D_{i+1,j+1} S^0_{i+1,j+1} + E_1 \tag{22}$$

Each term in Eqs. (21) and (22) is an 11 x 11 matrix

4. EFFECT OF ANGLE OF INCLINATION ON DEFLECTION OF THE INCLINED PLATE

For the purpose of this paper let $B = 0,$ which implies $B_x = 0$ and $L_x + \varepsilon/2 \le t$. Also, $M_f - \rho h = M_1$ (mass); and $0 \le \theta \le \pi/2$. Equation (3) becomes:

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + kW + M\,\frac{\partial \Delta_t}{\partial t} = \rho h \frac{\partial^2 W}{\partial t^2} - M_L g \sin\theta \tag{23}$$

$$\rho h \frac{\partial^2 W}{\partial t^2} - \left[\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + M\,\frac{\partial \Delta_t}{\partial t} \right] - kW = M_L g \sin\theta \tag{24}$$

since $\Delta_t = \dfrac{\partial W}{\partial t}$, $\dfrac{\partial \Delta_t}{\partial t} = \dfrac{\partial^2 W}{\partial t^2}$ and ρh is a mass.
Therefore, Eq. (24) becomes

$$-\left(\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} \right) - kW = M_L g \sin\theta \tag{25}$$

When $\theta = 0^\circ$,

$$W = -\frac{1}{k}\left(\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} \right) \tag{26}$$

When $\theta = 30^\circ$,

$$W = -\frac{M_L g}{2k} - \frac{1}{k}\left(\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} \right) \tag{27}$$

When $\theta = 60^\circ$,

$$W = -\frac{\sqrt{3} M_L g}{2k} - \frac{1}{k}\left(\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} \right) \tag{28}$$

When $\theta = 90^\circ \left(= \pi/2 \right)$,

$$W = -\frac{M_L g}{k} - \frac{1}{k}\left(\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} \right) \tag{29}$$

From Eq. (15), if $B = 0$, the applied load becomes

$$P(x, y, t) = -M_L g \sin\theta \tag{30}$$

When $\theta = 0^\circ$,

$$P = 0 \tag{31}$$

When $\theta = 30^\circ$,

$$P = -\frac{1}{2} M_L g \tag{32}$$

When $\theta = 60^\circ$,

$$P = -\frac{\sqrt{3}}{2} M_L g \tag{33}$$

When $\theta = 90^\circ$,

$$P = -M_L g \tag{34}$$

5. RESULTS DISCUSSION

The numerical calculations were carried out for a simply supported rectangular inclined plate resting on a Pasternak foundation and subject to a moving load. Damping effect was neglected.

In Fig. 3, the deflection of the plate for different values of K is presented. It is observed that the foundation stiffness have effect on the deflection of the plate. The highest value of the foundation stiffness K, produces the maximum deflection and the lowest value of stiffness produces the minimum deflection. In Fig. 4, the deflection of the plate for different values of G, is plotted as a function of time. Evidently, it can be noticed that the response amplitude of the plate continuously supported by a subgrade is less than that of the plate not resting on any elastic subgrade (i:e. K=0, G=0). It can also be seen that as K and G increase the response amplitude decreases. It is also observed that there is no clear cut difference between the deflection of non – Mindlin and rotatory plates. In other words, the effect of rotatory inertia is minimal when compared with the effect of shear deformation.

In Fig. 4, the deflection of the plate for different values of K and G, keeping the contact area, Arp, constant, is plotted as a function of time. Evidently, it can be noticed that the response amplitude of the plate continuously supported by a subgrade is less than that of the plate not resting on any elastic subgrade (i:e. K=0, G=0). It can also be seen that as K and G increase the response amplitude decreases. Deflection profiles of the Mindlin plate for various values of the contact area Arp (Arp=0.02, 0.125 and 0.5) are shown in Figs. 4, 5 and 6 respectively. In Fig. 4, the response curves of the plate is shown for K=0 and with the contact area Arp, as a parameter. The corresponding profiles for K=100 and K=200 are depicted in Figs. 5 and 6 respectively. It is found from these figures that as Arp increases, the response maximum amplitude increases for fixed values of K and G. For various values of the foundation reaction modulus K, the deflection of the plate for the various values of the subgrade's shear modulus G (i.e G=0, G=0.09 and G=0.9), considered were calculated and are plotted in Figs. 7, 8 and 9 as function of time. Specifically in figure 7, the deflection profile of the Mindlin plate is depicted for K=0 and with the subgrade's shear modulus G as a parameter, The corresponding curves for K=100 and 200 are shown in Figs. 8 and 9 respectively. Clearly, from the figures, the response maximum amplitude decreases with an increase in the value of G for fixed values of K, Arp and Up.

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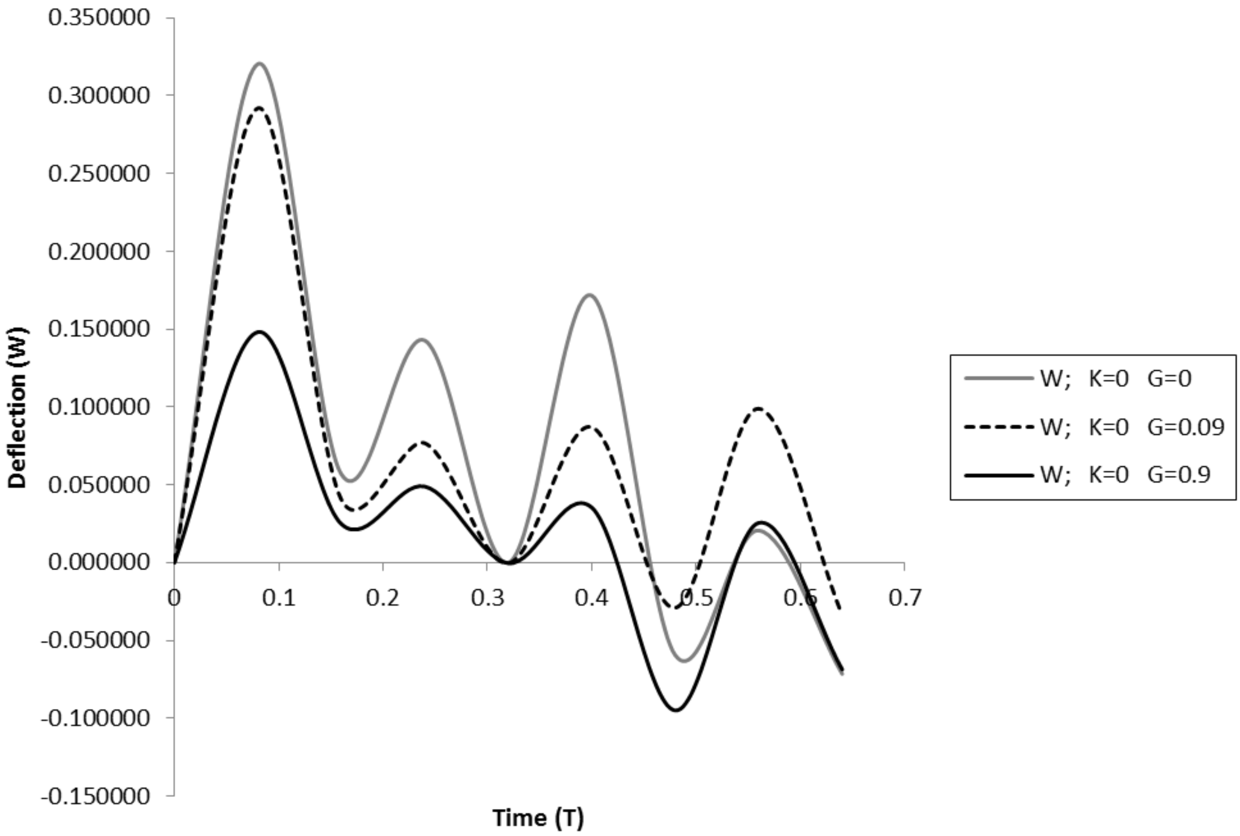

Figure 3. Deflection of plate at various foundation modulus and different times

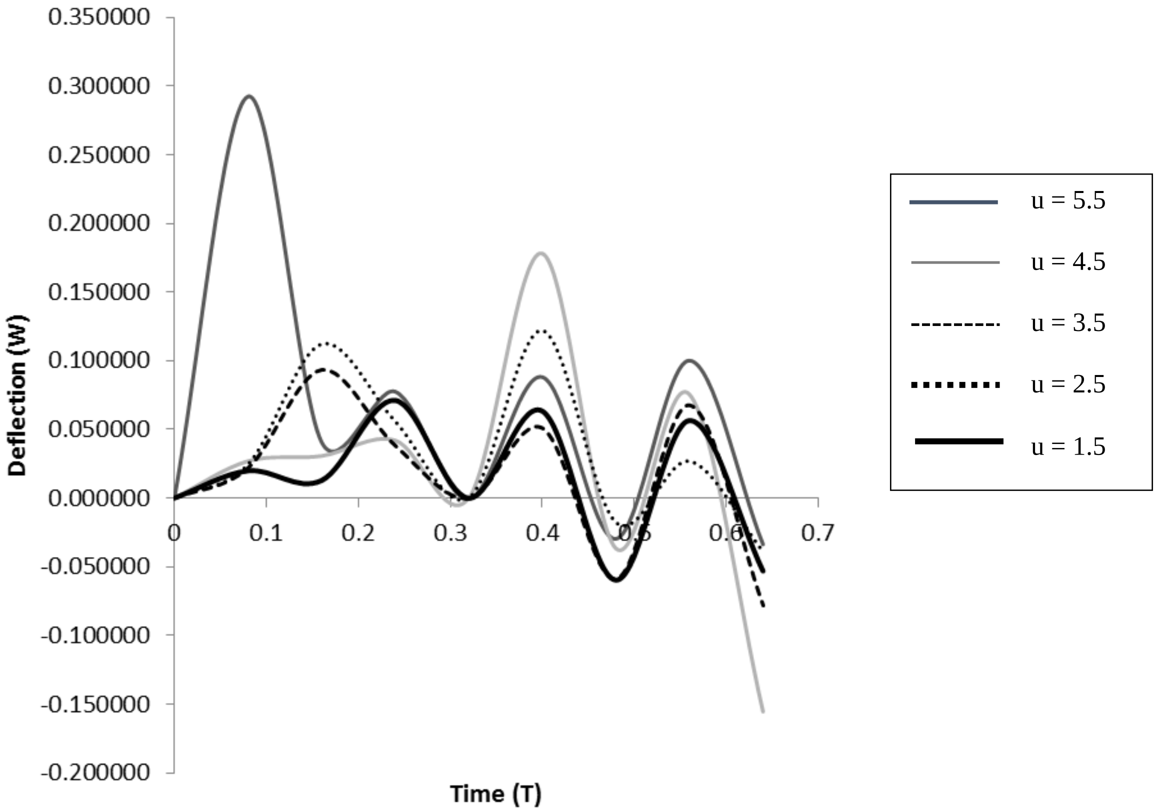

Figure 5. Deflection of the plate at various values of velocity and different times

6. CONCLUSION

The dynamic behaviour of a Mindlin plate carrying a uniform partially distributed moving load, supported by a Pasternak foundation, has been analysed. The non-dimensionalized equations of motion were transformed into equivalent finite difference ones, and then solved. Results have been have been presented not only for the deflection but also for the velocity, bending and twisting moments, shearing force for all instants of time and at selected space nodes. Hence all the components composing the dynamic response of the system have been obtained. The formulation for the Kirchoff plate is deduced by neglecting both effects of rotatory inertia and shear deformation. A numerical example of simply supported rectangular plate is presented. It is shown that the elastic subgrade, on which the Mindlin plate rests has a significant effect on the dynamic response of the plate to a partially distributed load. The effect of rotatory inertia and shear deformation on the dynamic response of the Mindlin plate to the moving load give a more realistic results for practical application, especially when such plate is considered to rest on a foundation.

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