 ISSN:2449-075X

Graph-Theoretical Approach for Solving Loss Allocation Problems in Interconnected Power Grids

A. S. Alayande†, C. O. A. Awosope†† and Ademola Abdulkareem††

†Department of Electrical Engineering, Faculty of Engineering and the Built Environment,

Tshwane University of Technology, Private Bag X680, Pretoria 0001, Staatsartillerie Road, Pretoria South Africa

††Department of Electrical & Information Engineering, College of Engineering, Covenant University, Canaanland, Ota, Nigeria

alayandeas@tut.ac.za,coaawosope@yahoo.co.uk, adekaree@gmail.com

***Abstract*— A Fair and transparent allocation of transmission loss to network generators and loads has been a major challenge for efficient transmission pricing in the open access environment. In this paper, the application of graph theory in solving loss allocation problems in highly interconnected and large-sized practical power networks is investigated. The relevant mathematical formulations for solving such problems in power networks are presented. A General Allocation Factor (GAF) matrix, which relies on Kirchoff's laws, for any given power network is formulated. The transmission line loss is obtained, based on the ac power-flow solution, using Newton Raphson iterative technique. The transmission line losses that are obtained are allocated to the generators within the network according to GAF matrix. The efficiency of the approach is demonstrated using a s standard IEEE 5-bus system and tested on a practical Nigerian 28-bus network. The results obtained, from the simulations, show the effectiveness of the method in solving loss allocation problems in real-time power networks.**

***Index Terms*—** Generator Contribution, Transmission Pricing, Graph Theory, General Allocation Factor Matrix, Kirchoff's laws, Loss Allocation**.**

1. INTRODUCTION

The total transmission loss within power systems accounts for up to ten percent of the total generation within the systems which amounts to millions of dollars every year [1]. The allocation of transmission losses to the network participants in a fair and transparent manner is therefore a major concern to power system regulators, operators, researchers and engineers in recent times. This is necessary because efficient allocation of resources within power networks demands a fair and transparent transmission pricing. Furthermore, the solution to such a problem becomes more tedious due to the nonlinear nature of the problem formulation most especially in large-sized practical power networks. In solving this problem, it is necessary to determine the contribution of each generator and load to the network losses. Different approaches have been proposed and documented in solving the problem. Unfortunately, there is no unique method of solution to the problem [2]. Moreover, the existing approaches to the problem and their applications to large-scale practical power networks are yet to be demonstrated holistically. As proposed in [3], the network losses are usually allocated to the generators and loads using a well-known incremental transmission loss coefficient. The limitation of this approach is hinged on the fact that it is solely dependent on the slack bus within the network and no loss is assigned to the slack bus. Application of proportional sharing method with graph theoretical approach and its application is demonstrated in [2]. Although, this approach gives promising results, it assumes that power at nodal inflows is shared proportionally between nodal outflows. In other words, this approach does not present a justifiable relationship between the power flow tracing and the electrical behaviour of the network. In [1], a Z-bus approach for loss allocation based on the power-flow solution is presented. The formulation of the approach is based on complex network impedance and the nodal injections [3]. In [4], game theoretic approach is proposed, which is found to be an acceptable and independent solution tool that satisfies the individual network players. However, it is computationally demanding as it requires handling a large amount of data for the solution of a single case in practical power systems. In [5], optimization approach is considered with loading conditions of the system. However, the time required to obtain the results may be of significant value. This may take longer time in large practical networks.

An attempt is therefore made in this paper to investigate the viability of graph theoretical-based approach, based on a solved power-flow analysis, in solving network loss allocation problem in large practical power networks. Section 2 presents the relevant mathematical formulations of the graph theoretical-based approach to the solution of transmission loss allocation problem. Section 3 gives the description of the standard IEEE 5-bus network as well as the Nigerian practical power system studied. The numerical illustrations of the approach are also presented in this section. The results and discussion of results are presented in section 4 while section 5 concludes the study.

2. LOSS ALLOCATION BASED ON GRAPH THEORY APPROACH

The mathematical formulation of the graph theoretical-based solution to transmission loss allocation problems within power systems is network dependent. This makes it to be much easier for the tracing of the power flow within the network. It is, therefore, possible to determine the participation of each network participant to the transmission losses based on the power-flow solutions whose detail has been well documented in the literature. The application of graph theory approach [4] to large practical power networksis revisited in this paper.

A power system can be considered as a complex network consisting of several nodes and links. The Kirchoff's matrix for such a network graph G can be defined by

$$K_{ij}(G) = \begin{cases} d^{-}(v_i) & for\ i = j \\ -a_{ij} & for\ i \neq j \end{cases} \quad (1)$$

where $d^{-}(v_i)$ is the in-degree of bus i

a_{ij} is the $(i, j)^{th}$ element in the adjacency matrix A, which is defined as

$$A_{ij}(G) = \begin{cases} 1 & if\ buses\ i\ and\ j\ are\ connected\ and\ bus\ i\ is\ directed\ towards\ bus\ j \\ 0 & otherwise \end{cases} \quad (2)$$

In this paper, a GAF matrix of a network graph G whose elements are determined based on the adjacency matrix given in equation (1), is defined as

$$GAF_{ij}(G) = \begin{cases} -P_{ij} & for\ i \neq j\ and\ P_{ij} > 0 \\ P_{Ti} & for\ i = j \\ 0 & otherwise \end{cases} \quad (3)$$

where

$$P_{Ti} = P_{gi} + \sum_{k=1}^{n} P_{ki}$$

$$\text{for } P_{ij} > 0,\ j = 1,2,\ldots,n \quad (4)$$

Close observation of equation (4) shows the following characteristics about GAF matrix:

i. it is a square and invertible matrix.
ii. the active power generated at bus k equals the algebraic sum of the kth column elements of GAF matrix.
iii. the active power demanded at bus k equals the algebraic sum of the kth row elements of GAF matrix.

Based on the above stated characteristics of the GAF matrix, we can write

$$GAF^{T} \times U = P_G \quad (5)$$

$$GAF \times U = P_L \quad (6)$$

where

U is a unit vector

P_G is the vector of generator outputs and

P_L is the vector of the network loads.

The individual load on the system is given by

$$P_L = P_{LL} U \quad (7)$$

Using (5) in (7) gives

$$P_L = DP_G \quad (8)$$

where D is the allocation factor matrix given by

$$D = P_{LL}(GAF)^{-1} \quad (9)$$

where

$$P_{LL} = diag(P_{L1}, P_{L2}, P_{L3}, \ldots, P_{Ln}) \quad (10)$$

The allocation of the transmission losses to the network participants, as determined from the power-flow solution, can therefore be obtained from the matrix D given in (9). In other words, matrix D gives the contribution of the generators and loads to the losses across the transmission links within the network.

The allocation of transmission line losses associated with the branch connecting any two buses j and k to any network participant whose location is at any bus i, is given by

$$P_{Li\to j-k} = D_{ij} \times P_{j\to k} \quad (11)$$

where $P_{Li\to j-k}$ is the loss allocated to load bus i by the line connecting buses j and k and $P_{j\to k}$ is the active power loss on the line connecting buses j and k.

Therefore, the total loss allocated to any network participant located at bus i equals the sum of all the losses allocated to that participant by every link within the network.

3. NUMERICAL ILLUSTRATION

Two numerical examples are considered as case studies in this paper; a simple standard IEEE 5-bus system and a practical 28-bus Nigerian power network.

4. RESULTS AND DISCUSSION

Case Study 1: Standard IEEE 5-Bus Network

The standard IEEE 5-bus system considered consists of two generators placed at buses 1 and 2 while buses 3, 4, and 5 are load buses. Table 1 presents the line data for the standard IEEE 5-bus system.

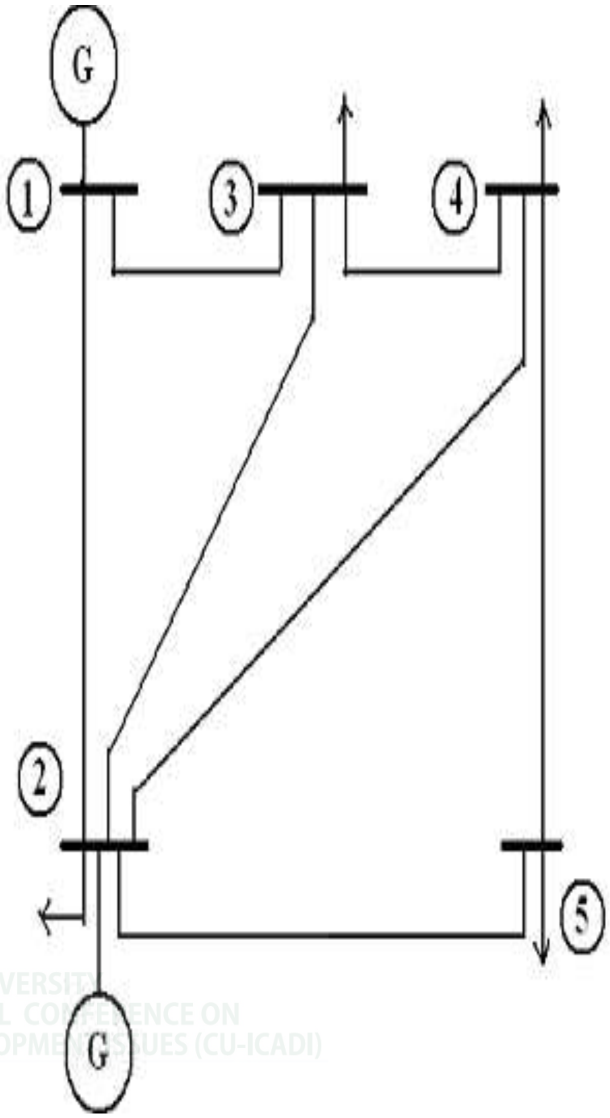

Figure 1: One-line diagram for the standard IEEE 5-bus network

Table 1: Transmission Line Data for the standard IEEE 5-bus network

Line	Resistance (Per Unit)	Reactance (Per Unit)	Suceptance (Per Unit)
1-2	0.0200	0.0600	0.0300
1-3	0.0800	0. 2400	0.0250
2-3	0.0600	0.1800	0.0200
2-4	0.0600	0.1800	0.0200
2-5	0.0400	0.1200	0.0150
3-4	0.0100	0.0300	0.0100
4-5	0.0800	0.2400	0.0250

Table 2 presents the converged power-flow results for the standard IEEE 5-bus network obtained after three iterations using Newton-Raphson iterative method. The simulation results obtained when the approach is applied are presented in tables 3 and 4. Tables 3 presents the results obtained for the General Allocation Factor (GAF) Matrix for the network under consideration. The results of the GAF matrix presented in table 3 are determined based on the structural interconnections of the buses and the admittances of individual lines within the network.

Table 2: Power-flow Solution for the standard IEEE 5-bus network

Bus	Voltage Magnitude	Voltage angle	Generator		Load	
	Per Unit	Radians	MW	MVAR	MW	MVAR
1	1.0600	0.000	0.578	0.500	0.000	0.000
2	1.0300	-0.0073	0.908	7.666	0.000	0.000
3	1.0110	-0.0576	0.000	0.000	0.450	0.149
4	1.0093	-0.0613	0.000	0.000	0.400	0.050
5	1.0010	-0.0704	0.000	0.000	0.600	0.100

Tables 4 gives the results obtained for the transmission line loss allocation to the generators G1 and G2 within the network under study.

From the 5-by-5 GAF matrix results given in table 3, it can be seen that the sum of each column equals 1. This indicates that the contribution of all the generators to each load is 100%, which is logically true. Therefore, structurally, it can be inferred that 100 percent of the L1 will be supplied by G1. In a similar manner, 23.5% of L2 will be supplied by G1 and 76.5% will be supplied by G2.

Table 3: General Allocation Factor (GAF) Matrix for the standard IEEE 5-bus network

		Load Bus				
		L1	L2	L3	L4	L5
Generators Bus	G1	1.0000	0.2350	0.6188	0.3468	0.2431
	G2	0.0000	0.7650	0.3812	0.6532	0.7569
	None	0.0000	0.0000	0.0000	0.0000	0.0000
	None	0.0000	0.0000	0.0000	0.0000	0.0000
	None	0.0000	0.0000	0.0000	0.0000	0.0000

However, considering the data for the case under study, the network loads are on buses 3, 4 and 5 and no load is attached to buses 1 and 2. Hence, it implies that G1 contributes 61.88%, 34.68% and 24.31% of its output power to loads at buses 3, 4 and 5 respectively while G2 contributes 38.12%, 65.32% and 75.69% of its total power to loads connected at buses 3, 4 and 5 respectively.

From the results presented in table 4, it can be seen that the highest transmission line loss of 1.6 MW is observed on the line 2-5 out of which the least power loss of 0.3761 MW is allocate to G1 and the remaining 1.2239 MW is allocated to G2.

Table 4: Line loss Allocation for the standard IEEE 5-bus network

Line	Total Line Loss from power-flow solution	Allocation of Transmission Line Loss to Generators (MW)		Total Line loss Allocated to Generators (MW)
	(MW)	G1	G2	G1 + G2
1-2	0.8000	0.8000	0.0000	0.8000
1-3	1.4000	1.4000	0.0000	1.4000
2-3	0.5000	0.1175	0.3825	0.5000
2-4	0.6000	0.1410	0.4590	0.6000
2-5	1.6000	0.3761	1.2239	1.6000
3-4	0.0000	0.0000	0.0000	0.0000
4-5	0.0000	0.0000	0.0000	0.0000

Furthermore, higher portion of the losses within the network is allocated to G1 with 2.8346 MW out of which the highest part with 1.4 MW is allocated to line 1-4. In a similar manner, 2.0654 MW of the total network losses is supplied by G2 out of which 1.2239 MW is allocated to line 2-5.

Case Study 2: Nigerian 28-Bus Network

A practical Nigerian 28-bus network, whose structure consists of 10 generator buses and 18 load buses. The bus codes LG1 to LG10 in the first column of table 5 represent the network generator buses while the bus codes L11 to L28 represent the network load buses.

The power-flow solution for the practical Nigerian 28-bus network, using Newton-Raphson method, converges after five iterations. The converged power-flow results obtained for the network are also presented in table 5.

As can be observed for the case of the standard IEEE 5-bus network, the base matrix is the GAF matrix. In the same manner, for the Nigerian 28-bus network, the GAF matrix is determined which also serves as the basis for the network under consideration. The network loss obtained from the power-flow solution is then allocated to the network generators. Based on GAF matrix for the Nigerian 28-bus system, the simulation results obtained for the allocation of transmission line loss to the network generators are presented in table 6 presents.

It can be seen from table 6 that the transmission line connecting Okpai and Calabar, has the highest power loss of 41.7230MW allocated to it and it is ranked number 1. The transmission line connecting Benin and Onitsha has the least part of the loss of 0.040MW allocated to it and it is ranked number 31. It can be observed that the burden of the cost for the loss allocated to Okpai-Calabar will be borne on the Okpai GS based on table 6. In a similar manner, cost associated with the transmission loss allocated to Benin-Onitsha will be responsible for by the generators G2 (Delta) with 0.0237MW and G4 (Sapele) with 0.0163 MW respectively.

The participation of the generators in the apportionment of total transmission line loss is shown graphically in figure 2.

Table 5: Power-flow Solution for the Nigerian 28-bus network

Bus Code	Bus Name	Voltage Magnitude	Voltage angle
		Per Unit	Radians
LG1	**Egbin**	1.050	0.00
LG2	**Delta**	1.030	21.341
LG3	**Okpai**	1.040	59.306
LG4	**Sapele**	1.020	17.894
LG5	**AES**	1.030	8.003
LG6	**Afam**	1.040	24.412
LG7	**Calabar**	1.020	35.045
LG8	**Shiroro**	1.030	23.034
LG9	**Kainji**	1.050	37.875
LG10	**Jebba**	1.040	32.103
L11	**Ajah**	1.044	-0.599
L12	**Akangba**	1.003	2.254
L13	**Ikj-West**	1.006	3.227
L14	**Ajaokuta**	1.030	14.317
L15	**Aladja**	1.022	19.939
L16	**Benin**	1.014	14.881
L17	**Jebba**	1.038	31.583
L18	**Ayede**	1.001	5.264
L19	**Jos**	1.027	25.366
L20	**Kaduna**	1.024	19.304
L21	**Osogbo**	1.007	12.406
L22	**Kano**	1.001	10.823
L23	**Alaoji**	1.019	23.034
L24	**New Haven**	1.004	11.547
L25	**Onitsha**	1.010	14.294
L26	**Katampe**	1.009	18.540
L27	**Birnin Kebbi**	1.010	32.356
L28	**Gombe**	1.027	18.905

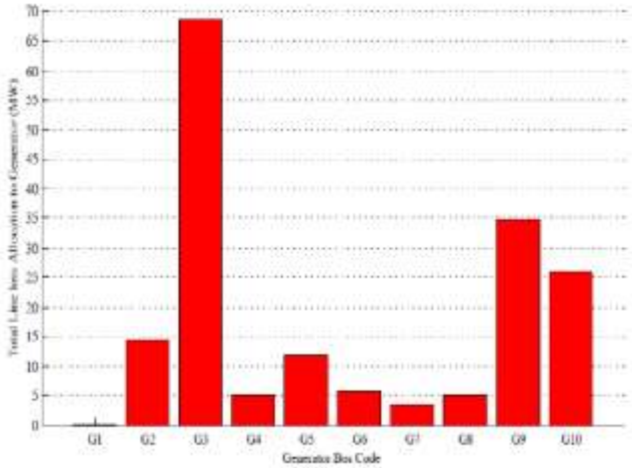

Figure 2: Allocation of Transmission line loss to generators for the Nigerian 28-bus network

It can be seen that the highest percentage of the total loss contributed by all the lines within the network is allocated to generator G3, which corresponds to Okpai with a loss of 68.6367 MW. In a similar way, the least percentage of the total loss is apportioned to generator G1 (Egbin) with a loss allocation of 0.0633MW. In other words, based on this analysis, it is much easier to determine the cost allocation of the network losses to the generators.

Table 6: Transmission line loss allocation to network generators based on GAF matrix for the Nigerian 28-bus system

Bus-Bus	Line Loss Allocation (MW)										Line Loss (MW)	Loss Ranking
	G1	G2	G3	G4	G5	G6	G7	G8	G9	G10		
Egbin GS– Ajah	0.0633	0.0505	0.0000	0.0347	0.2357	0.0000	0.0000	0.0000	0.0441	0.0436	0.4720	26
Ikj-West - Akangba	0.0000	0.1025	0.0000	0.0704	0.4782	0.0000	0.0000	0.0000	0.0895	0.0884	0.8290	25
Ikj-West - Egbin GS	0.0000	0.4773	0.0000	0.3281	2.2279	0.0000	0.0000	0.0000	0.4171	0.4116	3.8620	14
Benin - Ikj-West	0.0000	3.9348	0.0000	2.7052	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	6.6400	8
Ayede - Ikj-West	0.0000	0.0182	0.0000	0.0125	0.0000	0.0000	0.0000	0.0000	0.1954	0.1929	0.4190	28
Osogbo - Ikj-West	0.0000	0.1972	0.0000	0.1356	0.0000	0.0000	0.0000	0.0000	2.1205	2.0927	4.5460	11
Benin - Ajaokuta	0.0000	0.0480	0.0000	0.0330	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0810	30
Delta GS – Benin	0.0000	5.6810	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	5.6810	10
Delta GS – Aladja	0.0000	1.0230	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0230	23
Aladja – Sapele GS	0.0000	0.9470	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.9470	24
Benin – Onitsha	0.0000	0.0237	0.0000	0.0163	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0400**	**31**
Benin - Osogbo	0.0000	0.2027	0.0000	0.1393	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.3420	29
Sapele GS – Benin	0.0000	1.3268	0.0000	1.2702	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	2.5970	16

Osogbo - Ayede	0.0000	0.2601	0.0000	0.1788	0.0000	0.0000	0.0000	0.0000	2.7959	2.7593	5.9940	9
Kainji GS - B' Kebbi	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.6370	0.0000	1.6370	21
Jebba - Osogbo	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	16.6087	16.3913	33.0000	2
Afam GS – Alaoji	0.0000	0.0000	0.0000	0.0000	0.0000	1.9120	0.0000	0.0000	0.0000	0.0000	1.9120	20
Alaoji – Onitsha	0.0000	0.0000	3.1158	0.0000	0.0000	3.2977	0.4154	0.0000	0.0000	0.0000	6.8290	7
Onitsha - N' Haven	0.0000	0.0486	0.4791	0.0334	0.0000	0.5071	0.0639	0.0000	0.0000	0.0000	1.1320	22
Jos - Gombe	0.0000	0.0000	1.7426	0.0000	0.0000	0.0000	0.2324	0.0000	0.0000	0.0000	1.9750	19
Jebba GS – Jebba	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.4570	0.4570	27
Jebba – Shiroro	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	2.1757	2.1473	4.3230	13
Kainji GS – Jebba	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	7.3060	0.0000	7.3060	6
Jos – Kaduna	0.0000	0.0000	2.2297	0.0000	0.0000	0.0000	0.2973	0.0000	0.0000	0.0000	2.5270	17
Kaduna – Kano	0.0000	0.0000	1.6597	0.0000	0.0000	0.0000	0.2213	1.5985	0.4558	0.4498	4.3850	12
Shiroro GS - Kaduna	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.3316	0.3797	0.3747	2.0860	18
Shiroro GS–Katampe	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	2.0223	0.5766	0.5691	3.1680	15
Calabar GS - Alaoji	0.0000	0.0000	9.8956	0.0000	0.0000	0.0000	1.3194	0.0000	0.0000	0.0000	11.2150	3
Calabar GS – Jos	0.0000	0.0000	7.7912	0.0000	0.0000	0.0000	1.0388	0.0000	0.0000	0.0000	8.8300	4
Okpai GS – Calabar GS	0.0000	0.0000	41.7230	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	**41.7230**	**1**
AES GS - Ikj-West	0.0000	0.0000	0.0000	0.0000	8.7740	0.0000	0.0000	0.0000	0.0000	0.0000	8.7740	5
Total	0.0633	14.3413	68.6367	4.9576	11.7159	5.7168	3.5885	4.9524	34.8019	25.9775		

5. CONCLUSION

In this paper, the application of a graph theoretical-based approach for solving loss allocation problem has been presented. The relevant mathematical formulations based on the traditional power-flow equations are presented. Application of this approach to both the standard IEEE 5-bus network and a practical network of Nigerian 28-bus system is investigated. The simulation results obtained with MATLAB environment as the simulation tool show the strength of the method in handling loss allocation problem among power network participants. The method is simple for solving loss allocation problem within interconnected power grids. This method could, therefore, serve as a good price signal for a reasonable cost allocation and transmission loss pricing within practical power systems in deregulated electricity markets.

References

[1] A. J. Conejo, F. D. Galiana, and I. Kockar, "Z-Bus Loss Allocation" *IEEE Trans. On Power Systems*, Vol. 16, no. 1, February 2001.

[2] V.S.C. Lim, T.K. Saha, J.D.F. McDonald, Comparative effectiveness of loss allocation methods for providing signals to affect market operation, Australian Journal of Electrical and Electronics Engineering, vol. 3, issue 2, 2007.

[3] A. J. Conejo, J. M. Arroyo, N. Alguaci and A. L. Guijarro, "Transmission Loss Allocation: A Comparison of Different Practical Algorithms" IEEE Transactions on Power Systems, vol. 17, no. 3, pp. 571-576, August 2002.

[4] B. Khan andG. Agnihotri, "Optimal Transmission Network Usage and Loss Allocation Using Matrices Methodology and Cooperative Game Theory," International Journal of Eelectrical, Computer, Energetic, Electronic and Communication Engineering, Vol.7, No 2, pp 196-204, 2013.

[5] A.R. Abhyankar, S. A. Soman and S.A. Khaparde, "Optimization Approach to Real Power Tracing:Application to Transmission Fixed Cost Allocation" *IEEE Trans. On Power Systems*, Vol. 21, no. 3, August 2006.