Declaration

I, Oghonyon, Jimevwo Godwin, (Matric. Number: CUGP070206) declare that this research was carried out by me under the supervision of Prof. Solomon A. Okunuga of the Department of Mathematics, University of Lagos, Lagos and Dr. Nicholas A. Omoregbe of the Department of Computer and Information Sciences, Covenant University, Ota, Ogun State. I attest that the thesis has not been presented either wholly or partly for the award of any degree elsewhere. All sources of data and schorlarly information used in this thesis are duly acknowledged.

Signature.....

Date.....

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Certification

This is to certify that this research was carried out by **Mr Oghonyon, Jimevwo Godwin,(CUGP070206)**, in the Department of Mathematics, School of Natural and Applied Sciences, College of Science and Technology, Covenant University, Ota, Ogun State, Nigeria.

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Dedication

To my lovely wife, Mrs Oghonyon Blessing, my son , Omemena Divine Oghonyon, my daughter, Ahoghome Marvelous Oghonyon and the entire Oghonyon's family.

Acknowledgement

Thanks and so much thanks to God the Father, Jesus Christ the Son and the Holy Spirit for all insight and inspiration throughout this research work. With Him this research work has become a reality.

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Abstract

The numerical solutions of general third order initial value problems of ordinary differential equations have been studied in this research work. A new class of block multistep methods capable of solving general third order IVPs of ODEs using variable step size technique have been developed. Collocation and interpolation of power series as the approximate solution is adopted. The block multistep method was intensified by the introduction of continuous scheme in order to circumvent the limitation created by reducing to systems of first order ODEs. The new class of variable step-size method has the advantage to control and minimize error, determine and vary the step size as well as decide the prescribed tolerance level to ascertain the maximum errors. Some theoretical properties of the block multistep methods such as order of the scheme, zero stability, consistency and determination of the region of absolute stability of the scheme have been conducted and presented. Numerical examples on nonstiff IVPs have been used to test the performance of the methods, in addition, comparing the maximum error as the prescribed tolerance parameter level is reduced in the method. The newly developed methods have been written as mathematical program and expressed in form of mathematical language which can run simultaneously when implemented. The newly formulated variable step-size block multistep methods perform better when compared with other existing methods as the prescribed tolerance parameter level got smaller and smaller.

Furthermore, the newly developed methods possess the attribute to control and decide on the estimate of the actual step size that will guarantee an improved results with better maximum errors. This, in particular, is seen as an advantage of the variable step size method over other existing methods approximated with fixed step size. Finally, the idea of predictor-corrector methods used by various researchers to predict and correct estimates has been extended in the newly proposed method to change/decide on suitable step size, determine the prescribed tolerance level and error control/minimization.

Contents

List of Tables

List of Figures

List of Abbreviations

TOL - Tolerance

PTL - Prescribed Tolerance Level

MTD - Method Employed

b - End of Interval

TS - Total Steps Taken

$MAXE$ - Magnitude of the Maximum Errors of the Computed Solution

FCN - Total Function Calls

$TLO5$ - Tolerance Level of Order 5

$TLO6$ - Tolerance Level of Order 6

$TLO7$ - Tolerance Level of Order 7

ES - Exact Solutions

$CSO5$ - Computed Solutions of Order 5

$CSO6$ - Computed Solutions of Order 6

$CSO7$ - Computed Solutions of Order 7

$MEO5$ - Maximum Errors of Order 5

$MEO6$ - Maximum Errors of Order 6

$MEO7$ - Maximum Errors of Order 7

$B1^{st}SITN$ - Beginning of the First Sets of Iteration

$E1^{st}SITN$ - End of the First Sets of Iteration

$B2^{nd}SITN$ - Beginning of the Second Sets of Iteration

$E2^{nd}SITN$ - End of the Second Sets of Iteration

$B3^{rd}SITN$ - Beginning of the Third Sets of Iteration

$E3^{rd}SITN$ - End of the Third Sets of Iteration

54VS - Implementation of Five-Step Explicit and Four-Step Implicit Block Multistep Methods Derived Earlier Using Variable Step Size method.

65VS - Implementation of Six-Step Explicit and Five-Step Implicit Block Multistep Methods Derived Earlier Using Variable Step Size Method.

76VS - Implementation of Seven-Step Explicit and Six-Step Implicit Block Multistep Methods Derived Earlier Using Variable Step Size method.

Olabode and Yusuph (2009) - Numerical Results in Olabode and Yusuph (2009)

Zanariah *et al.* (2012) - Numerical Results in Zanariah *et al.* (2012)

Awoyemi *et al.* (2014) - Numerical Resutls in Awoyemi *et al.* (2014)

New Results (2015) - Numerical Results in Oghonyon (2015)

List of Appendices