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On Response of Elastic Isotropic Damped Shear Highway Bridge Supported by Sub-grade to Uniform Partially Distributed MovingVehicle

[1*]Agarana M.C, [2]Gbadeyan J.A, [3]Emetere M.E.

[1]Department of Mathematics, Covenant University, Ota, Ogun State, Nigeria.
[2]Mathematics Department, University of Ilorin, Ilorin, Kwara State, Nigeria.
[3]Department of Computer Science and Management Information System, Covenant University, Ota, Ogun State, Nigeria.

Abstract
Plates and Plate-like structures resting on elastic foundation, such as Winkler or Pasternak foundation, have wide application in modern Engineering practices. Areas of application include vibration of railway and highway bridges, as well as effects of moving loads on highway pavements. In this paper we attempt to analyse the effect of shear deformation, damping and Winkler foundationon the vibration of a typical highway bridgethat is elastic and rectangular, as a result of moving vehicle on it. The moving vehicle (load) is assumed to be uniform partially distributed, while the Bridge (plate) is isotropic. We used MATLAB, in conjunction with a computer program developed, to solve the resulting set of algebraic system of equations obtained from the conversion of the coupled partial differential governing equations viafinite difference method. The desired solutions are obtained. The results are consistent with the ones in literature. In particular, deflection is lower with shear deformation when compared with results in literature where the effect of shear deformation was neglected. Also, there is a significant effect of the damping and foundation on deflection of the bridge, as these two factors reduce the vibration of the bridge under a moving vehicle.

Keywords: Damped Shear Highway Bridge, Sub-grade, Uniform Partially distributed moving vehicle

Introduction
The solution of problem of carrying out an analysis of plate resting on elastic foundation, which is often encountered in the analysis of bridges and other geotechnical structures, demands modelling of the mechanical behaviour of the bridge[3,11,15]. Alsothe mechanical behaviour of the soil as elastic subgrade,and the form of interaction between the bridge and the soil are very important in the analysis[6,11]. Several foundation models have been reported in the literature [7,16] and investigations on the static deflection, the dynamic response and the dynamic stability of plates on elastic foundations have been carried out [1,15,16]. Many researchersuse the Winkler model for soil structure interaction in the static and dynamic analysis of plates resting on elastic foundations where the vertical surface displacement of the plate is assumed to be proportional at any point to the contact pressure at that point [2,4,5,6]. In Winkler model, it is assumed that the foundation soil consists of linear elastic springs which are closely spaced and independent of each other [1].The importance of moving load problem manifested in several fields of applied mathematics, Engineering, applied physics and transportation.[1,3,8,11,12,] Bridges, guideways, overhead cranes, cableways, rails, roadways, runways, tunnels and pipelines are examples of structural elements.[1,3,6,] They support moving masses. Various kinds of problems associated with moving loads, especially the medium on which they move, have attracted the attention of researchers since 1897. Recently, developments and results can be found in state-of-the-art reviews [2,4,6,14] The dynamic behaviour[5,7,] of an elastic shear highway bridgeresting on a Winkler foundation and traversed by uniform partially distributed moving load is consideredin this paper. The elastic bridge is the shear's rectangular bridge. The effect of rotatory inertia is neglected. The study of such problem is of practical importance because of its applications in Engineering and other fields.[13,16] For example, this study is relevant when considering the reliability, safety and performance of modern highway bridges over which loads, like vehicles,move. In this analysis, both damping and the gravitational effects of the moving load are taken into account. The effect of shear deformation on the response of the damped bridge resting on a Winkler foundation under the action of partially distributed moving vehicle is investigated. The bridge is modelled as a plate while the moving vehicle is modelled as a normal moving load [1,9.10].Numerical discussions of the deflections of the bridge on Winkler foundation of various stiffness are also presented.

Formulation of Problem
A highway bridge, modelled as a rectangular plate, with a moving small vehicle (moving load) and different boundary conditions is considered. The load is relatively small, so its inertia can be neglected, and is moving along the mid-space on the surface of the bridge, supported by a Winkler foundation [1,2,6].

Assumption:
The following assumptions are made[2,16]:
- The plate is of constant cross-section
- The moving load moves with a constant speed
- The moving load is guided in such a way that it keeps contact with theplate throughout the motion
- The plate is continuously supported by a Winkler foundation
- The moving load is a partially distributed moving load
- The rectangular shear plate is elastic.

Initial Conditions

W (x, y, o) = 0 = $\frac{\partial W}{\partial T}$ (x, y, 0)

Boundary Conditions.

W (x, y, t) = M_X (x, y, t) = ψ_Y (x, y, t) = 0, for x = 0 and x = a
W (x, y, t) = M_Y (x, y, t) = ψ_X (x, y, t) = 0, for y = 0 and y = b
Where M_X and M_y are bending moments in the x – and y – directions respectively, ψ_X (x, y, t) and ψ_y (x, y, t) are local rotations in the x – and y – directions respectively. $W(x, y, t)$ is the traverse displacement of the plate at time t.

Problem Solution

The set of dynamic equilibrium equations which govern the behaviour of Mindlin's plate (with the effects both rotatory inertia and shear deformation) supported by Pasternak foundation and traversed by a partially distributed moving load may be written as [2,14,16];

$$Q_x - \frac{\partial M_x}{\partial x} - \frac{\partial M_{xy}}{\partial y} = \frac{\rho h^3}{12}\frac{\partial^2 \varphi_x}{\partial T^2} + \frac{\rho_L h_1^3}{12}\Big[\frac{\partial^2 \varphi_x}{\partial T^2} + \mathbf{U}\frac{\partial^2 \varphi_x}{\partial_x \partial T^2} + \frac{\mathbf{U}}{D(\upsilon^2-1)}\Big\{\frac{\partial M_x}{\partial T} + \mathbf{U}\frac{\partial M_x}{\partial x}\Big\} - \frac{\mathbf{U}\upsilon}{D(\upsilon^2-1)}\Big\{\frac{\partial M_y}{\partial T} + \mathbf{U}\frac{\partial M_y}{\partial y}\Big\}\Big]\mathrm{B} \tag{1}$$

$$Q_y - \frac{\partial M_{xy}}{\partial x} - \frac{\partial M_y}{\partial y} = \frac{\rho h^3}{12}\frac{\partial^2 \varphi_x}{\partial T^2} + \frac{\rho_L h_1^3}{12}\Big[\frac{\partial^2 \varphi_x}{\partial T^2} + \mathbf{U}\frac{\partial^2 \varphi_x}{\partial_x \partial T^2} + \frac{\mathbf{U}}{D(\upsilon^2-1)}\Big\{\frac{\partial M_x}{\partial T} + \mathbf{U}\frac{\partial M_x}{\partial x}\Big\} - \frac{\mathbf{U}\upsilon}{D(\upsilon^2-1)}\Big\{\frac{\partial M_x}{\partial T} + \mathbf{U}\frac{\partial M_x}{\partial x}\Big\}\Big]\mathrm{B} \tag{2}$$

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + kW + (M_f - \rho h)s\frac{\partial D_T}{\partial T} + \frac{M_L}{A}\Big[g + \frac{\partial D_T}{\partial T} + \mathbf{U}\frac{\partial D_T}{\partial T} + \mathbf{G}\Big(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y}\Big) + \Big\{\frac{\partial \psi_x}{\partial T} + \frac{\mathbf{U}}{D(\upsilon^2-1)}\mathbf{M}_x - \frac{\mathbf{U}\upsilon}{D(\upsilon^2-1)}\mathbf{M}_y\Big\} - \frac{\mathbf{U}}{\propto Gh}\Big\{\frac{\partial Q_x}{\partial T} + \mathbf{U}\frac{\partial Q_x}{\partial x}\Big\}\Big]\mathrm{B} = \rho h\frac{\partial^2 \varphi_x}{\partial T^2} \tag{3}$$

where ψ_x and ψ_y are local rotations in the x – and y – directions respectively. M_x and M_y bending moments in the x- and y- directions respectively, M_{xy} is the twitting moments, Q_x and Q_y are the traversed Shearing forces in x – and y – directions respectively, h and h_1 are thickness of the plate and load respectively,s is the viscous damping coefficient,ρ and ρ_L are the densities of the plate and the load per unit volume respectively. W(x,y,T) is the traverse displacement of the plate at time T, P(x,y,T) is the applied dynamic load (force) and the last terms in equation (1) and (2) account for inertia effects of the load in x – and y – directions respectively. It is the velocity of a load (M_L) of rectangular dimensions E by U with one of its lines of symmetry moving along $Y=Y_1$ .Theplate is L_X and L_Y in dimensions and ξ= UT + $\varepsilon/2$ as shown in figure1, also B = $B_X B_Y$, where $B_x =$

$$H\Big(x-\xi+\frac{\varepsilon}{2}\Big) - H\Big(x-\xi-\frac{\varepsilon}{2}\Big)\begin{cases} 1 - H\big(x-\xi+\frac{\varepsilon}{2}\big) \dots\dots\dots 0 < 1 < \frac{\varepsilon}{2} \\ \dots\dots\dots \frac{\varepsilon}{2} < 1 < \frac{L_x}{u} \\ H\big(\xi+\frac{\varepsilon}{2}\big) \dots\dots \frac{L_x}{u} \le T < (L_x+\varepsilon)/U \\ 0 \dots\dots (L_x+\varepsilon)/U \le T \end{cases} \tag{4}$$

$$B_y = H\Big(y - y_1 + \frac{\mu}{2}\Big) - H\Big(y - y_1 - \frac{\mu}{2}\Big) \tag{5}$$

H (x) is the Heaviside function defined as

$$H(x) = \begin{cases} 1 & x>0 \\ 0.5 & x=0 \\ 0 & X<0 \end{cases} \tag{6}$$

K is the foundation stiffness, Gis the foundation Shear modulus and M_f is the mass of the foundation.

Now considering the situation in which effect of rotatory inertia on the plate is neglected - shear plate, and supported by the simplest elastic foundation – Winkler foundation, the equations (1), (2) and (3) become, respectively [1,2]:

$$Q_x - \frac{\partial M_x}{\partial x} - \frac{\partial M_{xy}}{\partial y} = \frac{\rho_L h_1^3}{12}\Big[\frac{\partial^2 \varphi_x}{\partial T^2} + \mathbf{U}\frac{\partial^2 \varphi_x}{\partial_x \partial T^2} + \frac{\mathbf{U}}{D(\upsilon^2-1)}\Big\{\frac{\partial M_x}{\partial T} + \mathbf{U}\frac{\partial M_x}{\partial x}\Big\} - \frac{\mathbf{U}\upsilon}{D(\upsilon^2-1)}\Big\{\frac{\partial M_y}{\partial T} + \mathbf{U}\frac{\partial M_y}{\partial y}\Big\}\Big]\mathrm{B} \tag{7}$$

$$Q_y - \frac{\partial M_{xy}}{\partial x} - \frac{\partial M_y}{\partial y} = \frac{\rho_L h_1^3}{12}\Big[\frac{\partial^2 \varphi_x}{\partial T^2} + \mathbf{U}\frac{\partial^2 \varphi_x}{\partial_x \partial T^2} + \frac{\mathbf{U}}{D(\upsilon^2-1)}\Big\{\frac{\partial M_x}{\partial T} + \mathbf{U}\frac{\partial M_x}{\partial x}\Big\} - \frac{\mathbf{U}\upsilon}{D(\upsilon^2-1)}\Big\{\frac{\partial M_x}{\partial T} + \mathbf{U}\frac{\partial M_x}{\partial x}\Big\}\Big]\mathrm{B} \tag{8}$$

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + kW + (M_f - \rho h)s\frac{\partial D_T}{\partial T} + \frac{M_L}{A}\Big[g + \frac{\partial D_T}{\partial T} + \mathbf{U}\frac{\partial D_T}{\partial T} + {} + U\Big\{\frac{\partial \psi_x}{\partial T} + \frac{\mathbf{U}}{D(\upsilon^2-1)}\mathbf{M}_x - \frac{\mathbf{U}\upsilon}{D(\upsilon^2-1)}\mathbf{M}_y\Big\} - \frac{\mathbf{U}}{\propto Gh}\Big\{\frac{\partial Q_x}{\partial T} + \mathbf{U}\frac{\partial Q_x}{\partial x}\Big\}\Big]\mathrm{B} = 0 \tag{9}$$

The equations for the bending moments, twisting moments and Shear force are given as follows [2]:

$$M_x = -D\Big(\frac{\partial \psi_x}{\partial x} + \upsilon\frac{\partial \psi_y}{\partial y}\Big) \tag{10}$$

$$M_y = -D\Big(\frac{\partial \psi_y}{\partial y} + \upsilon\frac{\partial \psi_x}{\partial x}\Big) \tag{11}$$

$$M_{xy} = \Big(\frac{1-\upsilon}{2}\Big) D\Big(\frac{\partial \psi_x}{\partial x} + \upsilon\frac{\partial \psi_y}{\partial y}\Big) \tag{12}$$

$$Qx = -K^2 G_1 h\Big(\boldsymbol{\psi}_x - \frac{\partial W}{\partial x}\Big) \tag{13}$$

$$Qy = -K^2 G_1 h\Big(\boldsymbol{\psi}_y - \frac{\partial W}{\partial y}\Big) \tag{14}$$

Where G_1 is the modulus of rigidity of the plate, D is the flexural rigidity of the plate defined by D = $\frac{1}{12}Eh^3(1-\upsilon^2)$ =

$G_1h^3/6(1-\boldsymbol{v})$ for isotopic plate k^2 is the Shear correction factor and $\boldsymbol{v}$ is the Poisson's ratio of the plate. Also, in equation (1), (2) and (3);

$$D_T = \frac{\partial W}{\partial T} \tag{15}$$

The set of partial differential equations (7) – (15), are the partial differential equations to be solved for the following eleven dependent variables: Q_x, Q_y, M_x, M_y, M_{xy}, $\psi_{x,t}$, $\psi_{y,t}$, D_t and W.

The Non – Dimensional Form

The following variables: Q_x, Q_y, M_x, M_y, M_{xy}, $\psi_{x,t}$, $\psi_{y,t}$, D_t and W, in the above equations are the unknowns. These variables and others can be written in a non – dimensional form as follows:

$$q_x = \frac{Q_x}{\propto Gh} \tag{16}$$

$$q_y = \frac{Q_y}{\propto Gh} \tag{17}$$

$$m_x = \frac{M_x}{\propto Gh\gamma Lx} \tag{18}$$

$$m_y = \frac{M_y}{\propto Gh\gamma Lx} \tag{19}$$

$$m_{xy} = \frac{M_{xy}}{\propto Gh\gamma Lx} \tag{20}$$

$$W = \frac{W}{h} \tag{21}$$

$$dt = \frac{L_x D_T}{\gamma hc} \tag{22}$$

$$\varphi_{xt} = \frac{L_x \psi x,t}{\gamma^2 C} \tag{23}$$

$$\varphi_{yt} = \frac{L_x \psi y,t,}{\gamma^2 C} \tag{24}$$

where $\psi_{x,t} = \frac{\partial \psi_x}{\partial T}, \psi_{y,t} = \frac{\partial \psi_y}{\partial T}, D_T = \frac{\partial W}{\partial T}$

For the parameters: a= $\frac{A}{L^2{}_x}, m = \frac{M_L}{\rho h L^2{}_x}, g_n = \frac{L_x}{\gamma^3 c^2}, u = \frac{U}{\gamma c}, x = \frac{X}{L_x}, y = \frac{Y}{L_x}, t = \frac{T}{L_x/\gamma c}, \quad s = \frac{S}{\gamma c}$

The dimensionless form of equations (7) – (15) using equations (16) – (24) are:

$$\propto Ghq_x - \propto Gh\gamma \frac{\partial m_x}{\partial x} - \propto Gh\gamma \frac{\partial m_{xy}}{\partial y} = \frac{\rho L h_1^3}{a}\Big[\frac{\gamma^3 c^2}{L_x}\frac{\partial \varphi_{xt}}{\partial t} + \frac{\partial^4 c^3 u}{L^2{}_x}\frac{\partial \varphi_{xt}}{\partial t} + \frac{\gamma cu}{D(v^2-1)}\Big\{\frac{\alpha G h \gamma^2 c}{L_x}\frac{\partial m_x}{\partial t} + \frac{\gamma^2 cu\alpha Gh}{L_x}\frac{\partial m_x}{\partial x}\Big\} - \frac{\gamma Guv}{D(v^2-1)}\Big\{c\alpha G h\gamma^2 \frac{\partial m_y}{\partial t} + \gamma^2 cu\alpha G \frac{\partial m_y}{\partial x}\Big\}\Big] B_n \tag{25}$$

$$\propto Ghq_y - \propto Gh\gamma \frac{\partial m_{xy}}{\partial x} - \propto Gh\gamma \frac{\partial m_y}{\partial y} = \frac{\rho L h_1^3}{12}\Big[\frac{\gamma^3 c^2 u}{L^2{}_x}\frac{\partial \varphi_{xt}}{\partial x} + \frac{\gamma^4 c^3 u}{L^2{}_x}\frac{\partial \varphi_{yt}}{\partial x} + \frac{\gamma cu}{D(v^2-1)}\Big\{\alpha G h\gamma^2 c \frac{\partial m_y}{\partial t} + \alpha G h\gamma^2 cu \frac{\partial m_y}{\partial t}\Big\} - \frac{\gamma cuv}{D(v^2-1)}\Big\{\alpha G h\gamma^2 c\frac{\partial m_x}{\partial t} + \alpha G h\gamma^3 cu \frac{\partial m_x}{\partial y}\Big\}\Big] B_n \tag{26}$$

$$\frac{\alpha Gh}{L_x}\frac{\partial q_x}{\partial x} + \frac{\alpha Gh}{L_x}\frac{\partial q_y}{\partial y} + KhW + \frac{\rho h^2 \gamma \quad c \quad s}{L^2{}_x}\frac{\partial dt}{\partial t} - \frac{\rho hm}{a}\Big[\frac{\gamma^3 c^2}{L_x} g_n + \frac{\gamma^2 c^2 h}{L^2{}_x}\frac{\partial dt}{\partial t} + \frac{\gamma^3 c^3 hu}{L^2{}_x}\frac{\partial dt}{\partial t} + \gamma cu \Big\{\frac{\gamma^2 c}{L_x}\varphi_{xt} + \frac{\gamma cu}{D(v^2-1)}\alpha G h\gamma L_x M_x - \frac{\gamma cUv}{D(v^2-1)}\alpha G h \gamma L_x M_y\Big\} - \frac{\gamma cu}{\alpha Gh}\Big\{\frac{\alpha G h\gamma c}{L_x}\frac{\partial q_x}{\partial x} + \frac{\alpha cu\alpha Gh}{L_x}\frac{\partial q_x}{\partial x}\Big\}\Big] B_n = 0 \tag{27}$$

$$\alpha G h\gamma^2 c\frac{\partial m_x}{\partial t} = \frac{D\gamma^2 c}{L^2{}_x}\frac{\partial \varphi_{xt}}{\partial y} - \frac{Dv\gamma^3 c}{L^2{}_x}\frac{\partial \varphi_{yt}}{\partial y} \tag{28}$$

$$\alpha G h\gamma^2 c\frac{\partial m_y}{\partial t} = \frac{D\gamma^2 c}{L^2{}_x}\frac{\partial \varphi_{yt}}{\partial y} - \frac{Dv\gamma^3 c}{L^2{}_x}\frac{\partial \varphi_{yt}}{\partial y} \tag{29}$$

$$\alpha G h\gamma^2 c\frac{\partial m_{xy}}{\partial t} = \frac{D(1-v)}{2}\Big[\frac{\gamma^2 c}{L^2{}_x}\frac{\partial \varphi_{xt}}{\partial y} + \frac{\gamma^2 c}{L^2{}_x}\frac{\partial \varphi_{yt}}{\partial x}\Big] \tag{30}$$

$$\frac{\alpha G h\gamma c}{L_x}\frac{\partial q_x}{\partial t} = \alpha h G\Big[\frac{\gamma^2 c}{L_x}\varphi_{xt} - \frac{\gamma^2 c}{L^2{}_x}\frac{\partial dt}{\partial t}\Big] \tag{31}$$

$$\frac{\alpha G h\gamma c}{L_x}\frac{\partial q_y}{\partial t} = \alpha h G\Big[\frac{\gamma^2 c}{L_x}\varphi_{yt} - \frac{\gamma hc}{L^2{}_x}\frac{\partial dt}{\partial y}\Big] \tag{32}$$

$$\frac{\gamma hc}{L_x} dt = \frac{h\gamma c}{L_x}\frac{\partial w}{\partial t} \tag{33}$$

Where $\gamma = \frac{h}{L_x}$ and $B_n = B_{xn} B_{yn}$

$$B_{xn} = \begin{cases} 1 - H\left(x - \xi_n + \frac{\varepsilon_n}{2}\right) - - - - - 0 < 1 < \frac{\varepsilon_n}{u} \\ H\left(x - \xi_n + \frac{\varepsilon_n}{2}\right) - H\left(x - \xi_n - \frac{\varepsilon_n}{2}\right) - - - - - \frac{\varepsilon_n}{u} < 1 < \frac{2_x}{u} \\ H\left(\xi_n + \frac{\varepsilon_n}{2}\right) - - - - - \frac{L_x}{u} \le T < \frac{(L+\varepsilon_n)}{u} \\ 0 - - - - - - \frac{(L_x+\varepsilon_n)}{u} \le 1 \end{cases} \tag{34}$$

$$B_{yn} = H\Big\{y - y_1 + \frac{\mu_n}{2}\Big\} - H\Big\{y - y_1 - \frac{\mu_n}{2}\Big\} \tag{35}$$

$$\left.\begin{aligned} \varepsilon n &= \frac{\varepsilon}{L_x} \\ \mu_n &= \frac{\mu}{L_x} \\ \xi_n &= \frac{\xi}{L_x} \end{aligned}\right\} \tag{36}$$

H(x) is a defined by

$$H(x) = \begin{cases} 1 & x > 0 \\ 0.5 & x = 0 \\ 0 & x < 0 \end{cases} \quad (37)$$

Writing equations (25) – (33) in a more concise form, we have [7]

$$\frac{N_1 N_2}{\rho hc} q_x - \frac{N_1 N_2}{\rho c}\frac{\partial M_x}{\partial x} - \frac{N_1 N_2}{\rho c}\frac{\partial M_{xy}}{\partial y} = \frac{\rho L h_1^3}{12}\left(\frac{N_1}{L^2 x\rho h}\frac{\partial \varphi_{xt}}{\partial t} + \frac{N_1 CU}{\rho L_x^3}\frac{\partial M_x}{\partial t} + \frac{N_3 U}{L^3 x\rho h}\frac{\partial M_x}{\partial t} + \frac{N_3 U}{\rho h L_x^2}\frac{\partial M_x}{\partial x} - \frac{N_3 UV}{\rho h L_x^2}\frac{\partial M_y}{\partial t} - \frac{N_3 U^2 V}{\rho h L_x^2}\frac{\partial M_y}{\partial x}\right) B_n \quad (38)$$

$$\frac{N_1 N_2}{\rho \gamma c} q_y - \frac{N_1 N_2}{\rho c}\frac{\partial M_{xy}}{\partial x} - \frac{N_1 N_2}{\rho c}\frac{\partial M_y}{\partial y} = \frac{\rho L h_1^3}{12}\left(\frac{N_1}{\rho h L_x^2}\frac{\partial \varphi_{yt}}{\partial t} + \frac{N_1 U}{\rho h L_x^2}\frac{\partial \varphi_{xt}}{\partial x} + \frac{N_3 U}{\rho h L_x^3}\frac{\partial M_y}{\partial t} + \frac{N_3 U^2}{\rho h L_x^2}\frac{\partial y}{\partial y} - \frac{N_3 UV}{\rho h L_x^2}\frac{\partial M_x}{\partial t} - \frac{N_3 U^2 V}{\rho h L_x^2}\frac{\partial M_x}{\partial y}\right) B_n \quad (39)$$

$$\frac{N_1 N_2}{\rho \gamma c}\frac{\partial q_x}{\partial x} + \frac{N_1 N_2}{\rho \gamma c}\frac{\partial q_y}{\partial y} + \frac{N_1 K s L_x}{\rho \gamma^4 c^3} W + \left(\frac{N_1}{L_x a} + \frac{\gamma N_1 M}{L_x a} B_n + \frac{N_1 M \gamma C U}{L_x a}\right) d\frac{\partial dt}{\partial t} = \frac{-M}{a}\left[\frac{N_1 U}{L_x} g_n + \frac{N_1 U}{L_x^2}\varphi_{xt}\frac{N_3 U^2}{L_x} M_x - \frac{N_2 U^3 V}{L_x} M_y - \frac{N_1 U}{L_{xy}}\frac{\partial q_x}{\partial x} + \frac{N_1 U^2}{L_x \gamma}\frac{\partial q_x}{\partial x}\right] B_n \quad (40)$$

$$\frac{\partial m_x}{\partial t} = -N_1\frac{\partial \varphi_{xt}}{\partial x} - VN_1\frac{\partial \varphi_{yt}}{\partial y} \quad (41)$$

$$\frac{\partial m_y}{\partial t} = N_1\frac{\partial \varphi_{yt}}{\partial y} - VN_1\frac{\partial \varphi_{xt}}{\partial x} \quad (42)$$

$$\frac{\partial m_{xy}}{\partial t} = N_1\frac{(1-\nu)}{2}\left(\frac{\partial \varphi_{xt}}{\partial x} + \frac{\partial \varphi_{yt}}{\partial y}\right) \quad (43)$$

$$\frac{\partial q_x}{\partial t} = \gamma\left(\varphi_{xt} - \frac{\partial t}{\partial x}\right) \quad (44)$$

$$\frac{\partial q_y}{\partial t} = \gamma\left(\varphi_{yt} - \frac{\partial dt}{\partial y}\right) \quad (45)$$

$$dt = \frac{\partial w}{\partial t} \quad (46)$$

Where

$$\left.\begin{aligned} N_1 &= \frac{D}{\alpha G h L^2 x} \\ N_2 &= \frac{\alpha G}{\gamma^2 C} \\ N_3 &= \frac{1}{(V^2-1)} \\ R &= \frac{\rho_L h_1^3}{\rho h^3} \end{aligned}\right\} \quad (47)$$

Equations (38) – (46) are the non-dimensional first order governing differential equations to be solved for the following nine dependent variables

$q_x, q_y, m_x, m_y, m_{xy}, w, dt, \varphi_{xt}$ *and* φ_{yt}

The non-dimensional boundary conditions are:

$$\left.\begin{aligned} dt = m_x = \varphi_{yt} = 0 \ (at\ x = 0\ and\ x = 1) \\ dt = m_y = \varphi_{xt} = 0\left(at\ y = 0\ and\ y = {}^{b}/_{2}\right) \end{aligned}\right\} \quad (48)$$

Finite Difference Algorithm for the Shear plate

Equations (38) - (46) is solved using a numerical method based on the finite difference algorithm. These equations are to be transformed into their equivalent algebraic form. The finite difference definition of first order partial derivative of a function E(x,y,t) with respect to x,y and t respectively are as follows:[5,7]

$$\frac{\partial E}{\partial t} = \frac{1}{4r}\left(E_{i+1,j+1}^{k+1} + E_{i+1,j}^{k+1} + E_{i,j+1}^{k+1} + E_{i,j}^{k+1} - E_{i+1,j+1}^{k} - E_{i+1,j}^{k} - E_{i,j+1}^{k} - E_{i,j}^{k}\right) \quad (49)$$

$$\frac{\partial E}{\partial x} = \frac{1}{4h}\left(E_{i+1,j+1}^{k+1} + E_{i+1,j}^{k+1} - E_{i,j+1}^{k+1} - E_{i,j}^{k+1} + E_{i+1,j+1}^{k} + E_{i+1,j}^{k} - E_{i,j+1}^{k} - E_{i,j}^{k}\right) \quad (50)$$

$$\frac{\partial E}{\partial y} = \frac{1}{4k}\left(E_{i+1,j+1}^{k+1} + E_{i+1,j}^{k+1} - E_{i,j+1}^{k+1} - E_{i,j}^{k+1} + E_{i+1,j+1}^{k} + E_{i+1,j}^{k} - E_{i,j+1}^{k} - E_{i,j}^{k}\right) \quad (51)$$

Where E is the function value of the centre of a grid, which is well approximated by the average of its values at the grid nodes [5]

$$E\left(x+\frac{h}{2}, y+\frac{k}{2}, t+\frac{r}{2}\right) = \frac{1}{8}\left(E_{i+1,j+1}^{k+1} + E_{i+1,j}^{k+1} + E_{i,j+1}^{k+1} + E_{i,j}^{k+1} + E_{i+1,j+1}^{k} + E_{i+1,j}^{k} + E_{i,j+1}^{k} + E_{i,j}^{k}\right) \quad (52)$$

Hence,

$$\frac{\partial q_x}{\partial x}=\frac{1}{4h}\left(q^{k+1}_{x_{i+1,j+1}}+q^{k+1}_{x_{i+1,j}}-q^{k+1}_{x_{i,j}}-q^{k+1}_{x_{i+1,j+1}}+q^{k}_{x_{i+1,j+1}}+q^{k}_{x_{i+1,j}}-q^{k}_{x_{i,j+1}}-q^{k}_{x_{i,j}}\right)\tag{53}$$

$$\frac{\partial qy}{\partial y}=\frac{1}{4k}\left(q^{k+1}_{y_{i+1,j+1}}+q^{k+1}_{y_{i+1,j}}-q^{k+1}_{y_{i,j}}-q^{k+1}_{y_{i+1,j}}+q^{k}_{y_{i,j+1}}+q^{k}_{x_{i+1,j}}-q^{k}_{y_{1+i,j}}-q^{k}_{x_{i,j}}\right)\tag{54}$$

$$\frac{\partial dt}{\partial t}=\frac{1}{4r}\left(d^{k+1}_{t_{i+1,j+1}}+d^{k+1}_{t_{i+1,j}}+d^{k+1}_{t_{i,j+1}}+d^{k+1}_{t_{i,j}}-d^{k}_{t_{i+1,j+1}}-d^{k}_{t_{i+1,j}}-d^{k}_{t_{i,j+1}}-d^{k}_{t_{i,j}}\right)\tag{55}$$

$$\frac{\partial m_{xy}}{\partial x}=\frac{1}{4h}\left(m^{k+1}_{xy_{i+1,j+1}}+m^{k+1}_{xy_{i+1,j}}-m^{k+1}_{xy_{i,j}}-m^{k+1}_{xy_{i+1,j+1}}+m^{k}_{xy_{i+1,j+1}}+m^{k}_{xy_{i+1,j}}-m^{k}_{xy_{i,j+1}}-m^{k}_{xy_{i,j}}\right)\tag{56}$$

$$\frac{\partial m_{y}}{\partial y}=\frac{1}{4k}\left(m^{k+1}_{y_{i+1,j+1}}+m^{k+1}_{y_{i+1,j}}-m^{k+1}_{y_{i+1,j}}-m^{k+1}_{y_{i,j}}+m^{k}_{y_{i+1,j+1}}+m^{k}_{y_{i,j+1}}-m^{k}_{y_{i+1,j}}-m^{k}_{y_{i,j}}\right)\tag{57}$$

$$\frac{\partial m_{x}}{\partial t}=\frac{1}{4r}\left(m^{k+1}_{x_{i+1,j+1}}+m^{k+1}_{x_{i+1,j}}+m^{k+1}_{x_{i,j+1}}+m^{k+1}_{x_{i,j}}-m^{k}_{x_{i+1,j+1}}-m^{k}_{x_{i,j+1}}-m^{k}_{x_{i+1,j}}-m^{k}_{x_{i,j}}\right)\tag{58}$$

$$\frac{\partial \varphi_{yt}}{\partial t}=\frac{1}{4r}\left(\varphi^{k+1}_{yt_{i+1,j+1}}+\varphi^{k+1}_{yt_{i+1,j}}+\varphi^{k+1}_{yt_{i,j+1}}+\varphi^{k+1}_{yt_{i,j}}-\varphi^{k}_{yt_{i+1,j+1}}-\varphi^{k}_{yt_{i+1,j}}-\varphi^{k}_{yt_{i,j+1}}-\varphi^{k}_{yt_{i,j}}\right)\tag{59}$$

$$\frac{\partial \varphi_{xt}}{\partial t}=\frac{1}{4h}\left(\varphi^{k+1}_{xt_{i+1,j+1}}+\varphi^{k+1}_{xt_{i+1,j}}-\varphi^{k+1}_{xt_{i,j+1}}-\varphi^{k+1}_{xt_{i,j}}+\varphi^{k}_{xt_{i+1,j+1}}+\varphi^{k}_{xt_{i+1,j}}-\varphi^{k}_{xt_{i,j+1}}-\varphi^{k}_{xt_{i,j}}\right)\tag{60}$$

$$\frac{\partial q_x}{\partial t}=\frac{1}{4r}\left(q^{k+1}_{x_{i+1,j+1}}+q^{k+1}_{x_{i+1,j}}+q^{k+1}_{x_{i,j}}+q^{k+1}_{x_{i,j+1}}-q^{k}_{x_{i+1,j+1}}-q^{k}_{x_{i+1,j}}-q^{k}_{x_{i,j+1}}-q^{k}_{x_{i,j}}\right)\tag{61}$$

$$\frac{\partial m_{x}}{\partial x}=\frac{1}{4h}\left(m^{k+1}_{x_{i+1,j+1}}+m^{k+1}_{x_{i+1,j}}-m^{k+1}_{x_{i,j+1}}-m^{k+1}_{x_{i,j}}+m^{k}_{x_{i+1,j+1}}+m^{k}_{x_{i,j+1}}+m^{k}_{x_{i+1,j}}-m^{k}_{x_{i,j}}\right)\tag{62}$$

$$\frac{\partial m_{x}}{\partial y}=\frac{1}{4k}\left(m^{k+1}_{x_{i+1,j+1}}-m^{k+1}_{x_{i+1,j}}+m^{k+1}_{x_{i,j+1}}-m^{k+1}_{x_{i,j}}+m^{k}_{x_{i+1,j+1}}+m^{k}_{x_{i,j+1}}-m^{k}_{x_{i+1,j}}-m^{k}_{x_{i,j}}\right)\tag{63}$$

$$\frac{\partial m_{y}}{\partial t}=\frac{1}{4k}\left(m^{k+1}_{y_{i+1,j+1}}+m^{k+1}_{y_{i+1,j}}+m^{k+1}_{y_{i+1,j}}+m^{k+1}_{y_{i,j}}-m^{k}_{y_{i+1,j+1}}-m^{k}_{y_{i,j+1}}-m^{k}_{y_{i+1,j}}-m^{k}_{y_{i,j}}\right)\tag{64}$$

$$\frac{\partial m_{xy}}{\partial y}=\frac{1}{4k}\left(m^{k+1}_{xy_{i+1,j+1}}+m^{k+1}_{xy_{i+1,j}}-m^{k+1}_{xy_{i,j}}-m^{k+1}_{xy_{i+1,j+1}}+m^{k}_{xy_{i+1,j+1}}+m^{k}_{xy_{i+1,j}}-m^{k}_{xy_{i,j+1}}-m^{k}_{xy_{i,j}}\right)\tag{65}$$

$$\frac{\partial m_{y}}{\partial x}=\frac{1}{4h}\left(m^{k+1}_{y_{i+1,j+1}}+m^{k+1}_{y_{i+1,j}}-m^{k+1}_{y_{i+1,j}}-m^{k+1}_{y_{i,j}}+m^{k}_{y_{i+1,j+1}}+m^{k}_{y_{i,j+1}}-m^{k}_{y_{i+1,j}}-m^{k}_{y_{i,j}}\right)\tag{66}$$

$$\frac{\partial \varphi_{xt}}{\partial t}=\frac{1}{4r}\left(\varphi^{k+1}_{xt_{i+1,j+1}}+\varphi^{k+1}_{xt_{i+1,j}}+\varphi^{k+1}_{xt_{i,j+1}}+\varphi^{k+1}_{xt_{i,j}}-\varphi^{k}_{xt_{i+1,j+1}}-\varphi^{k}_{xt_{i+1,j}}-\varphi^{k}_{xt_{i,j+1}}-\varphi^{k}_{xt_{i,j}}\right)\tag{67}$$

$$\frac{\partial \varphi_{yt}}{\partial y}=\frac{1}{4k}\left(\varphi^{k+1}_{yt_{i+1,j+1}}+\varphi^{k+1}_{yt_{i+1,j}}-\varphi^{k+1}_{yt_{i,j+1}}-\varphi^{k+1}_{yt_{i,j}}+\varphi^{k}_{yt_{i+1,j+1}}+\varphi^{k}_{yt_{i+1,j}}-\varphi^{k}_{yt_{i,j+1}}-\varphi^{k}_{yt_{i,j}}\right)\tag{68}$$

$$\frac{\partial m_{xy}}{\partial t}=\frac{1}{4r}\left(m^{k+1}_{xy_{i+1,j+1}}+m^{k+1}_{xy_{i+1,j}}+m^{k+1}_{xy_{i,j}}+m^{k+1}_{xy_{i+1,j+1}}-m^{k}_{xy_{i+1,j+1}}-m^{k}_{xy_{i+1,j}}-m^{k}_{xy_{i,j+1}}-m^{k}_{xy_{i,j}}\right)\tag{69}$$

$$\frac{\partial dt}{\partial x}=\frac{1}{4h}\left(d^{k+1}_{t_{i+1,j+1}}+d^{k+1}_{t_{i+1,j}}-d^{k+1}_{t_{i,j+1}}-d^{k+1}_{t_{i,j}}+d^{k}_{t_{i+1,j+1}}+d^{k}_{t_{i+1,j}}-d^{k}_{t_{i,j+1}}-d^{k}_{t_{i,j}}\right)\tag{70}$$

$$\frac{\partial dt}{\partial y}=\frac{1}{4k}\left(d^{k+1}_{t_{i+1,j+1}}+d^{k+1}_{t_{i+1,j}}-d^{k+1}_{t_{i,j+1}}-d^{k+1}_{t_{i,j}}+d^{k}_{t_{i+1,j+1}}+d^{k}_{t_{i+1,j}}-d^{k}_{t_{i,j+1}}-d^{k}_{t_{i,j}}\right)\tag{71}$$

$$\frac{\partial qy}{\partial t}=\frac{1}{4r}\left(q^{k+1}_{y_{i+1,j+1}}+q^{k+1}_{y_{i+1,j}}+q^{k+1}_{y_{i,j}}+q^{k+1}_{y_{i+1,j}}+q^{k}_{y_{i,j+1}}+q^{k}_{x_{i+1,j}}-q^{k}_{y_{1+i,j}}-q^{k}_{x_{i,j}}\right)\tag{72}$$

$$\frac{\partial q_x}{\partial x}=\frac{1}{4h}\left(q^{k+1}_{x_{i+1,j+1}}+q^{k+1}_{x_{i+1,j}}-q^{k+1}_{x_{i,j}}-q^{k+1}_{x_{i+1,j+1}}+q^{k}_{x_{i+1,j+1}}+q^{k}_{x_{i+1,j}}-q^{k}_{x_{i,j+1}}-q^{k}_{x_{i,j}}\right)\tag{73}$$

$$\frac{\partial \varphi_{xt}}{\partial y} = \frac{1}{4k}\left(\varphi^{k+1}_{xt_{i+1,j+1}} + \varphi^{k+1}_{xt_{i+1,j}} - \varphi^{k+1}_{xt_{i,j+1}} - \varphi^{k+1}_{xt_{i,j}} + \varphi^{k}_{xt_{i+1,j+1}} + \varphi^{k}_{xt_{i+1,j}} - \varphi^{k}_{xt_{i,j+1}} - \varphi^{k}_{xt_{i,j}}\right) \tag{74}$$

$$\frac{\partial \varphi_{yt}}{\partial x} = \frac{1}{4h}\left(\varphi^{k+1}_{yt_{i+1,j+1}} + \varphi^{k+1}_{yt_{i+1,j}} - \varphi^{k+1}_{yt_{i,j+1}} - \varphi^{k+1}_{yt_{i,j}} + \varphi^{k}_{yt_{i+1,j+1}} + \varphi^{k}_{yt_{i+1,j}} - \varphi^{k}_{yt_{i,j+1}} - \varphi^{k}_{yt_{i,j}}\right) \tag{75}$$

$$\frac{\partial W}{\partial t} = \frac{1}{4r}\left(w^{k+1}_{i+1,j+1} + w^{k+1}_{i+1,j} + w^{k+1}_{i,j+1} + w^{k+1}_{i,j} - w^{k}_{i+1,j+1} - w^{k}_{i+1,j} - w^{k}_{i,j+1} - w^{k}_{i,j}\right) \tag{76}$$

$$\varphi_{xt} = \frac{1}{8}\left(\varphi^{k+1}_{xt_{i+1,j+1}} + \varphi^{k+1}_{xt_{i+1,j}} + \varphi^{k+1}_{xt_{i,j+1}} + \varphi^{k+1}_{xt_{i,j}} + \varphi^{k}_{xt_{i+1,j+1}} + \varphi^{k}_{xt_{i+1,j}} + \varphi^{k}_{xt_{i,j+1}} + \varphi^{k}_{xt_{i,j}}\right) \tag{77}$$

$$\varphi_{yt} = \frac{1}{8}\left(\varphi^{k+1}_{yt_{i+1,j+1}} + \varphi^{k+1}_{yt_{i+1,j}} + \varphi^{k+1}_{yt_{i,j+1}} + \varphi^{k+1}_{yt_{i,j}} + \varphi^{k}_{yt_{i+1,j+1}} + \varphi^{k}_{yt_{i+1,j}} + \varphi^{k}_{yt_{i,j+1}} + \varphi^{k}_{yt_{i,j}}\right) \tag{78}$$

$$w = \frac{1}{8}\left(w^{k+1}_{i+1,j+1} + w^{k+1}_{i+1,j} + w^{k+1}_{i,j+1} + w^{k+1}_{i,j} + w^{k}_{i+1,j+1} + w^{k}_{i+1,j} + w^{k}_{i,j+1} - w^{k}_{i,j}\right) \tag{79}$$

$$m_x = \frac{1}{8}\left(m^{k+1}_{x_{i+1,j+1}} + m^{k+1}_{x_{i+1,j}} + m^{k+1}_{x_{i,j+1}} + m^{k+1}_{x_{i,j}} + m^{k}_{x_{i+1,j+1}} + m^{k}_{x_{i,j+1}} + m^{k}_{x_{i+1,j}} + m^{k}_{x_{i,j}}\right) \tag{80}$$

$$m_y = \frac{1}{8}\left(m^{k+1}_{y_{i+1,j+1}} + m^{k+1}_{y_{i+1,j}} + m^{k+1}_{y_{i,j+1}} + m^{k+1}_{y_{i,j}} + m^{k}_{y_{i+1,j+1}} + m^{k}_{y_{i,j+1}} + m^{k}_{y_{i+1,j}} + m^{k}_{y_{i,j}}\right) \tag{81}$$

$$q_x = \frac{1}{8}\left(q^{k+1}_{x_{i+1,j+1}} + q^{k+1}_{x_{i+1,j}} + q^{k+1}_{x_{i,j}} + q^{k+1}_{x_{i+1,j+1}} + q^{k}_{x_{i+1,j+1}} + q^{k}_{x_{i+1,j}} + q^{k}_{x_{i,j+1}} + q^{k}_{x_{i,j}}\right) \tag{82}$$

3.3 The algebraic form of the non-dimensional first order governing differential equations

The substitution of the definition of the above finite difference scheme into equations (38) – (46) and multiplying both sides of the resulting equations by 4h, setting h/r = u and h/k = H, yields respectively[1,2]:

$$
\begin{aligned}
&\frac{N_1 N_2}{\rho\gamma c}\left(q^{k+1}_{x_{i+1,j+1}} + q^{k+1}_{x_{i+1,j}} - q^{k+1}_{x_{i,j}} - q^{k+1}_{x_{i,j+1}}\right)\\
&+\frac{N_1 N_2}{\rho\gamma c} H\left(q^{k+1}_{y_{i+1,j+1}} + q^{k+1}_{y_{i+1,j}} + q^{k+1}_{y_{i,j+1}} - q^{k+1}_{y_{i,j}}\right)\\
&+\frac{N_1 K L_x}{\rho\gamma^2 c^2}\frac{h}{2}\left(w^{k+1}_{xy_{i+1,j+1}} + w^{k+1}_{xy_{i+1,j}} - w^{k+1}_{xy_{i,j+1}} - w^{k+1}_{xy_{i,j}}\right)\\
&+\left(\frac{N_1}{L_x} + \frac{\gamma N_1 m}{L_x a} B_n + \frac{N_1 m\gamma c u}{L_x a} B_n\right) u\left(d^{k+1}_{t_{i+1,j+1}} + d^{k+1}_{t_{i+1,j}} + d^{k+1}_{t_{i,j+1}} + d^{k+1}_{t_{i,j}}\right)\\
&+[\frac{N_1 u m}{L_x^2 a}\frac{h}{2}\left(\varphi^{k+1}_{xt_{i+1,j+1}} + \varphi^{k+1}_{xt_{i+1,j}} + \varphi^{k+1}_{xt_{i,j+1}} + \varphi^{k+1}_{xt_{i,j}}\right)\\
&-\frac{N_3 u^2 m v}{L_x a}\frac{h}{2}\left(m^{k+1}_{x_{i+1,j+1}} + m^{k+1}_{x_{i+1,j}} + m^{k+1}_{x_{i,j+1}} + m^{k+1}_{x_{i,j}}\right)\\
&-\frac{N_1 m u^2}{L_x a\lambda}\left(q^{k+1}_{x_{i+1,j+1}} + q^{k+1}_{x_{i+1,j}} + q^{k+1}_{x_{i,j+1}} + q^{k+1}_{x_{i,j}}\right)\\
&+\frac{N_3 u^2 m}{L_x^2 a}\frac{h}{2}\left(m^{k+1}_{y_{i+1,j+1}} + m^{k+1}_{y_{i+1,j}} + m^{k+1}_{y_{i,j+1}} + m^{k+1}_{y_{i,j}}\right)\\
&+\frac{N_1 m u^2}{L_x\gamma a} H\left(q^{k+1}_{x_{i+1,j+1}} + q^{k+1}_{x_{i+1,j}} - q^{k+1}_{x_{i,j+1}} - q^{k+1}_{x_{i,j}}\right)]\mathrm{B}_n\\
&=-\frac{N_1 N_2}{\rho\gamma c}\left(q^{k}_{x_{i+1,j+1}} + q^{k}_{x_{i+1,j}} - q^{k}_{x_{i,j}} - q^{k}_{x_{i,j+1}}\right)\\
&+\frac{N_1 N_2}{\rho\gamma c} H\left(q^{k}_{y_{i+1,j+1}} + q^{k}_{y_{i+1,j}} - q^{k}_{y_{i,j+1}} - q^{k}_{y_{i,j}}\right)\\
&+\frac{N_1 K L_x}{\rho\gamma^3 c^2}\frac{h}{2}\left(w^{k}_{xy_{i+1,j+1}} + w^{k}_{xy_{i+1,j}} + w^{k}_{xy_{i,j+1}} + w^{k}_{xy_{i,j}}\right)\\
&-\left(\frac{N_1}{L_x} + \frac{\gamma N_1 m}{L_x a} B_n + \frac{N_1 m\gamma c u}{L_x a} B_n\right) u\left(-d^{k}_{t_{i+1,j+1}} - d^{k}_{t_{i+1,j}} - d^{k}_{t_{i,j+1}} - d^{k}_{t_{i,j}}\right)\\
&-4\mathrm{h}\frac{m}{a}\frac{N_1}{L_x}\mathrm{g}_n\mathrm{B}_n - [\frac{N_1 u m}{L_x^2 a}\frac{h}{2}\left(\varphi^{k}_{xt_{i+1,j+1}} + \varphi^{k}_{xt_{i+1,j}} + \varphi^{k}_{xt_{i,j+1}} + \varphi^{k}_{xt_{i,j}}\right)\\
&+\frac{N_3 u^2 m}{L_x a}\frac{h}{2}\left(m^{k}_{x_{i+1,j+1}} + m^{k}_{x_{i+1,j}} + m^{k}_{x_{i,j+1}} + m^{k}_{x_{i,j}}\right)\\
&-\frac{N_3 m v u^2}{L_x a\lambda}\left(q^{k+1}_{x_{i+1,j+1}} + q^{k+1}_{x_{i+1,j}} + q^{k+1}_{x_{i,j+1}} + q^{k+1}_{x_{i,j}}\right)\\
&-\frac{N_3 u^2 m}{L_x a}\frac{h}{2}\left(m^{k}_{y_{i+1,j+1}} + m^{k}_{y_{i+1,j}} + m^{k}_{y_{i,j+1}} + m^{k}_{y_{i,j}}\right)\\
&-\frac{N_1 m u^2}{L_x\gamma a}\left(q^{k}_{x_{i+1,j+1}} + q^{k}_{x_{i+1,j}} + q^{k}_{x_{i,j+1}} + q^{k}_{x_{i,j}}\right)\\
&-\frac{N_1 m u^2}{L_x\gamma a}\left(q^{k}_{x_{i+1,j+1}} + q^{k}_{x_{i+1,j}} - q^{k}_{x_{i,j+1}} - q^{k}_{x_{i,j}}\right)]\mathrm{B}_n \qquad (83)
\end{aligned}
$$

$$\frac{N_1N_2}{\rho hc}\frac{h}{2}\left(q_{x_{i+1,j+1}}^{k+1}+q_{x_{i+1,j}}^{k+1}+q_{x_{i,j}}^{k+1}+q_{x_{i,j+1}}^{k+1}\right)$$
$$+\frac{N_1N_2}{\rho c}\left(m_{x_{i+1,j+1}}^{k+1}+m_{x_{i+1,j}}^{k+1}+m_{x_{i,j+1}}^{k+1}-m_{x_{i,j}}^{k+1}\right)$$
$$-\frac{N_1N_2}{\rho c}H\left(m_{xy_{i+1,j+1}}^{k+1}+m_{xy_{i+1,j}}^{k+1}-m_{xy_{i,j+1}}^{k+1}-m_{xy_{i,j}}^{k+1}\right)$$
$$+\frac{\rho_L h_1^2}{12}[\frac{N_1u}{L_x^2\rho h}\left(\varphi_{xt_{i+1,j+1}}^{k+1}+\varphi_{xt_{i+1,j}}^{k+1}+\varphi_{xt_{i,j+1}}^{k+1}+\varphi_{xt_{i,j}}^{k+1}\right)$$
$$+\frac{N_1cu^2}{L_x^2\rho}\left(\varphi_{xt_{i+1,j+1}}^{k+1}+\varphi_{xt_{i+1,j}}^{k+1}-\varphi_{xt_{i,j+1}}^{k+1}-\varphi_{xt_{i,j}}^{k+1}\right)$$
$$+\frac{N_3u^2}{L_x^3\rho h}\left(m_{x_{i+1,j+1}}^{k+1}+m_{x_{i+1,j}}^{k+1}+m_{x_{i,j+1}}^{k+1}+m_{x_{i,j}}^{k+1}\right)$$
$$+\frac{N_3u}{L_x^3\rho h}\left(m_{x_{i+1,j+1}}^{k+1}+m_{x_{i+1,j}}^{k+1}+m_{x_{i,j+1}}^{k+1}+m_{x_{i,j}}^{k+1}\right)$$
$$+\frac{N_3u^2v}{L_x^2\rho h}\left(m_{y_{i+1,j+1}}^{k+1}+m_{y_{i+1,j}}^{k+1}+m_{y_{i,j+1}}^{k+1}+m_{y_{i,j}}^{k+1}\right)$$
$$+\frac{N_3u^2v}{L_x^2\rho h}\left(m_{y_{i+1,j+1}}^{k+1}+m_{y_{i+1,j}}^{k+1}-m_{y_{i,j+1}}^{k+1}-m_{y_{i,j}}^{k+1}\right)]\mathrm{B}_n$$
$$=-\frac{N_1N_2}{\rho hc}\frac{h}{2}\left(q_{x_{i+1,j+1}}^{k}+q_{x_{i+1,j}}^{k}+q_{x_{i,j}}^{k}+q_{x_{i,j+1}}^{k}\right)$$
$$+\frac{N_1N_2}{\rho c}\left(m_{x_{i+1,j+1}}^{k}+m_{x_{i+1,j}}^{k}-m_{x_{i,j+1}}^{k}-m_{x_{i,j}}^{k}\right)$$
$$+\frac{N_1N_2}{\rho c}H\left(m_{xy_{i+1,j+1}}^{k}+m_{xy_{i+1,j}}^{k}+m_{xy_{i,j+1}}^{k}+m_{xy_{i,j}}^{k}\right)$$
$$+\frac{\rho_L h_1^2}{12}[\frac{N_1u}{L_x^2\rho h}\left(-\varphi_{xt_{i+1,j+1}}^{k}-\varphi_{xt_{i+1,j}}^{k}-\varphi_{xt_{i,j+1}}^{k}-\varphi_{xt_{i,j}}^{k}\right)$$
$$+\frac{N_1cu^2}{L_x^2\rho}\left(\varphi_{xt_{i+1,j+1}}^{k}-\varphi_{xt_{i+1,j}}^{k}-\varphi_{xt_{i,j+1}}^{k}-\varphi_{xt_{i,j}}^{k}\right)$$
$$+\frac{N_3u^2}{L_x^2\rho h}\left(m_{x_{i+1,j+1}}^{k}-m_{x_{i+1,j}}^{k}-m_{x_{i,j+1}}^{k}-m_{x_{i,j}}^{k}\right)$$
$$+\frac{N_3u}{L_x^3\rho h}\left(m_{x_{i+1,j+1}}^{k}+m_{x_{i+1,j}}^{k}-m_{x_{i,j+1}}^{k}-m_{x_{i,j}}^{k}\right)$$
$$-\frac{N_3u^2v}{L_x^3\rho h}\left(m_{y_{i+1,j+1}}^{k}-m_{y_{i+1,j}}^{k}-m_{y_{i,j+1}}^{k}-m_{y_{i,j}}^{k}\right)$$
$$-\frac{N_3u^2v}{L_x^2\rho h}\left(m_{y_{i+1,j+1}}^{k}+m_{y_{i+1,j}}^{k}-m_{y_{i,j+1}}^{k}-m_{y_{i,j}}^{k}\right)]\mathrm{B}_n \quad (84)$$

$$
\begin{aligned}
&-\frac{N_1N_2}{\rho\gamma c}\frac{h}{2}\left(q_{x_{i+1,j+1}}^{k+1}+q_{x_{i+1,j}}^{k+1}+q_{x_{i,j}}^{k+1}+q_{x_{i,j+1}}^{k+1}\right)\\
&+\frac{N_1N_2}{\rho c}\left(m_{xy_{i+1,j+1}}^{k+1}+m_{xy_{i+1,j}}^{k+1}+m_{xy_{i,j+1}}^{k+1}-m_{xy_{i,j}}^{k+1}\right)\\
&+\frac{N_1N_2}{\rho c}H\left(m_{y_{i+1,j+1}}^{k+1}+m_{y_{i,j+1}}^{k+1}-m_{y_{i+1,j}}^{k+1}-m_{y_{i,j}}^{k+1}\right)\\
&+\frac{\rho_L h_1^3}{12}[\frac{N_1u}{L_x^2\rho h}\left(\varphi_{yt_{i+1,j+1}}^{k+1}+\varphi_{yt_{i+1,j}}^{k+1}+\varphi_{yt_{i,j+1}}^{k+1}+\varphi_{yt_{i,j}}^{k+1}\right)\\
&+\frac{N_1u}{L_x^2\rho h}\left(\varphi_{xt_{i+1,j+1}}^{k+1}+\varphi_{xt_{i+1,j}}^{k+1}-\varphi_{xt_{i,j+1}}^{k+1}-\varphi_{xt_{i,j}}^{k+1}\right)\\
&+\frac{N_3u}{L_x^2\rho h}\left(m_{y_{i+1,j+1}}^{k+1}+m_{y_{i+1,j}}^{k+1}+m_{y_{i,j+1}}^{k+1}+m_{y_{i,j}}^{k+1}\right)\\
&+\frac{N_3u^2}{L_x^3\rho h}H\left(m_{y_{i+1,j+1}}^{k+1}+m_{y_{i,j+1}}^{k+1}-m_{y_{i+1,j}}^{k+1}-m_{y_{i,j}}^{k+1}\right)\\
&-\frac{N_3u^2\nu}{L_x^2\rho h}\left(m_{x_{i+1,j+1}}^{k+1}+m_{x_{i+1,j}}^{k+1}+m_{x_{i,j+1}}^{k+1}+m_{x_{i,j}}^{k+1}\right)\\
&-\frac{N_3u^2\nu}{L_x^2\rho h}\left(m_{x_{i+1,j+1}}^{k+1}+m_{x_{i,j+1}}^{k+1}-m_{x_{i+1,j}}^{k+1}-m_{x_{i,j}}^{k+1}\right)]\mathrm{B}_n\\
&=\frac{N_1N_2}{\rho\gamma c}\frac{h}{2}\left(q_{x_{i+1,j+1}}^{k}+q_{x_{i+1,j}}^{k}+q_{x_{i,j}}^{k}+q_{x_{i,j+1}}^{k}\right)\\
&+\frac{N_1N_2}{\rho c}\left(m_{xy_{i+1,j+1}}^{k}+m_{xy_{i+1,j}}^{k}-m_{xy_{i,j+1}}^{k}-m_{xy_{i,j}}^{k}\right)\\
&+\frac{N_1N_2}{\rho c}H\left(m_{y_{i+1,j+1}}^{k}+m_{y_{i,j+1}}^{k}-m_{y_{i+1,j}}^{k}-m_{y_{i,j}}^{k}\right)\\
&+\frac{\rho_L h_1^3}{12}[\frac{N_1u}{L_x^2\rho h^2}\left(\varphi_{yt_{i+1,j+1}}^{k}-\varphi_{yt_{i+1,j}}^{k}-\varphi_{yt_{i,j+1}}^{k}-\varphi_{yt_{i,j}}^{k}\right)\\
&+\frac{N_1u}{L_x^2\rho h}\left(\varphi_{xt_{i+1,j+1}}^{k}+\varphi_{xt_{i+1,j}}^{k}-\varphi_{xt_{i,j+1}}^{k}-\varphi_{xt_{i,j}}^{k}\right)\\
&+\frac{N_3u^2}{L_x^2\rho h}\left(-m_{y_{i+1,j+1}}^{k}-m_{y_{i+1,j}}^{k}-m_{y_{i,j+1}}^{k}-m_{y_{i,j}}^{k}\right)\\
&+\frac{N_3u^2}{L_x^2\rho h}H\left(m_{y_{i+1,j+1}}^{k}+m_{y_{i,j+1}}^{k}-m_{y_{i+1,j}}^{k}-m_{y_{i,j}}^{k}\right)\\
&-\frac{N_3u^2\nu}{L_x^2\rho h}\left(-m_{x_{i+1,j+1}}^{k}-m_{x_{i+1,j}}^{k}-m_{x_{i,j+1}}^{k}-m_{x_{i,j}}^{k}\right)\\
&-\frac{N_3u^2\nu}{L_x^2\rho h}H\left(m_{x_{i+1,j+1}}^{k}+m_{x_{i,j+1}}^{k}-m_{x_{i+1,j}}^{k}-m_{x_{i,j}}^{k}\right)]\mathrm{B}_n
\end{aligned}
\tag{85}
$$

$$\begin{aligned}
&u\left(m_{x_{i+1,j+1}}^{k+1}+m_{x_{i+1,j}}^{k+1}+m_{x_{i,j+1}}^{k+1}+m_{x_{i,j}}^{k+1}\right)\\
&+N_1\left(\varphi_{xt_{i+1,j+1}}^{k+1}+\varphi_{xt_{i+1,j}}^{k+1}-\varphi_{xt_{i,j+1}}^{k+1}-\varphi_{xt_{i,j}}^{k+1}\right)\\
&+vHN_1\left(\varphi_{yt_{i+1,j+1}}^{k+1}+\varphi_{yt_{i+1,j}}^{k+1}+\varphi_{yt_{i,j+1}}^{k+1}+\varphi_{yt_{i,j}}^{k+1}\right)\\
&=+u\left(m_{x_{i+1,j+1}}^{k}+m_{x_{i+1,j}}^{k}+m_{x_{i,j+1}}^{k}+m_{x_{i,j}}^{k}\right)\\
&+N_1\left(-\varphi_{xt_{i+1,j+1}}^{k}-\varphi_{xt_{i+1,j}}^{k}+\varphi_{xt_{i,j+1}}^{k}+\varphi_{xt_{i,j}}^{k}\right)\\
&+vHN_1\left(-\varphi_{yt_{i+1,j+1}}^{k}-\varphi_{yt_{i+1,j}}^{k}+\varphi_{yt_{i,j+1}}^{k}+\varphi_{yt_{i,j}}^{k}\right)
\end{aligned}\tag{86}$$

$$\begin{aligned}
&u\left(m_{y_{i+1,j+1}}^{k+1}+m_{y_{i+1,j}}^{k+1}+m_{y_{i,j+1}}^{k+1}+m_{y_{i,j}}^{k+1}\right)\\
&+HN_1\left(\varphi_{yt_{i+1,j+1}}^{k+1}+\varphi_{yt_{i+1,j}}^{k+1}-\varphi_{yt_{i,j+1}}^{k+1}-\varphi_{yt_{i,j}}^{k+1}\right)\\
&+vN_1\left(\varphi_{xt_{i+1,j+1}}^{k+1}+\varphi_{xt_{i+1,j}}^{k+1}-\varphi_{xt_{i,j+1}}^{k+1}-\varphi_{xt_{i,j}}^{k+1}\right)\\
&=+u\left(m_{x_{i+1,j+1}}^{k}+m_{x_{i+1,j}}^{k}+m_{x_{i,j+1}}^{k}+m_{x_{i,j}}^{k}\right)\\
&+HN_1\left(-\varphi_{yt_{i+1,j+1}}^{k}-\varphi_{yt_{i+1,j}}^{k}+\varphi_{yt_{i,j+1}}^{k}+\varphi_{yt_{i,j}}^{k}\right)\\
&+vN_1\left(-\varphi_{xt_{i+1,j+1}}^{k}-\varphi_{xt_{i+1,j}}^{k}+\varphi_{xt_{i,j+1}}^{k}+\varphi_{xt_{i,j}}^{k}\right)
\end{aligned}\tag{87}$$

$$\begin{aligned}
&u\left(m_{xy_{i+1,j+1}}^{k+1}+m_{xy_{i+1,j}}^{k+1}-m_{xy_{i,j+1}}^{k+1}+m_{xy_{i,j}}^{k+1}\right)\\
&+N_1\left(\frac{1-v}{2}\right)\left(\varphi_{yt_{i+1,j+1}}^{k+1}+\varphi_{yt_{i+1,j}}^{k+1}-\varphi_{yt_{i,j+1}}^{k+1}-\varphi_{yt_{i,j}}^{k+1}\right)\\
&-N_1H\left(\frac{1-v}{2}\right)\left(\varphi_{xt_{i+1,j+1}}^{k+1}+\varphi_{xt_{i+1,j}}^{k+1}-\varphi_{xt_{i,j+1}}^{k+1}-\varphi_{xt_{i,j}}^{k+1}\right)\\
&=u\left(m_{xy_{i+1,j+1}}^{k}+m_{xy_{i+1,j}}^{k}+m_{xy_{i,j+1}}^{k}+m_{xy_{i,j}}^{k}\right)\\
&+N_1\left(\frac{1-v}{2}\right)\left(-\varphi_{yt_{i+1,j+1}}^{k}-\varphi_{yt_{i+1,j}}^{k}+\varphi_{yt_{i,j+1}}^{k}+\varphi_{yt_{i,j}}^{k}\right)\\
&+N_1H\left(\frac{1-v}{2}\right)\left(-\varphi_{xt_{i+1,j+1}}^{k}-\varphi_{xt_{i+1,j}}^{k}+\varphi_{xt_{i,j+1}}^{k}+\varphi_{xt_{i,j}}^{k}\right)
\end{aligned}\tag{88}$$

$$
\begin{aligned}
&u\left(q_{x_{i+1,j+1}}^{k+1}+q_{x_{i+1,j}}^{k+1}+q_{x_{i,j+1}}^{k+1}+q_{x_{i,j}}^{k+1}\right)\\
&+\gamma\left(d_{t_{i+1,j+1}}^{k+1}+d_{t_{i+1,j}}^{k+1}-d_{t_{i,j+1}}^{k+1}-d_{t_{i,j}}^{k+1}\right)\\
&-\gamma\frac{h}{2}\left(\varphi_{xt_{i+1,j+1}}^{k+1}+\varphi_{xt_{i+1,j}}^{k+1}+\varphi_{xt_{i,j+1}}^{k+1}+\varphi_{xt_{i,j}}^{k+1}\right)\\
&=u\left(q_{x_{i+1,j+1}}^{k}+q_{x_{i+1,j}}^{k}+q_{x_{i,j+1}}^{k}+q_{x_{i,j}}^{k}\right)\\
&-\gamma\left(d_{t_{i+1,j+1}}^{k}+d_{t_{i+1,j}}^{k}-d_{t_{i,j+1}}^{k}-d_{t_{i,j}}^{k}\right)\\
&+\gamma\frac{h}{2}\left(\varphi_{xt_{i+1,j+1}}^{k}+\varphi_{xt_{i+1,j}}^{k}+\varphi_{xt_{i,j+1}}^{k}+\varphi_{xt_{i,j}}^{k}\right)
\end{aligned}
\tag{89}
$$

$$
\begin{aligned}
&u\left(q_{y_{i+1,j+1}}^{k+1}+q_{y_{i+1,j}}^{k+1}+q_{y_{i,j+1}}^{k+1}+q_{y_{i,j}}^{k+1}\right)\\
&+\gamma H\left(d_{t_{i+1,j+1}}^{k+1}+d_{t_{i+1,j}}^{k+1}-d_{t_{i,j+1}}^{k+1}-d_{t_{i,j}}^{k+1}\right)\\
&-\gamma\frac{h}{2}\left(\varphi_{yt_{i+1,j+1}}^{k+1}+\varphi_{yt_{i+1,j}}^{k+1}+\varphi_{yt_{i,j+1}}^{k+1}+\varphi_{yt_{i,j}}^{k+1}\right)\\
&=u\left(q_{y_{i+1,j+1}}^{k}+q_{y_{i+1,j}}^{k}+q_{y_{i,j+1}}^{k}+q_{y_{i,j}}^{k}\right)\\
&-\gamma H\left(d_{t_{i+1,j+1}}^{k}+d_{t_{i+1,j}}^{k}-d_{t_{i,j+1}}^{k}-d_{t_{i,j}}^{k}\right)\\
&+\gamma\frac{h}{2}\left(\varphi_{yt_{i+1,j+1}}^{k}+\varphi_{yt_{i+1,j}}^{k}+\varphi_{yt_{i,j+1}}^{k}+\varphi_{yt_{i,j}}^{k}\right)
\end{aligned}
\tag{90}
$$

$$
\begin{aligned}
&\frac{h}{2}\left(d_{t_{i+1,j+1}}^{k+1}+d_{t_{i+1,j}}^{k+1}-d_{t_{i,j+1}}^{k+1}-d_{t_{i,j}}^{k+1}\right)\\
&-u\left(w_{i+1,j+1}^{k+1}+w_{i+1,j}^{k+1}+w_{i,j+1}^{k+1}+w_{i,j}^{k+1}\right)\\
&=\frac{h}{2}\left(d_{t_{i+1,j+1}}^{k}+d_{t_{i+1,j}}^{k}+d_{t_{i,j+1}}^{k}+d_{t_{i,j}}^{k}\right)\\
&+u\left(w_{i+1,j+1}^{k}+w_{i+1,j}^{k}+w_{i,j+1}^{k}+w_{i,j}^{k}\right)
\end{aligned}
\tag{91}
$$

These set of algebraic equations to be solved may now be written in matrix form as follows[7].
$H_{i,j+1}\, S'_{i,j+1} + I_{i+1,j+i}\, S'_{i+1,j+1} = - G_{i,j}S'_{I,,j} - J_{i+1,j}\, S'_{i+1,j} + K_{i,\,j}\, S^{\circ}_{i,j} + L_{i,j\,+i}\,,\ S^{\circ}_{i,j+1}\, M_{i+1}\, S^{\circ}_{i+1,j} + N_{i+1,\,j+1}\, S^{\circ}_{i+1,j+1} + P_1$ (99)

i = 1, 2, 3 ..., N-1; j = 1, 2, 3, ... M -1
where N and M are the numbers of the nodal points along X and Y axes respectively
Each term in equations (99) is a 9×9 matrix. Which is in the form:

$$H_{i,j+1}S'_{i,j+1} =$$

$$\begin{pmatrix} u & 0 & 0 & 0 & 0 & -N_1 & -vHN_1 & 0 & 0 \\ 0 & u & 0 & 0 & 0 & -vN_1 & -HN_1 & 0 & 0 \\ 0 & 0 & u & 0 & 0 & -N_1e_1H & -N_1e_1 & 0 & 0 \\ 0 & 0 & 0 & u & 0 & -\frac{\gamma h}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & u & 0 & -\frac{\gamma h}{2} & 0 & -\gamma H \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -u & \frac{h}{2} \\ e_2 & (-e_2v) & 0 & \begin{pmatrix} . \\ A \\ . \end{pmatrix} & 0 & (\mathrm{B}) & 0 & e\gamma & u\begin{pmatrix} \frac{N_1}{L_x} \\ +e_8 \\ +e_8cu \end{pmatrix} \\ \begin{pmatrix} \gamma e_6 \\ -e_9u \\ +e_9 \end{pmatrix} & 0 & (e_6\gamma H) & (e_{12}h^2) & 0 & \begin{pmatrix} . \\ C \\ . \end{pmatrix} & 0 & 0 & 0 \\ \begin{pmatrix} e_9L_xvu \\ +e_9vuL_xH \end{pmatrix} & \begin{pmatrix} . \\ D \\ . \\ . \end{pmatrix} & (-e_6\gamma) & 0 & \left(\frac{e_6h}{2}\right) & (-e_{11}) & (-e_{10}u-e_{11}) & 0 & 0 \end{pmatrix} \begin{pmatrix} m_{x_{i,j+1}} \\ m_{y_{i,j+1}} \\ m_{xy_{i,j+1}} \\ q_{x_{i,j+1}} \\ q_{y_{i,j+1}} \\ \varphi_{xt_{i,j+1}} \\ \varphi_{yt_{i,j+1}} \\ w_{i,j+1} \\ d_{t_{i,i+1}} \end{pmatrix}$$

Where,

$$\begin{pmatrix} . \\ A \\ . \end{pmatrix} = \begin{pmatrix} -e_6 \\ -e_6u \\ -e_4u \end{pmatrix},$$

$$(B) = \left(\frac{e_4h^2\gamma}{2}L_x\right)$$

$$\begin{pmatrix} . \\ C \\ . \end{pmatrix} = \begin{pmatrix} -e_{10}u \\ -e_{11} \\ -he_{11}cu \end{pmatrix}$$

$$\begin{pmatrix} . \\ D \\ . \\ . \end{pmatrix} = \begin{pmatrix} -\gamma e_6H \\ -e_9L_xu \\ -e_9L_xu \\ -e_9L_xuH \end{pmatrix}$$

and

$$e_1 = \frac{1-v}{2}$$

$$e_2 = \frac{mhu^2 N_3}{2aL_x} B_n$$

$$e_4 = \frac{muhN_1}{2aL_x\gamma} B_n$$

$$e_6 = \frac{N_1 N_2}{\rho\gamma c}$$

$$e_8 = \frac{\gamma N_1 m}{L_x a} B_n$$

$$e_9 = B_n \frac{N_3 u h_1^3 \rho_L}{12\rho h L_x^3}$$

$$e_{10} = \frac{N_1 \gamma^2}{12}$$

$$e_{11} = B_n \frac{N_1 u h_1^3 \rho_L}{12 L_x^2 \rho h}$$

Result Discussion

The numerical calculations were carried out for a simply supported rectangular isotropic damped highway bridge (plate) resting on a Winkler foundation and subjected to a moving railway vehicle (load.). Shear deformation effect was considered, while the effect of rotatory inertia was neglected. For a specific value of the parameters, deflection of the highway bridge was calculated and plotted as a function of time. The following results were obtained: The response amplitude of the highway bridge resting on a Winkler foundation decreases with an increase in the value of K, the foundation's constant, for various time t, but fixed values of both the velocity and contact area of the moving partially distributed vehicle. (as we can see in figures 3). The response maximum amplitude decreases with an increase in the value of damping coefficient, for fixed value of K, Ar and U. (as we can see in figure 5). The response maximum amplitude of the shear highway bridge supported continuously by an elastic Winkler subgrade decreases with an increase in contact area (Ar). (as we can see in figure 2). Also for the same time range and fixed values of K and Ar, it is observed that the shear highway bridge has a larger value of the maximum amplitude than non- Mindlin, for all the values of velocity considered. (as we can see in figure 1).

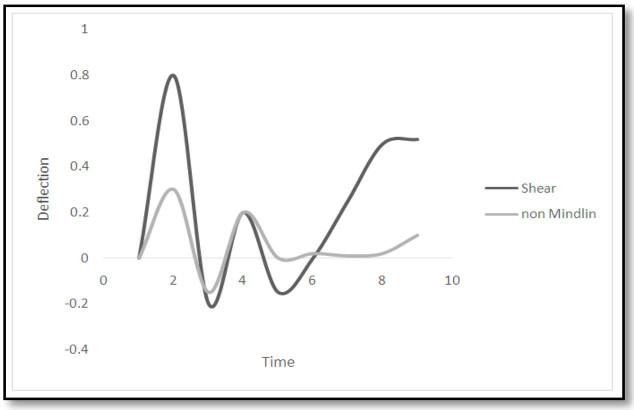

Figure 1: Deflection of the Shear and non Mindlin highway bridges for fixed K, and various values of time

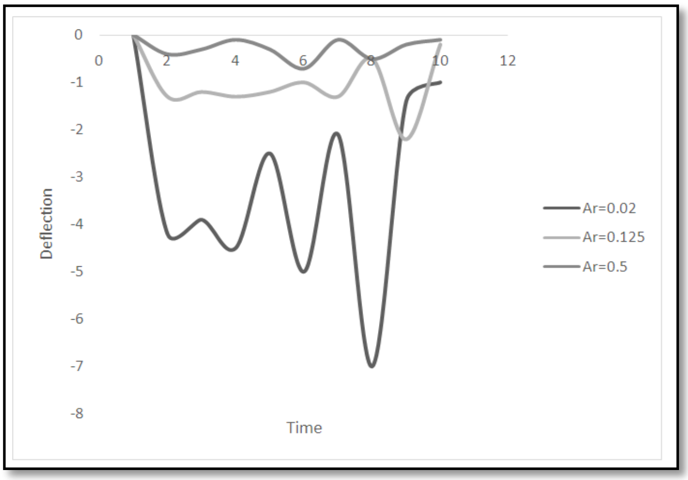

Figure 2: Deflection of the highway bridge for various values of contact area (Ar) and time

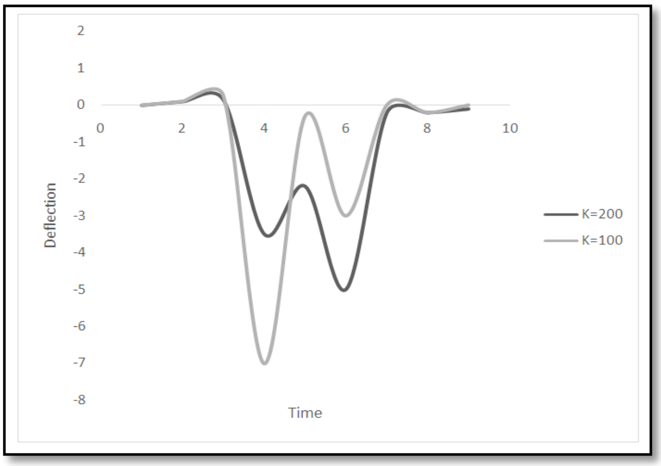

Figure 3: Deflection of the highway bridge for various values of foundation stiffness (K) and time

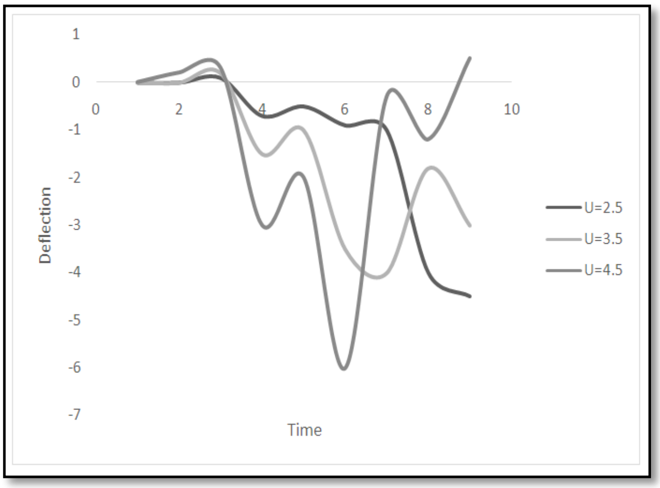

Figure 4: Deflection of the highway bridge for various values velocity (U)and time

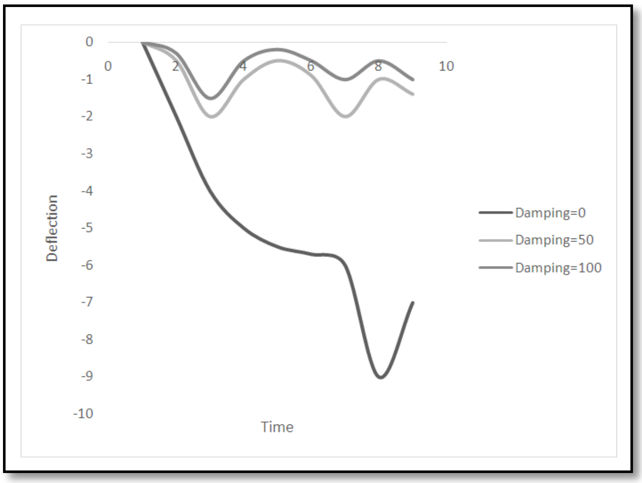

Figure 5: Deflection of the highway bridge for various values of damping coefficient and time

Conclusion

The structure of interest was a railway bridge modelled as an isotropic shear rectangular elastic highway bridge, on Winkler foundation, under the influence of a uniform partially distributed moving vehicle. The problem was to determine the dynamic response of the whole system.Finite Difference technique was adopted in solving the resulting first order coupled partial differential equations obtained from the governing equations, for the simply supported highway bridge.The effects of the foundation, damping and shear deformation on the dynamic response of the isotropic highway bridge to the moving vehicle, give more realistic results for practical application.

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