Application of Differential Transform Method to Vibration Analysis of Damped Railway Bridge on Pasternak Foundation under Moving Train

Agarana M.C. and Ede A.N.

Abstract— **In this paper, we used differential transform method (DTM) to analyse free vibration of a railway bridge, modelled as an orthotropic rectangular plate, supported by Pasternak foundation. The effect of damping was considered. The present method transformed the governing equation to its algebraic form. Solution form to similar equation was adopted. The results obtained are in agreement with the ones in literature, and it shows that the technique introduced is easy to apply to such differential equation governing the vibration of such plates.**

Index Terms— **Pasternak foundation, Vibration, Damping, Differential transform method**

I. INTRODUCTION

All branches of transport have experienced great advances characterized by increasing weight and high speed of vehicles including railway vehicles [9,10]. Railway bridges vibrate as loads move on them [1,10]. In this work, the railway bridge was modelled as a rectangular plate. The dynamic behaviour of Plates, as structural elements, highly influence overall performance of a structure [1,9,10]. The purpose of this paper is to implement the DTM to the fourth order differential equation governing the free vibration of damped orthotropic plates [1,7,8]. Most of the other methods used in solving such problem, are computationally intensive. On the other hand, DTM is relatively simple [2,3,4,5]. It involves the transformation of differential equations to their algebraic forms [3,6].

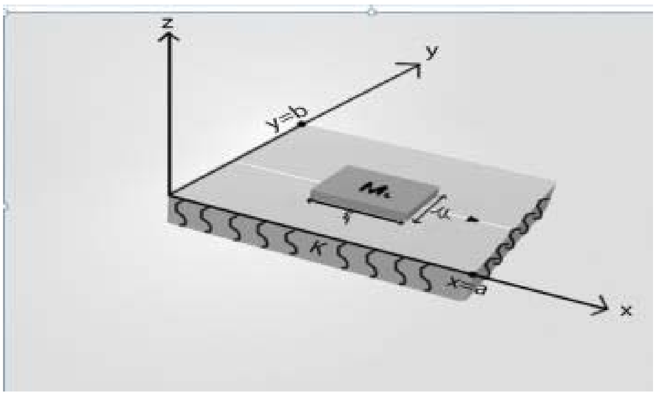

Figure 1. A moving load on a plate supported by Pasternak foundation

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II FORMULATION OF PROBLEM

The equation governing the vibration of damped simply supported orthotropic plate resting on Pasternak foundation subject to a moving load is given by [1,9];

$$\alpha_1 \frac{\partial^4 w}{\partial x^4} + 2\alpha_2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \alpha_3 \frac{\partial^4 w}{\partial y^4} = -Kw - m\frac{\partial^2 w}{\partial t^2} - 2m\gamma \frac{\partial w}{\partial t} - G_1\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) \quad (1)$$

where

w = w(x,y,t) is the deflection of the plate at the point (x, y).

t = time in seconds

E = Young's modulus

m = mass density per unit area

H = thickness of plate

v = velocity

K, G_1 = foundation stiffness

γ = viscous damping coefficient

α_1 = flexural rigidity in the x direction

α_3 = flexural rigidity in the y direction

α_2 = effective torsional rigidity

A. *Initial and boundaryConditions*

The following initial and boundary conditions are used [1,9]

$$w(x, y, 0) = w_t(x, y, 0) = 0$$

$$w(0, y, t) = w(a, y, t) = w_{xx}(0, y, t) = w_{xx}(a, y, t) = 0$$

$$w(x, 0, t) = w(x, b, t) = w_{yy}(x, 0, t) = w_{yy}(x, b, t) = 0$$

III. METHOD OF SOLUTION

The Taylor series expansion of a function f(x), about x=0, is given by [4]:

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$$f(x)=\sum_{x=0}^{\infty}(x-x_0)^k F_k$$

where $F_k=\frac{1}{k!}\left[\frac{d^k f}{dx^k}\right]_{x=x_0}$

F_k is called the k[th] order differential transform of f(x) about the point x = x_0. Usually, the series is truncated to finite number of terms for practical problems. For this work we used eight terms. Assuming the two opposite edges Y = 0 and Y = 1 to be simply supported, the deflection function can be expressed as [1,4,5]

$$W=\overline{w}(X)\sin(m\pi Y) \quad (2)$$

Substituting equation (4) into equation (1) leads to [4,5,6]

$$\alpha_1\frac{d^4\overline{w}}{dx^4}-2\alpha_2 m^2\pi^2\frac{d^2\overline{w}}{dx^2}-\alpha_3 m^4\pi^4\overline{w}+K\overline{w}+$$
$$G_1\frac{d^2\overline{w}}{dx^2}-G_1 m^2\pi^2\overline{w}=0 \quad (3)$$

Taking the differential transform of equation (5) at x_0 = 0 and using the differential transforms of some of the fundamental functions reported in [2,3,4,5,6], we have

$$\alpha_1\frac{k+4!}{k!}\overline{w}_{k+4}-(2\alpha_2 m^2\pi^2)\frac{k+2!}{k!}\overline{w}_{k+2}+$$
$$G_1\frac{k+2!}{k!}\overline{w}_{k+2}-(\alpha_3 m^4\pi^4+G_1 m^2\pi^2-K)\overline{w}_k=0 \quad (4)$$

Equation (6) can be written in a more concise form as :

$$\alpha_1\frac{k+4!}{k!}\overline{w}_{k+4}-(2\alpha_2 m^2\pi^2-G_1)\frac{k+2!}{k!}\overline{w}_{k+2}-$$
$$(\alpha_3 m^4\pi^4+G_1 m^2\pi^2-K)\overline{w}_k=0 \quad (4)$$

The boundary conditions was also transformed in similar way.

Now equation (7) can be rewritten in the form; [4],

$$\frac{d^4w}{dx^4}-2(m\pi\lambda)^2\frac{d^2w}{dx^2}-(\Omega^2-m^4\pi^4\lambda^4-K)\overline{w}=0 \quad (5)$$

where

$$2m^2\pi^2=\frac{G_1}{\alpha_1\lambda^2-\alpha_2}$$

$$\text{and}\quad (m\pi\lambda)^4=\frac{K+\alpha_1\Omega^2-\alpha_1K-G_1m^2\pi^2}{\alpha_3\alpha_1}$$

The general solution of equation (8) can be expressed as [4,6]

$$\overline{w}_k=\frac{d_1R_1^k+d_2R_2^k}{k!},\ \text{k=0,1,2,3...}\infty \quad (6)$$

where

$$R_1^k=(m\pi\lambda)^2+\sqrt{\Omega^2-K}$$
$$R_2^k=(m\pi\lambda)^2-\sqrt{\Omega^2-K}$$

and

$\lambda=\frac{a}{b}$, Ω^2 is the frequency parameter, a and b are the length and breadth of the plate respectively.

G_1 and K are the Pasternak foundation moduli, d_1 and d_2 are parameters.

IV. RESULT AND DISCUSSION

For the numerical work, the fourth order differential equation (1) was solved using DTM. The values of various parameters used are: m = 7, V = 3.5, 4.5, 5.5, a=1, b=2, d_1=1, d_2=2, γ = 0.02, 0.34, 0.55, G_1= 0, 0.09, 0.9, 0.18, E = 2.109 x 10^7, Ω^2 =101 and K=0,1,2,3,4,5.

It can be seen from Figure 2 that as the foundation modulus increases the deflection of the damped orthotropic plate decreases. This implies that the foundation reduces deflection of the plate. Also Figure 3 - 5 show the effect of damping on the deflection of orthotropic rectangular plate. It can be seen from the Figures that the higher the damping coefficient the lower the maximum amplitude. The damping effect also reduces the deflection of a plate under a moving load.

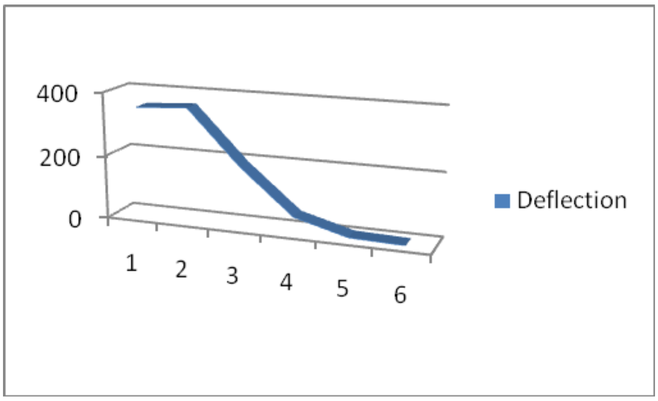

Figure 2. Deflection of the plate as the foundation modulus increases for various time t.

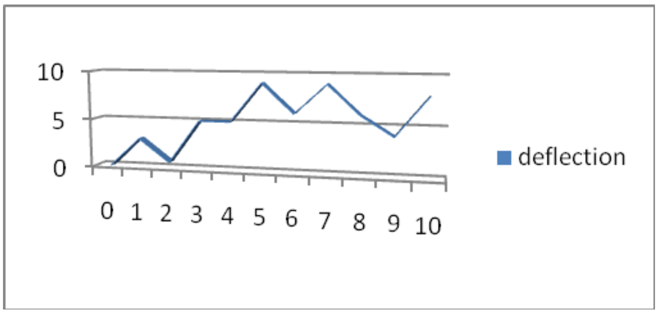

Figure 3. Deflection of the plate when damping coefficient is 0.02, for various time t

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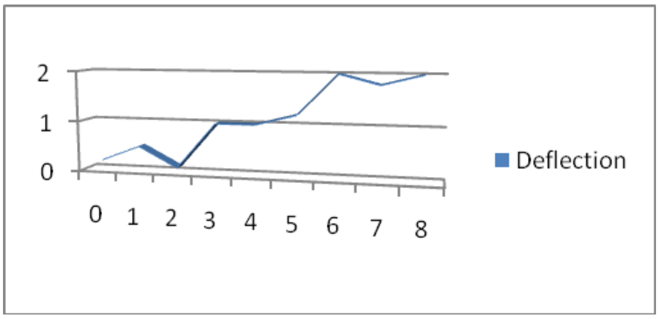

Figure 4. Deflection of the plate when damping coefficient is 0.34, for various time t

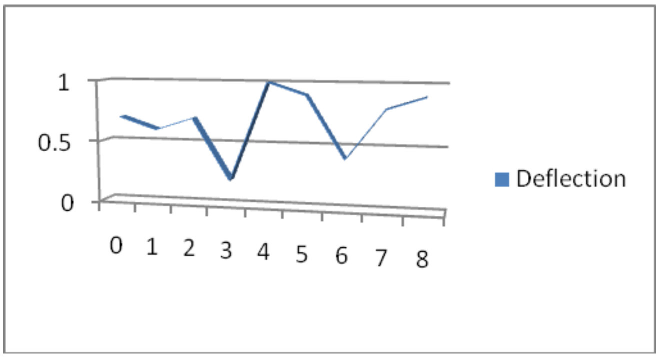

Figure 5. Deflection of the plate when damping coefficient is 0.55, for various time t

V. CONCLUSION

Application of differential transform method to the analysis of transverse vibration of orthotropic damped rectangular railway bridge supported by Pasternak foundation was carried out. The results obtained revealed that both the foundation modulus and damping have effects on the deflection of the railway bridges which was modelled as the plate in this work. These results and others obtained are consistent with the ones in the literature, as evident in some works referenced.

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