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Monte Carlo Simulation of Bottomhole Coordinates in Directional Drilling

Rowland Jerry Okechukwu EKEOCHA

ABSTRACT [ENGLISH/ANGLAIS]

Increasing interest is now focused on directional survey and on the uncertainty inherent in the calculation of the bottom hole coordinates because many wells are drilled from offshore platforms and urban drill sites. These uncertainties may result from errors due to the limitation on instrument accuracy, unconcentric positioning of the tools with the borehole and may include reading errors. It is therefore necessary to develop methods for determining the cumulative effect of these errors on the bottomhole coordinates and reducing such effects. In this study, Monte Carlo simulation method is adopted as a technique of achieving this objective. The inclination angle, Ii and the azimuthal angle, Ai are treated as random variables with their associated probability density function $P(I_i)$ and $P(A_i)$ respectively. These represent the distributions of the potential survey measurements in a given well. The parameter Si which represents the linear segment constituting the curvilinear axis of the wellbore is treated deterministically since its values can be measured accurately. Treating Ii and Ai as random variables in a wellbore model is justified by the fact that both of them, though deterministic, can hardly ever be measured accurately in any subsurface survey. The results of the study show that the probability of the estimated bottomhole positions being true is nearly unity for vertical well and slightly less for directional well.

Keywords: Bottomhole coordinates, directional drilling, Monte Carlo simulation

RÉSUMÉ [FRANÇAIS/FRENCH]

L'intérêt croissant se concentre maintenant sur l'enquête directionnelle et sur l'incertitude inhérente dans le calcul des coordonnées d'un puits parce que de nombreux puits sont forés à partir de plates-formes près du littoral et des sites de forage en milieu urbain. Ces incertitudes peuvent résulter d'erreurs dues au manque de précision de l'instrument, au positionnement non concentrique des outils avec forage et aux erreurs de lecture. Il est donc nécessaire de développer des méthodes de détermination de l'effet cumulatif de ces erreurs sur les coordonnées du puits et de réduire ces effets. Dans cette étude, la méthode de simulation de Monte Carlo est adoptée pour atteindre cet objectif. L'angle d'inclinaison, Ii et l'angle azimutal, Ai sont traités comme des variables aléatoires avec leur densité de probabilité associée à la fonction P (I_i) et P (A_i), respectivement. Celles-ci représentent les distributions des potentielles mesures d'enquête dans un puits donné. Le paramètre Si qui représente le segment linéaire constituant l'axe curviligne du puits est traité de façon déterministe puisque ses valeurs peuvent être mesurées avec précision. Le traitement de Ii et Ai comme des variables aléatoires dans un modèle de puits est justifiée par le fait que tous les deux, quoique déterminant, peuvent difficilement être mesurée avec précision dans toute l'étude du sous-sol. Les résultats de l'étude montrent que la probabilité des positions estimées dans le fond du puits étant vrai est voisin de l'unité pour le puits vertical et un peu moins pour le puits directionnel.

Mots-clés: Coordonnées de fond de puits, forage directionnel, la simulation de Monte Carlo

Affiliations:
Department of Mechanical Engineering, University of Nigeria, Nsukka, NIGERIA

Address for Correspondence/ Adresse pour la Correspondance:
ekeochaj@yahoo.com

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INTRODUCTION

Directional drilling is defined by Schlumberger [1] as the science of deviation of a wellbore along a planned path (trajectory) to a target located at a given distance and direction from the vertical. Drilling from offshore and urban drill sites has given rise to directional survey. Accurate computation of the bottomhole coordinates has been attempted through several methods which include the acceleration and compensated acceleration methods, the radius of curvature method, the tangential method, the angle-averaging method, the minimum curvature method and the mercury method [2, 3]. These methods are deterministic approaches to the computation of the bottomhole coordinates in a directional survey. Each method has its own assumptions which are usually the contributing sources of error. These single-case computations are generally misleading in terms of decision making especially when one looks critically at the assumptions made during the development of the methods and the attendant errors accruing from the measurements of the inclination and azimuthal angles by survey tools like the whipstock gyroscope. Positional accuracy is very important in directional drilling involving the following situations: boundary restrictions, avoiding the intersection of two or more wellbores under a platform, bringing a relief well besides a blowing well, location of formation for proper geological definition with respect to a contour or discontinuity and computing net reservoir thickness [4-8]. To achieve these objectives, a method has to be devised that will simultaneously obtains the wellbore coordinates and determines the degree of uncertainty associated with the computed coordinates.

This in turn serves as guide to the economic viability of the drilling operation. In this study, Monte Carlo simulation technique is adopted in the computation of the bottomhole coordinates.

The inclination and azimuthal angles are generated as uniformly distributed random variables (from the Fibonacci sequence between zero and unity) and substituted into the models for the bottomhole coordinates which are assumed to be binomially distributed. To completely define (determine) the bottomhole coordinates, the expected value (mean) variance, covariance (spread), correlation coefficient and the probability of locating the coordinates are also computed.

Finally, Monte Carlo simulation technique offers a means of planning ahead a directional survey. The main limitation of the techniques is its lack of check on model error as it only accounts for the statistical errors through the statistical test for randomness of the generated values for the inclination and azimuthal angles. However, a better choice of model may reduce the model errors considerably which in turn ensures that the pay zone can be located from any reference point on the surface.

MATERIALS AND METHODS

The methodology involves the use of computer to perform the following operations:

i. Generate uniformly distributed random variables between zero and unity from the Fibonacci sequence to represent the inclination and azimuthal angles in radians.

ii. Compute the bottomhole coordinates in the X, Y and Z directions using the following wellbore models [9].

$$X = \sum_{i=1}^{N} X_i = \sum_{i=1}^{N} S_i \sin I_i \sin A_i$$

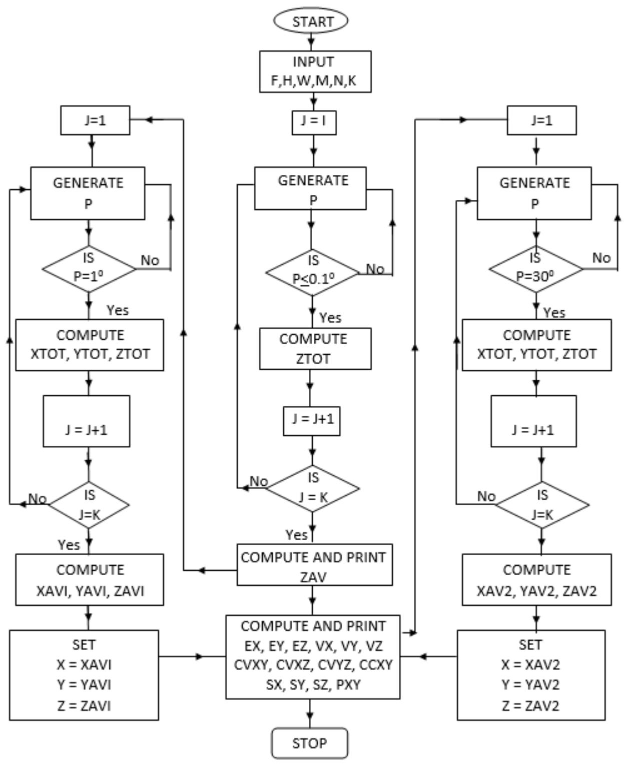
$$Y = \sum_{i=1}^{N} Y_i = \sum_{i=1}^{N} S_i \sin I_i \cos A_i$$

$$Z = \sum_{i=1}^{N} Z_i = \sum_{i=1}^{N} S_i \cos I_i$$

Where

X, Y, Z = Bottomhole coordinates in the x, y, z directions
I_i = Inclination angle from the vertical in radians
A_i = Azimuthal angle from true North in radians
S_i = Linear segment representing the wellbore axis, ft.
N = Number of linear segments.

Figure 1: This figure shows a flow chart of the computer programme.



The next step is to compute the limits of random variables I_i and A_i. The limits are defined by Willier and Lieberman [10] as follows:

$$a_i = \text{Max}(I_i - 0.0043625, 0)$$

$$b_i = I_i + 0.0043625$$

$$\left.\begin{aligned} c_i &= A_i - 3.141 \\ d_i &= A_i + 3.141 \end{aligned}\right\} I_i \leq 0.0043625$$

$$\left.\begin{aligned} c_i &= A_i - 0.349 \\ d_i &= A_i + 0.349 \end{aligned}\right\} 0.0043625 \leq I_i \leq 0.01745$$

$$\left.\begin{aligned} c_i &= A_i - 0.0349 \\ d_i &= A_i + 0.0349 \end{aligned}\right\} I_i \geq 0.01745$$

Where

a_i, b_i = lower and upper limits of I_i respectively
c_i, d_i = lower and upper limits of A_i respectively

We computed the expected value (mean), variance, covariance (spread) and the correlation coefficient of the dependent variables x, y and z.

The expected values which correspond with our intuitive idea of average are given by the following equations [9]:

$$E(X_i) = \frac{S_i (\mathrm{Cos}\, C_i - \mathrm{Cos}\, d_i)(\mathrm{Cos}\, a_i - \mathrm{Cos}\, b_i)}{(d_i - c_i)(b_i - a_i)}$$

$$E(Y_i) = \frac{S_i (\mathrm{Cos}\, a_i - \mathrm{Cos}\, b_i)(\mathrm{Sin}\, d_i - \mathrm{Sin}\, c_i)}{(d_i - c_i)(b_i - a_i)}$$

$$E(Z_i) = \frac{S_i (\mathrm{Sin}\, b_i - \mathrm{Sin}\, a_i)}{b_i - a_i}$$

Where

$(E(X_i), E(Y_i), E(Z_i)$ are the expected values for variables X_i, Y_i and Z_i respectively.

We computed the variance which is a measure of spread [9]:

$$V(X_i) = \frac{S_i^2 (d_i - c_i - \mathrm{Sin}\, d_i\, \mathrm{Cos}\, d_i + \mathrm{Sin}\, c_i\, \mathrm{Cos}\, c_i)(b_i - a_i - \mathrm{Sin}\, b_i\, \mathrm{Cos}\, b_i + \mathrm{Sin}\, a_i\, \mathrm{Cos}\, a_i)}{4(d_i - c_i)(b_i - a_i)} - (E(X_i))^2$$

$$V(Y_i) = \frac{S_i^2 (d_i - c_i + \mathrm{Sin}\, d_i\, \mathrm{Cos}\, d_i - \mathrm{Sin}\, c_i\, \mathrm{Cos}\, c_i)(b_i - a_i - \mathrm{Sin}\, b_i\, \mathrm{Cos}\, b_i + \mathrm{Sin}\, a_i\, \mathrm{Cos}\, a_i)}{4(d_i - c_i)(b_i - a_i)} - (E(Y_i))^2$$

$$V(Z_i) = \frac{S_i^2 (b_i - a_i + \mathrm{Sin}\, b_i\, \mathrm{Cos}\, b_i - \mathrm{Sin}\, a_i\, \mathrm{Cos}\, a_i)}{2(b_i - a_i)} - (E(Z_i))^2$$

Where $V(X_i)$, $V(Y_i)$, $V(Z_i)$ = Variance of Variables X_i, Y_i and Z_i respectively

We computed the covariance of two random variables, which is another measure of spread [9]:

$$\mathrm{Cov}\,(x_i, y_i) = \frac{S_i^2 (b_i - a_i\, \mathrm{Sin}\, b_i\, \mathrm{Cos}\, b_i + \mathrm{Sin}\, a_i\, \mathrm{Cos}\, a_i)(\mathrm{Cos}\, 2c_i - \mathrm{Cos}\, 2d_i))}{8(d_i - c_i)(b_i - a_i)} - E(x_i)\, E(y_i)$$

$$\mathrm{Cov}\,(x_i, z_i) = \frac{S_i^2 (\mathrm{Cos}\, c_i - \mathrm{Cos}\, d_i)(\mathrm{Cos}\, 2a_i - \mathrm{Cos}\, 2b_i)}{4(d_i - c_i)(b_i - a_i)} - E(x_i)\, E(z_i)$$

$$\mathrm{Cov}\,(y_i, z_i) = \frac{S_i^2 (\mathrm{Cos}\, a_i - \mathrm{Cos}\, 2b_i)(\mathrm{Sin}\, d_i - \mathrm{Sin}\, c_i)}{4(d_i - c_i)(b_i - a_i)} - E(y_i)\, E(z_i)$$

Where $\mathrm{Cov}\,(x_i, y_i)$ = Covariance of variables x_i and y_i and so on.

One then computes the correlation coefficient which gives the linear relationship of two random variables [9]:

$$CCx_i\, y_i = \frac{\mathrm{Cov}\,(x_i\, y_i)}{[V(x_i)\, V(y_i)]^{1/2}}$$

$$CCx_i\, z_i = \frac{\mathrm{Cov}\,(x_i\, z_i)}{[V(x_i)\, V(z_i)]^{1/2}}$$

$$CCy_i\, z_i = \frac{\mathrm{Cov}\,(y_i\, z_i)}{[V(y_i)\, V(z_i)]^{1/2}}$$

Where $CCx_i\, y_i$ = Correlation coefficient of x_i and y_i.

Finally, we computed the probability that the bottomhole coordinates are true by the following equation [11]:

$$P(\lambda^2) = 1 - e^{-[\lambda^2/2(1-\rho^2)]}$$

Where $P(\lambda^2)$ = Joint probability that the bottomhole positions x, y are true; ρ = Correlation coefficient of variables x and y.

The flow chart of the computer programme is presented in figure 1.

RESULTS

Monte Carlo simulation technique is applied to two hypothetical wells. The first well is vertical of very small angle of inclination and a total depth of 1800ft T.D. The second is a directional well of three stages. The first stage is a vertical hole with little or no inclination angle; the second stage is deviated or inclined by one degree to the vertical while the third stage is inclined at thirty degrees to the vertical. Each stage is 3000ft TD. It then means that the first kickoff point is at the depth of 3000ft while the second kickoff point is at 6000ft. There may be another kickoff point at the entry point of the pay zone (formation) with a near horizontal inclination and remaining within the reservoir until the desired bottomhole location is reached. This is known as horizontal drilling [12]. It is applicable to oil reservoir with poor matrix permeability in all directions, gas cap and water drive, otherwise it is not economically viable. However horizontal drilling is not considered in this study.

The following results are obtained from fifty simulations. The seeds for the generation of random variables are 0.3682 and 0.78458 for the inclination and azimuthal angles respectively [4]. All measurements are in imperial units which are generally accepted in the petroleum industry (10 feet = 3 meters).

Vertical Well

The average x coordinate = 3.2370ft
The average y coordinate = 6.0621ft
The average z coordinate = 1799.9640ft
The probability that the bivariate (x, y) is true = 1.000

Directional Well

First Stage (Vertical hole)

The average x value = 5.5061ft
The average y value = 10.0959ft
The average z value = 2999.9460ft
The probability that the bivariate (x, y) is true = 1.0000

Second Stage (One Degree Inclination)

The average x value = 12.3038ft
The average y value = 21.7171ft
The average z value = 2999.8220ft
The probability that the bivariate (x, y) is true = 1.0000

Third Stage (Thirty degree inclination)

The average x value = 682.0824ft
The average y value = 1235.0520ft
The average z value = 2615.5910ft
The bottomhole position of the directional well is as follows:
The x coordinate = 699.8923ft
The y coordinate = 1266.8650ft
The z coordinate = 8615.3590ft
The probability that the bivariate (x, y) is true = 1.0000

The expected value for the x coordinate = 24.7032ft
The expected value for the y coordinate = 24.0758ft
The expected value for the z coordinate = 49.0875ft

The variance of the x coordinate = 0.2515ft^2
The variance of the y coordinate = 0.2632ft^2
The variance of the z coordinate = 0.0239ft^2

The covariance of x and y = - 0.2263ft^2
The covariance of x and z = - 0.0400ft^2
The covariance of y and z = - 0.0395ft^2
Variables x and y are negatively correlated and the value of their correlation coefficient = - 0.8797

DISCUSSION

It can be observed that the deviation from the vertical increases in both the x and y directions as the inclination angle increases and decreases if otherwise. The deviation in the x direction increases with increasing azimuthal angle but decreases in the y direction as the azimuthal angle increases. The increase of both the inclination and azimuthal angles may cause severe dog-leg effect which in turn may lead to the stucking of pipe during drilling of a highly deviated well. Proper choice of inclination angle is therefore required for directional drilling to avert dog-leg phenomenon.

The values of the variance of the bottomhole coordinates in the directional well suggest that the computed bottomhole positions are not widely deviated from their mean values. This fact implies high probability for the computed bottomhole positions. The result shows that the probability is nearly unity. The high value of probability is also accounted for by the narrow limits of the variables I_i and A_i. Generally, the random variables I_i and A_i must pass some statistical test for randomness to ensure that the limits are narrow.

The expected values which correspond to our intuitive idea of average give the mean values for the x, y and z bottomhole positions while the variance and covariance are measures of spread. The correlation coefficient gives the linear relationship between the dependable variables x and y. Collectively, these values give a complete description of the bottomhole positions. The result shows that these values are favourable and fall within acceptable limits.

CONCLUSION

Uncertainty of the bottomhole positions increases as both the inclination and azimuthal angles increase. In deviated holes, this uncertainty becomes more significant in the direction at right angle to the deviated hole. This is in contrast to what obtains in a vertically drilled well where no problem is encountered in the computation of the bottomhole positions. Accordingly, the probability that the computed bottomhole coordinates, being true is nearly unity for vertical well and slightly less for directional well. Monte Carlo simulation technique recognizes these uncertainties and addresses them ahead of a directional survey. The trajectory control needed to drill the well correctly, the high quality hole required to successfully run and cement casing precisely in order to optimize production and maximize recovery are provided by expert directional drillers employing proven technologies and suitable drilling equipment. This equipment includes the drilling rig, jetting drill bit, whipstock devices, mud motors, packed hole assemblies and geosteering tool.

Finally, proper planning ahead of a directional drilling can be achieved by the use of Monte Carlo simulation technique which gives guide in the choice of inclination and azimuthal angles.

It is worthy to note that the simulation can be run by the use of modeling and simulation soft wares like MATLAB and SIMUL8.

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CONFLICT OF INTEREST

Nil.