A machine survival time-based maintenance workforce allocation model for production systems

Authors: **D.E. Ighravwe and S.A. Oke**

African Journal of Science, Technology, Innovation and Development, Volume 8, 2016 - Issue 5-6, pp. 457-466

Title: **A machine survival time-based maintenance workforce allocation model for production systems**

Authors: D.E. Ighravwe and S.A. Oke

Issue Date: 2016
Publisher: **African Journal of Science, Technology, Innovation and Development. (Published by Taylor and Francis)**

Abstract: Today's maintenance workforce operate in a complex business environment and rely on metrics that indirectly link equipment breakdown, fluctuating production rate, demand uncertainties and fluctuating raw material requirements. This has triggered a change of scope as well as the substance of maintenance workforce theory and practice and the necessary requirement to promote a full understanding of maintenance workforce optimisation of some seemingly non-polynomial hard problems. Theorising is essential on the near optimal solution techniques for the maintenance workforce problem. In this paper, a fuzzy goal programming model is proposed and used in formulating a single objective function for maintenance workforce optimisation with stochastic constraint consideration. The performance of the proposed model was verified using data obtained from a production system and simulated annealing (SA) as a solution method. The results obtained using SA and differential evolution (DE) was compared on the basis of computational time and quality of solution. We observed that the SA results outperform that of DE algorithm. Based on the results obtained, the proposed model has the capacity to generate reliable information for preventive and breakdown workforce maintenance planning.

Description: Research Article

A machine survival time-based maintenance workforce allocation model for production systems

[1,2] D.E. Ighravwe and [1,3]S.A. Oke
[1]Department of Mechanical Engineering, University of Lagos, Nigeria
[2]Department of Mechanical Engineering, Ladoke Akintola University of Technology, Nigeria
[3]Industrial and Production Engineering Unit, Department of Mechanical Engineering, Covenant University, Ota, Nigeria
Corresponding author: sa_oke@yahoo.com

Abstract

Today's maintenance workforce operate in a complex business environment and rely on metrics that indirectly link equipment breakdown, fluctuating production rate, demand uncertainties and fluctuating raw material requirements. This has triggered a change of scope as well as the substance of maintenance workforce theory and practice and the necessary requirement to promote a full understanding of maintenance workforce optimisation of some seemingly non-polynomial hard problems. Theorising is essential on the near optimal solution techniques for the maintenance workforce problem. In this paper, a fuzzy goal programming model is proposed and used in formulating a single objective function for maintenance workforce optimisation with stochastic constraint consideration. The performance of the proposed model was verified using data obtained from a production system and simulated annealing (SA) as a solution method. The results obtained using SA and differential evolution (DE) was compared on the basis of computational time and quality of solution. We observed that the SA results outperform that of DE algorithm. Based on the results obtained, the proposed model has the capacity to generate reliable information for preventive and breakdown workforce maintenance planning.

Keywords: Maintenance planning, meta-heuristics, workforce performance, raw material shortage, conditional probability, fuzzy logic theory

Introduction

Maintenance scholars argue that today's maintenance workforce operate in a complex business environment. The wide body of literature is increasingly being rooted in the understanding that the definition of maintenance has expanded beyond the traditional frontiers wherein maintenance is viewed as the technical repairs and restoration of broken-down machineries. In the current dispensation of evolving global practices, concepts of customer orientation and sustainability have overshadowed maintenance, portraying a picture of sustainable practices in maintenance and one that is customer-oriented.

In addition to this, government regulation on safety is compelling on maintenance, forcing maintenance units to deliver machines that are safe to operate to the production team and also guaranteeing the safety of the maintenance workforce itself during repair activities. Thus, for effectiveness, the maintenance manager must manage sustainability issues, customer satisfaction and quality-related issues, safety concerns and the complex humans that make the maintenance system to function. Consequently, the maintenance system is rolling a wheel of activities that is complex and crying for theorising of a deal of understanding and effectiveness by the maintenance manager.

In order to succeed, the maintenance workforce, which is an importance part of maintenance, relies on metrics that indirectly link equipment breakdown, fluctuating production rate, demand uncertainties and fluctuating raw material requirements. This has triggered a change of scope as well as the substance of maintenance workforce theory and practice and the necessary requirement to promote a full understanding of a maintenance workforce optimization of some seemingly up-hand problems. Theorising is essential on the near optimal solution techniques for the maintenance workforce problem. It is puzzling that despite the evidence that suggests the potential usefulness of near-optimal solutions for non-polynomial hard problems in maintenance scheduling, theoretical frameworks and practical lens have not yet been explored relevant to workforce planning, which is an aspect of maintenance scheduling. In particular, models and frameworks to enhance workforce planning for maintenance in a stochastic environment are missing.

In this paper, a fuzzy goal programming model is proposed and used in formulating a single objective function for maintenance workforce optimisation with stochastic constraint consideration. The performance of the proposed model was verified using data obtained from a production system and simulated annealing (SA) as a solution method. Workforce consideration affects the allocation of maintenance resources. Optimal allocation of maintenance resource is often directed at generating a minimum cost for operating maintenance systems. This benefit has encouraged a lot of researchers to focus more on development of models that optimises maintenance cost (Mansour, 2011, Ighravwe and Oke, 2014). Most of the optimisation models for maintenance cost optimisation are usually multi-objective.

Since the desired values of maintenance objectives are either minimum or maximum, the use of goal programming (GP) in generating compromise solution for maintenance model has proven to be a useful tool. To capture imprecise in model goals, the integration of GP and fuzzy logic has improved the generation of practicable results for multi-objective models. In maintenance system, imprecise in maintenance goals affects the desired values of total maintenance time, unavailability of machine, machine overall effectiveness, system survival time and system efficiency. When these goals are properly evaluated, the actual effectiveness of maintenance systems will be obtained.

Different studies have considered how to evaluate the performance of maintenance systems, with a view to justify the money spent for maintenance activities (Parida and Kumar, 2009; Ighravwe and Oke, 2015). Parida and Kumar (2009) presented comprehensive list of how to combine different maintenance performance at strategy, tactical and functional levels. Ighravwe and Oke (2015) presented an intelligent system for optimising maintenance key performance indices (KPI) using artificial neural network, differential evolution (DE), grey relational analysis and Taguchi method.

Despite the increased interest in maintenance performance analysis, a model which considered the survival time of production systems with respect to the contributions of maintenance workforce has not been reported. Also, no study has reported method on how to optimise number of machine breakdowns when considering maintenance workforce size and workloads to the best of our knowledge. Furthermore, the use of fuzzy goal programming approach for workforce-based multi-objective model for maintenance systems is sparse in literature.

The above mentioned knowledge gaps necessitated the need for this study. This study is aimed at presenting an optimisation which considered the effects of effective maintenance activities on system survival time and production time efficiency. Also, to determine a suitable solution method for solving the proposed model by comparing simulated annealing (SA) and differential evolution (DE) algorithm performance (Ahire *et al*., 2000; Ighravwe and Oke, 2014). In an attempt to address some of the gaps mentioned above, this study focuses on using different maintenance KPI in developing optimisation model. First, an expression which considered the effects of maintenance time and shortage in raw materials on total quantity of goods produced is considered. Second, the issue of improvement in failure rate of machines using workforce contribution is modelled. Lastly, the use of fuzzy logic approach to combine the machine survival time as a means for evaluating production system survival time is introduced.

A novelty of this study is its consideration of system survival time under production time efficiency and maintenance time considerations. Also, the application of SA and fuzzy goal programming method as a solution method for optimising maintenance workforce size and workloads as well as the number of machine breakdowns is another novelty of this study.

Optimisation model

This section presents the proposed model goals, discussions on the proposed model constraints, and the proposed fuzzy goal programming model. The notations used in describing the model goals and constraints are as follows:

Sets

K	total number of machines
M	total number of maintenance sections
N	total number of worker category
T	total number of planning periods

Parameters

$\circ_{ijk}$	contribution of workers in maintenance section *i* belonging to worker's category *j* to machine *k* failure rate improvement
$\bar{g}_{ik}$	average contribution of maintenance section *i* to machine *k* failure rate improvement factor
I_k	machine *k* failure rate improvement factor
«$_{\bar{g}_i}(\bar{g}_i)$	membership function for maintenance section *i* contribution to machine *k* failure rate improvement factor
I_s	system failure rate improvement factor
«$_{I_k}(I_k)$	membership function for machine *k* contribution to system failure rate improvement factor
u_{kt}	time loss on machine *k* due to raw material shortage at period *t*
P_{kt}	production time of machine *k* at period *t*
μ_t	proportion of preventive maintenance with to respect to total maintenance time at period *t*
$\eqsim_{kt}$	confidence interval for breakdown maintenance time for machine *k* at period *t*
$\widehat{W}_t$	total number of workers expected at period *t*
$\bar{c}_k$	unit cost of goods produced on machine *k*

Variables

NF_{kt}	number of failures for machine *k* at period *t*
$\overline{w}_{ijkt}$	amount of service time per day for a breakdown worker in maintenance section *i* belonging to worker category *j* that works on machine *k* at period *t*
$\overline{x}_{ijkt}$	number of breakdown workers in maintenance section *i* belonging to worker category *j* that works on machine *k* at period *t*
w_{ijkt}	amount of service time per day for a preventive worker in maintenance section *i* belonging to worker category *j* that works on machine *k* at period *t*
x_{ijkt}	number of preventive workers in maintenance section *i* belonging to worker category *j* that works on machine *k* at period *t*

The following assumptions were made during the development of the proposed model:

- The value of machine failure rates (a_{kt}) without effective preventive maintenance are known;
- Failure rates of machines are constant;
- Machines (production line) used for production activities are independent;
- The maintenance department are in-charge of preventive and breakdown maintenance activities;
- There is enough market for goods produced;
- The probability of a machine surviving at time $t + \Delta t$ is known; and
- The ratio of number of scheduled technicians for preventive maintenance activity to the number of technicians for breakdown maintenance activity is known.

Model Goals

This subsection provides details on the mathematical expressions for the four goals that are considered. The optimisation model that is proposed is aimed at maximising system survival time and production time efficiency as well as minimising total maintenance time and system unavailability.

***Maximisation of system survival time* (G_1)**

The curve for the machine survival without an effective maintenance programme is represented as curve L_1L_1 (Figure 1). The behaviour of a machine survival under an effective programme is represented as curve L_1L_2.

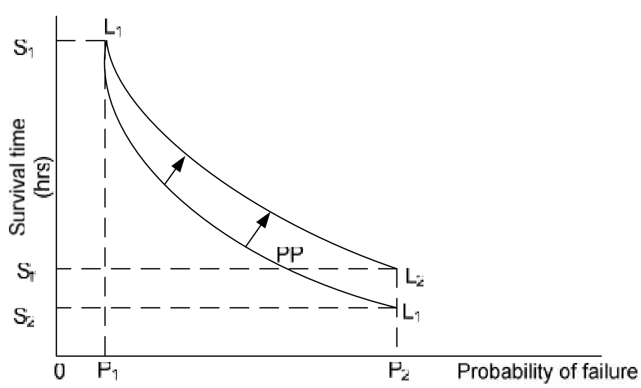

Figure 1: Change in system survival time

The one of the main benefits of an effective maintenance programme is to extend the survival time of machines from $L_1 L_1$ to $L_1 L_2$. Other benefits are: reduced machine unavailability, decrease in number of defective, increase in machine productivity, increase in revenue etc. To move from line $S_2 L_1$ to $S_f L_2$, organisations incur costs as a result of spare parts used, engaging of skilled workers and maintenance time. Based on the concept of overall inflation rate (Caretto, 2010), the average contribution of each section to machine's failure rate improvement is expressed as Equation (1). Since it is the duty of maintenance workers to prevent breakdown, their contribution to system survival is considered (Equation 2).

$$\bar{g}_{ik} = (1 + g_{ik})^{\frac{1}{T}} - 1 \tag{1}$$

$$g_{ik} = \frac{\sum_{j=1}^{N} (\circ_{ijk}) \sum_{t=1}^{T} x_{ijkt}}{\sum_{j=1}^{N} \sum_{t=1}^{T} x_{ijkt}} \tag{2}$$

$$0 \leq \circ_{ijk} \leq 1 \tag{3}$$

where $\circ_{ijk}$ is contribution of workers in maintenance section *i* belonging to worker's category *j* to machine *k* failure rate improvement, and $\bar{g}_{ik}$ is the average contribution of maintenance section *i* to machine *k* failure rate improvement factor.

The contribution of the various maintenance sections to machine failure rates varies from machine to machine. It has not been scientifically proven that the contribution of workers to machine failure rate improvement can be combined either multiplicatively or additively. The problem of combining the contributions of each maintenance section is addressed using fuzzy logic approach. This approach provides a means of converting linguistic specifications of decision makers into crisp values. The improvement factor for each machine is calculated using a weighted aggregate defuzzification scheme (Equation 4). The membership function for each maintenance section is obtained using Figure 2. Trapezoidal membership function is adopted because of its capacity to encompass uncertainty than triangular membership function (Shemshadi *et al*., 2011). The equations for low, medium and high membership functions are presented in Appendix 1.

$$I_k = \frac{\sum_{i=1}^{M} \text{«}_{\bar{g}_{ik}} (\bar{g}_{ik}) \bar{g}_{ik}}{\sum_{i=1}^{M} \text{«}_{\bar{g}_{ik}} (\bar{g}_{ik})} \tag{4}$$

where I_k is machine *k* failure rate improvement factor, and $\text{«}_{\bar{g}_i} (\bar{g}_i)$ is the membership function for maintenance section *i* contribution to machine *k* failure rate improvement factor.

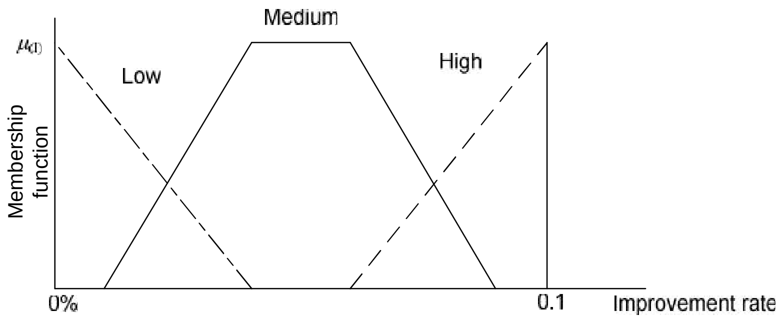

Figure 2: Membership function for maintenance section improvement factor

Similarly, the contribution of each machine to the system failure rate improvement is considered. The system failure rate improvement (λ_s) is expressed as Equation (5). The expect value for the system failure rate factor under an effective maintenance programme is expressed as Equation (6).

$$I_s = \frac{\sum_{k=1}^{K} \mu_{I_k}(I_k) I_k}{\sum_{k=1}^{K} \mu_{I_k}(I_k)} \qquad (5)$$

$$\lambda_s = \lambda_s (1 - I_s) \qquad (6)$$

where I_s is system failure rate improvement factor, and $\mu_{I_k}(I_k)$ is the membership function for machine *k* contribution to system failure rate improvement factor.

By using the concept of conditional probability (Equation 7), the expected survival time of a system at a given probability is estimated. The value of the expected system survival time at S_2 is expressed as Equation (9). The expected survival time (G_1) under an effective maintenance programme is expressed as Equation (10).

$$P\left(\frac{P_2}{P_1}\right) = \frac{F(S_2) - F(S_1)}{1 - F(P_1)} \qquad (7)$$

$$F(S_i) = 1 - e^{-\lambda S_i} \qquad (8)$$

$$S_2 = -\frac{\ln\left(e^{-\lambda S_1}\left(1 - P\left(\frac{S_2}{S_1}\right)\right)\right)}{\lambda} \qquad (9)$$

$$\text{Max } G_1 : \frac{\ln\left(e^{-(a_s(1+I_s))S_t}\left(P\left(\frac{P_2}{P_1}\right) - 1\right)\right)}{a_s(1+I_s)} \tag{10}$$

Maximisation of production time efficiency (G*$_2$*)

By extending a system survival time, it implies that the system (machine) will be available for production activities. Machine availability affects the value of manufacturing systems efficiency (Kardas, 2012). Production time efficiency is a function of the total amount of production time and available manufacturing time (OP_{kt}) for a system (Kardas, 2012). Production time is affected by preventive (PT_{kt}) and breakdown (BT_{kt}) maintenance time. Apart from maintenance activity which affects the amounts of available production, raw material shortage is another factor. With the knowledge of the expected minimum (U_1) and maximum (U_2) amounts of delay in production resulting from raw materials shortage, production time efficiency is expressed as Equation (11).

$$Max\, G_2 = \frac{\sum_{t=1}^{T}\sum_{k=1}^{K}\left(OP_{kt} - BT_{kt} - PT_{kt} - u_{kt}\right)}{\sum_{t=1}^{T}\sum_{k=1}^{K} OP_{kt}} \tag{11}$$

$$u_{kt} = u_{1k} + \left(1 - \bar{\sim}_{kt}\right)\left(u_{2k} - u_{1k}\right) \qquad \forall k,t \tag{12}$$

$$BT_{kt} = \frac{\sum_{i=1}^{M}\sum_{j=1}^{N} \bar{w}_{ijkt}\bar{x}_{ijkt}}{\sum_{i=1}^{M}\sum_{j=1}^{N} \bar{x}_{ijkt}} \qquad \forall k,t \tag{13}$$

$$PT_{kt} = \frac{\sum_{i=1}^{N}\sum_{j=1}^{M} w_{ijkt}x_{ijkt}}{\sum_{i=1}^{N}\sum_{j=1}^{M} x_{ijkt}} \qquad \forall k,t \tag{14}$$

where u_{kt} is time loss on machine *k* due to raw material shortage at period *t*, and $\bar{\sim}_{kt}$ is confidence interval for machine *k* stochastic constraint at period *t*.

Minimisation of system unavailability (G*$_3$*)

The availability of a machine (system) depends on its $MTBF_{kt}$ and $MTTR_{kt}$. The values of $MTTR_{kt}$ and $MTBF_{kt}$ affects a machine's failure rate (Rodrigues and Hatakeyama, 2006; Parida and Kumar, 2009). The interrelationships among $MTTF_{kt}$, $MTTR_{kt}$ and $MTBF_{kt}$ is expressed as Equation (15).

$$MTTF_{kt} = MTBF_{kt} - MTTR_{kt} \tag{15}$$

$$MTTF_{kt} = \frac{1}{\text{a}_{kt}} \qquad \forall k, t \qquad (16)$$

$$MTBF_{kt} = \frac{OT_{kt} - BT_{kt} - PT_{kt}}{NF_{kt}} \qquad \forall k \qquad (17)$$

$$MTTR_{kt} = \frac{BT_{kt}}{NF_{kt}} \qquad \forall k \qquad (18)$$

where NF_{kt} is number of failures of machine k at period t.

By using the interrelationships among $MTBF_{kt}$, $MTTR_{kt}$ and $MTTF_{kt}$, the unavailability of a system is expressed as Equation (19). For a maintenance system with machine failure improvement factor (I_k), the value of machine unavailability is expressed as Equation (20).

$$G_3 = 1 - \frac{MTBF_{kt}}{MTBF_{kt} + MTTR_{kt}} \qquad (19)$$

$$\text{Min G}_3 : \sum_{k=1}^{K} \sum_{t=1}^{T} \left(1 - \left(\frac{\frac{1}{\text{a}_{kt}(1+I_k)} + \frac{1}{\text{«}_{kt}}}{\frac{1}{\text{a}_{kt}(1+I_k)} + 2\frac{1}{\text{«}_{kt}}} \right) \right) \qquad (20)$$

Minimisation of total maintenance time **(*G_4*)**
The total amount of time required for maintenance activities is expressed as the average total amount of time used for maintenance activities by workers (Ighravwe and Oke, 2014). For a system that uses preventive and breakdown maintenance techniques in carrying out maintenance activities, the total maintenance time is expressed as Equation (21).

$$\text{Min G}_4 = \frac{\sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{K} \sum_{t=1}^{T} w_{ijkt} x_{ijkt}}{\sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{K} \sum_{t=1}^{T} x_{ijkt}} + \frac{\sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{K} \sum_{t=1}^{T} \overline{w}_{ijkt} \overline{x}_{ijkt}}{\sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{K} \sum_{t=1}^{T} \overline{x}_{ijkt}} \qquad (21)$$

Model Constraints
Information on the various constraints which are considered in the proposed model is outlined in this subsection.

Number of failures
By considering Equations (15) to (18), the number of machine failure is determined. The value of total time allocated for maintenance and production for a machine (OT_{kt}) is estimated using stochastic constraint (Ighravwe *et al*., 2015). Given that the value of OT_{kt} varies between a minimum (Q_1) and maximum (Q_2) values, the value of OT_{kt} at each period is determined using uniform distribution (Equation 22). Based on the

concept of I_k, the relationships among NF_{kt} with $MTTF_{kt}$, $MTTR_{kt}$ and $MTBF_{kt}$ is expressed as Equation (23). The reason for selecting uniform distribution to model stochastic constraints is that it reduces the problem of parameter variation to confidence interval ($\bar{\sim}_k$) determination.

$$OT_{kt} = Q_{1k} + \left(1 - \bar{\sim}_{kt}\right)\left(Q_{2k} - Q_{1k}\right) \qquad \forall k,t \qquad (22)$$

$$NF_{kt} \leq \text{a}_{kt}\left(1 + I_k\right)\left(OT_{kt} - PT_{kt} - 2BT_{kt}\right) \qquad \forall k,t \qquad (23)$$

Average system reliability

The average system reliability is determined using the total number of working days at period t ($\overline{N}_k$) and the total number of machines used for production activities. Since the machines are in parallel connection, the minimum expected average system reliability at period t is expressed as Equation (24).

$$1 - \prod_{k=1}^{K}\left(1 - e^{-\text{a}_k (1+I_k) P_{kt}}\right) \geq R_t \qquad \forall t \qquad (24)$$

$$P_{kt} = \frac{OT_{kt} - BT_{kt} - PT_{kt}}{\overline{N}_k} \qquad \forall k,t \qquad (25)$$

where P_{kt} is production time of machine k at period t.

Workload distribution

During maintenance activities, the amount of maintenance time allocated for preventive maintenance activities is often greater than the amount of maintenance time allocated for breakdown maintenance activity. The reason is that breakdown maintenance activity does not always occur as often as preventive maintenance activity. With the knowledge of the expected proportion of time for preventive maintenance activities (μ_t), the relationships between preventive and breakdown maintenance time is expressed as Equation (26).

$$\frac{\sum_{i=1}^{M}\sum_{j=1}^{N}\left(w_{ijkt}x_{ijkt} - \overline{w}_{ijkt}\overline{x}_{ijkt}\right)}{\sum_{i=1}^{M}\sum_{j=1}^{N} w_{ijkt}x_{ijkt}} \geq \mu_t \qquad \forall k,t \qquad (26)$$

The amount of maintenance time for preventive and breakdown maintenance activities in the system is directly related to the amount of time available for production activities (Equation 27). The amount of preventive maintenance time to be allocated to each maintenance section depends on the type of machine (semi-automated or manual) to be maintained. By using the concepts of relative importance ($¡_k$) of a machine with respect to preventive maintenance activities, Equation (28) is used in constraining the amount of maintenance time for each machine.

$$\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\left(w_{ijkt}x_{ijkt}+\overline{w}_{ijkt}\overline{x}_{ijkt}\right)\leq r\sum_{k=1}^{K}OP_{kt} \qquad \forall t \qquad (27)$$

$$\sum_{i=1}^{M}\sum_{j=1}^{N}w_{ijkt}x_{ijkt}\leq \text{¡}_{k}\cdot \qquad \forall k,t \qquad (28)$$

$$\cdot = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}w_{ijkt}x_{ijkt} \qquad (29)$$

where r is the proportion total maintenance time with respect to total amount of manufacturing time.

To model the amount of time used for restoring broken down machine to functional state, the minimum ($\overline{b}_{1ik}$) and maximum ($\overline{b}_{2ik}$) amount of time used by a section is considered. With the knowledge of minimum and maximum time for breakdown maintenance, uniform distribution is used in modelling the relationship between workers size for breakdown maintenance and the amount of time for breakdown maintenance (Equation 30).

$$\sum_{j=1}^{N}\overline{w}_{ijkt}\overline{x}_{ijkt}\leq\left(\overline{b}_{2ik}-\overline{b}_{1ik}\right)\left(1-\tilde{-}_{kt}\right)+\overline{b}_{1ik} \qquad \forall i,k,t \qquad (30)$$

In workforce planning, the size of workers required to manage the available workloads in a system is constrained using the maximum allowable workforce size (Ighravwe and Oke, 2014, Ighravwe *et al*., 2015). The workforce size constraint for preventive maintenance activities is expressed as Equation (31). The relationships the number of preventive and breakdown maintenance workers is expressed as Equation (32).

$$\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}x_{ijkt}\leq\widehat{W}_{t} \qquad \forall t \qquad (31)$$

$$\ddot{}\, x_{ijkt}\leq\overline{x}_{ijkt} \qquad \forall i,j,k,t \qquad (32)$$

where $\widehat{W}_{t}$ is total number of workers expected at period t, and ¨ is the ratio of preventive maintenance workers to breakdown maintenance workers.

Production capacity

The demand for product of an organisation often oscillates between a minimum and maximum values at different periods. Apart from machine problem which may results in increase in the number of defective goods produced, worker's skills and their state of mind affects organisation's ability to meet the demand for a product. The expression for the relationships among production volume, loss of production time, finished goods inventory and product demand in an imprecise environment is expressed as Equation (33). The minimum amount of finished goods inventory

(Belmokaddem *et al.*, 2008) expected at each period is used in controlling production volume (Equation 34).

$$\sum_{k=1}^{K}(OP_{kt} - BT_{kt} - PT_{kt} - u_k)R_k + I_{t-1} - I_t = D_t \qquad \forall t \tag{33}$$

$$I_T \geq I_{\min} \tag{34}$$

$$R_k = R_{1k} + (1 - \sim_{kt})(R_{2k} - R_{1k}) \tag{35}$$

$$D_t = D_1 + (1 - \sim_t)(D_2 - D_1) \tag{36}$$

where D_1 is minimum demand, D_2 is maximum demand, $\sim_t$ is the confidence interval for demand at period *t*, I_T is inventory at period *T*, $I_{\min}$ is minimum inventory, R_{1k} is minimum production rate of machine *k*, and R_{2k} is maximum production rate of machine *k*.

Workforce productivity and cost

The productivity of maintenance workforce (Ighravwe and Oke, 2014), which is a function of the quantity of goods produced and the cost of workers used for maintenance activities is expressed as Equation (37). To further control the number of workers used for the maintenance activities, the budget for workers' expenses is considered. We adopt the concepts of uniform distribution for workforce budget (Equation 38).

$$\frac{\sum_{k=1}^{K}\bar{c}_k(OP_{kt} - BT_{kt} - PT_{kt} - u_k)R_{kt}}{\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}c_{ijt}x_{ijkt}} \geq Pd_t \qquad \forall t \tag{37}$$

$$\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}c_{ijt}(x_{ijkt} + \bar{x}_{ijkt}) \leq (B_u - B_l)(1 - \sim_t) + B_l \qquad \forall t \tag{38}$$

where $\bar{c}_k$ is the unit cost of goods produced on machine *k*, B_u is the maximum technicians' budget, B_l is the minimum technicians' budget, and $\sim_t$ is the confidence interval for stochastic budget constraint (Ighravwe *et al.*, 2015).

Fuzzy Goal Programming

To handle the goals in the proposed model, fuzzy goal programming technique is used in converting each goal to a soft constraint by generating membership functions for the model goals. The membership function for the maximisation goals (Equations 10 and 14) is expressed as Equation (39). The membership function for the minimisation goals (Equations 20 and 21) is expressed as Equation (40).

$$\text{«}_{z_k}(x) = \begin{cases} 1 & \text{if} \quad Z_k(x) \geq b_k^u \\ 1 - \dfrac{b_k^u - Z_k(x)}{b_k^u - b_k^l} & \text{if} \quad b_k^l \leq Z_k(x) < b_k^u \\ 0 & \text{if} \quad Z_k(x) \leq b_k^u \end{cases} \tag{39}$$

$$\text{«}_{z_k}(x) = \begin{cases} 1 & \text{if} \quad Z_k(x) \leq b_k^l \\ 1 - \dfrac{Z_k(x) - b_k^l}{b_k^u - b_k^l} & \text{if} \quad b_k^l < Z_k(x) \leq b_k^u \\ 0 & \text{if} \quad Z_k(x) \geq b_k^u \end{cases} \tag{40}$$

where b_k^u is upper limit for the *k-th* fuzzy goal, and b_k^l is the lower limit for the *k-th* fuzzy goal.

The single objective function of the proposed model is expressed as Equation (41). The soft constraints for the crisp goals are expressed as Equations (42) and (43).

$$Max = \sum_{k=1}^{4} \text{«}_k \tag{41}$$

$$\text{«}_k \leq \text{«}_{z_k} \qquad k = 1,2,3,4 \tag{42}$$

$$\text{«}_k \leq \sim_k \qquad k = 1,2,3,4 \tag{43}$$

where «_k is the membership function for the *k*-th fuzzy goal, and $\sim_k$ is the desired achievement value for the *k*-th fuzzy goal.

Solution Methods

Maintenance workforce scheduling problem has been identified as a NP-hard problem. This implies that the use of exact approach to solve the proposed model will not be possible within reasonable time (Safaei *et al*., 2008). This motivated the use of meta-heuristics (SA and DE). This section presents brief descriptions of the solution methods used in solving the proposed model. SA implementation procedure and the description of DE are presented later in the paper.

***Simulated annealing* (*SA*)**

SA is a mathematical analogy to a cooling system is used to optimise nonlinear, multivariate combinatorial optimisation. SA implementation involves two basic steps: perturbing of solution and evaluation of the quality solution. The pseudo code for SA is presented as follows (Ledesma *et al*., 2008, Janiak and Lichtenstein, 2011):

Select random values for y_{ij0}, initial and final temperatures, cooling scheme and maximum iteration step
Evaluate $f(y_{ijg})$
While stoppage criterion is not satisfied do
  Perturb $y_{ijg+1} \leftarrow y_{ijg}$ using Equation (44)
  Evaluate $f(y_{ijg+1})$
  Calculate $\Delta = f(y_{ijg}) - f(y_{ijg+1})$
  If $\Delta \leq 0$
  y_{ijg+1} is the current solution
  Else
    If $U(0,1) \leq e^{\frac{-\Delta}{T}}$ then
    y_{ijg+1} is the current solution
    Else
    y_{ijg} is the current solution
    end if
  end if
  Reduce the system temperature using annealing scheme
Check stoppage criterion
End while

$$y_{ijg+1} = (1-¢)y_{ijg} + ¢R(y_{ij\min}, y_{ij\max}) \tag{44}$$

$$¢ = \frac{T_g}{T_0} \tag{45}$$

where T_0 is initial temperature, and T_g is current temperature at iteration step g (Ledesma *et al*., 2008).

***Differential evolution algorithm* (*DE*)**

DE algorithm belongs to a group of meta-heuristics known as evolutionary algorithms. Evolutionary algorithms are mathematical analogy of survival of human beings (Storn and Price, 1997; Engelbrecht, 2007). DE is a population-based stochastic search algorithm. The operation of DE algorithm involves mutation, crossover and selection of solutions. Mutant vectors are generated using mutation operation, while crossover operation generates trial vectors. During selection operation, target vectors are generated. This study applied the DE algorithm described below (Storn and Price, 1997; Engelbrecht, 2007).

Select random values for x_{ij0}, mutation probability (*f*), crossover probability (*CR*), population size and maximum iteration step
Evaluate $f(y_{ijg})$
While stoppage criterion is not satisfied do
  Generate mutant vectors using Equation (46)

Evaluate $f(y_{ijg+1})$
Generate trial vectors using crossover operation
If $rnd \leq CR \quad i = I_r$
$v_{ijg} = u_{ijg}$
Else
$v_{ijg} = y_{ijg}$
end if
Evaluate new solution
If $f(y_{ijg+1}) < f(y_{ijg})$
y_{ijg+1} is the current solution
Else
y_{ijg} is the current solution
Check stoppage criterion
End while

$$u_{ijg} = y_{ijg}^1 + f\left(y_{ijg}^2 - y_{ijg}^3\right) \tag{46}$$

$$y_{ijg} \neq y_{ijg}^1 \neq y_{ijg}^2 \neq y_{ijg}^3$$

where rnd is a random variable.

Model Application and Results

The proposed model was tested on a manufacturing system with two production lines (machines). The data obtained was complemented using Monte Carlos simulation technique. The expected system reliability at each period was 50%, while the expected ratio of total preventive maintenance time to preventive maintenance time was 50%. It expected that the total maintenance time on Machine I was at least 45% of the total maintenance time for the system. The minimum system reliability was 0.65.

The MTTF for Machine I was 720 min and Machine II had a MTTF value of 700 min. The delay caused by lack of material was simulated between 2 and 5% of total production time. The average system reliability was estimated based on a minimum breakdown per day. Given that the system will not fail within the first 10 hrs after preventive or breakdown maintenance, a conditional probability of 50% for the next hours of the system survival was considered.

The maximum workforce budget was ₦16, 800,000 and the minimum workforce budget was ₦15, 600,000. The maximum workforce size for the maintenance system was 52 workers. The ratio of the number of scheduled preventive maintenance workers to breakdown maintenance workers was 0.5 for any of the sections. The solution methods iteration were terminated using a maximum iteration step (100). The DE algorithm was implemented using a mutation probability of 0.1, crossover probability of 0.3 and population size of 30.

Linear cooling scheme was used in controlling the uphill movement of the SA solution. For the SA, the acceptance of new solution was based on Boltzmann-Gibbs distribution. The stoppage criterion for the SA algorithm was the minimum allowable temperature (Engelbrecht, 2007). The proposed model and solution methods were coded using VB.Net programming. The performance of the solution methods are analysed using the quality of solution and computational time for 30 runs (Figures 3 and 4). The SA algorithm result in terms of fitness function is preferable to that of DE (Table 1). By using the SA algorithm as a solution method, Pareto solution for the decision variables were generated (Tables 2 and 5).

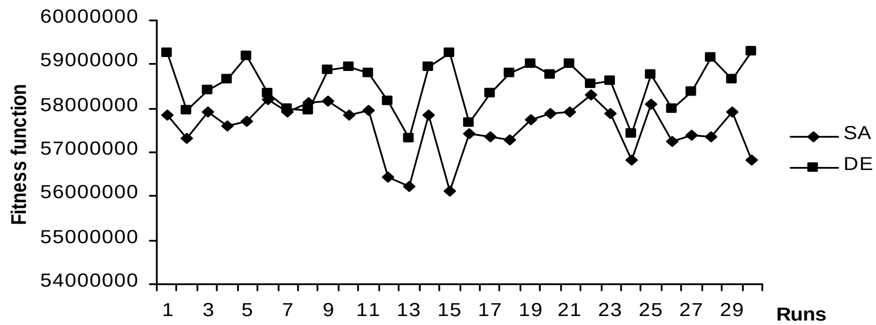

Figure 3: Fitness function of the solution methods at different runs

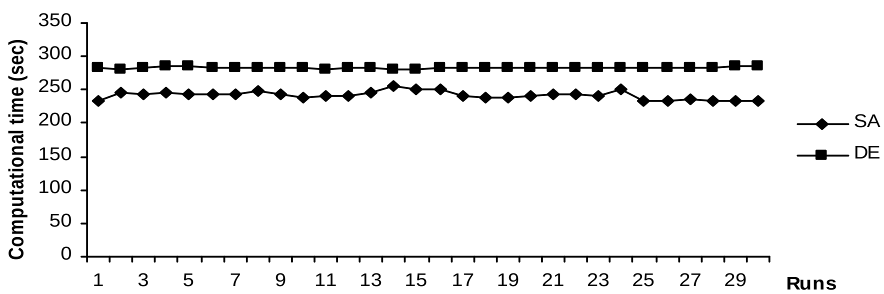

Figure 4: Computational time of the solution methods at different runs

Table 1: Performance of solution methods

Solution methods	Best solution	Worst solution	Average solution	Average computational time (sec)
SA	56,116,960.96	58,289,729.10	57,554,141.20	241.64
DE	57,326,369.00	59,305,816.00	58,547,651.00	282.63

Table 2: Pareto solution for the objective functions

Parameters	Objective 1	Objective 2	Objective 3	Objective 4
Crisp value	11.13 hr	73%	5%	15,089.74 hr
Membership function	1	1	1	1

Table 3: Worker's size distribution for maintenance activities

Periods	Machines	Preventive maintenance				Breakdown maintenance			
		x_{11kt}	x_{12kt}	x_{21kt}	x_{22kt}	$\overline{x}_{11kt}$	$\overline{x}_{12kt}$	$\overline{x}_{21kt}$	$\overline{x}_{22kt}$
$t = 1$		6	3	4	3	3	2	1	1
$t = 2$	k = I	6	2	5	3	4	2	3	2
$t = 3$		6	3	3	3	4	3	1	2
$t = 4$		6	3	4	2	3	2	2	2

$t = 1$		4	3	5	3	2	2	2	3
$t = 2$	k = II	4	4	6	3	3	3	3	2
$t = 3$		4	4	6	5	2	2	4	3
$t = 4$		5	3	4	5	3	2	2	2

Table 4: Number of failures for the machines

Machines	Period 1	Period 2	Period 3	Period 4
I	187	200	201	189
II	195	171	191	191

Table 5: Worker's service time distribution for maintenance activities

Periods	Machines	Preventive maintenance (hr)				Breakdown maintenance (hr)			
		w_{11kt}	w_{12kt}	w_{21kt}	w_{22kt}	$\overline{w}_{11kt}$	$\overline{w}_{12kt}$	$\overline{w}_{21kt}$	$\overline{w}_{22kt}$
$t = 1$		1,148.26	1,154.64	1,187.35	1,204.86	1,170.34	1,212.48	1,163.96	1,163.36
$t = 2$	k = I	1,145.49	1,220.47	1,167.96	1,200.10	1,203.35	1,195.26	1,225.72	1,146.31
$t = 3$		1,170.80	1,149.40	1,228.07	1,207.17	1,235.36	1,146.75	1,192.14	1,215.74
$t = 4$		1,171.65	1,235.74	1,185.70	1,175.86	1,225.47	1,144.37	1,183.14	1,177.48
$t = 1$		1,712.68	1,172.25	1,205.68	1151.4	1,163.02	1,215.20	1,206.34	1,162.52
$t = 2$	k = II	1,231.50	1,235.67	1,162.97	1169.43	1,178.04	1,220.67	1,226.71	1,150.03
$t = 3$		1,148.28	1,187.05	1,144.15	1185.82	1,178.21	1,173.32	1,149.45	1,190.81
$t = 4$		1,179.02	1,144.28	1,195.60	1205.82	1,184.21	1,214.62	1,178.86	1,200.97

Discussion of Results

The membership functions results for the objective functions showed that they have a complete membership function (Table 2). This implies that the SA was able to generate results that meet the decision makers' requirements. The effects of improvement in maintenance activities on the system survival time at a conditional probability of 0.5 improved the system survival time by 11.3% (Table 2). The average preventive and breakdown maintenance time was 3,772.44 hr per period. The proportion of time for the system availability was 95%, while the production time inefficiency of the system was 27%. This implies that the time used for maintenance activities and time loss due to raw material shortage was 27% of the total time available for manufacturing activities.

The total number of workers required for maintenance activities on Machine I (99 workers) was less than the number of workers for Machine II (108 workers). Analysis of the average number of workers for Machine I maintenance showed that a minimum of seven regular mechanical maintenance workers were for preventive maintenance activities (Table 3). The system required four regular mechanical workers for breakdown maintenance activities on Machine I. A minimum five regular and three casual electrical workers were required for electrical preventive maintenance activity at each period on Machine. The minimum number of regular electrical workers for breakdown maintenance activity on Machine I was two, while one casual worker will assist the breakdown regular electrical workers on Machine I.

The total number of electrical workers (37 workers) preventive maintenance activity on Machine II was more than that of the mechanical workers (31 workers). For breakdown maintenance activity, the number of mechanical workers required for Machine II was 19 workers. Machine II required 21 electrical workers for its breakdown maintenance activity, while at least three regular workers are schedule for either mechanical or electrical maintenance activity on Machine II. Equal numbers of casual workers (three workers) were for required for mechanical or electrical maintenance activity on Machine II. A minimum of five regular workers was required for mechanical preventive maintenance activity on Machine II, while six regular workers were required for electrical maintenance activity on Machine II at each period.

The average number of Machine I breakdown was between 31 and 34 breakdowns/month (Table 4). To optimally utilised these workers, at most 33 workers are required to be scheduled for Machine I breakdown maintenance activities (Table 3). Machine II required at most 40 workers to be schedule for breakdown maintenance activity for all the planning periods. These workers are expected to address breakdowns of between 27 and 33 breakdowns/month (Table 4). Based on the workload distribution (Table 5), a regular mechanical maintenance worker that is assigned to maintain Machine I is expected work for 194 hr/month. On Machine II, a regular mechanical worker scheduled for preventive maintenance is expected to work for 220 hr/month. The average monthly period which a casual mechanical worker assigned to Machine I to carry out preventive maintenance was 198 hr.

For breakdown maintenance activities, a regular mechanical worker will work for 202 hr/month, while a casual mechanical worker on Machine I will work for 196 hr/month. The amount of maintenance time a regular electrical maintenance worker for preventive and breakdown maintenance activities on Machine I will work was the same (199 hr/month). A casual electrical worker required about 200 hr/month working period during preventive maintenance activity on Machine I. A casual electrical worker scheduled for breakdown activity is expected to work for 195 hr/month on Machine I.

During preventive maintenance on Machine II, a regular mechanical worker is expected to work for 220 hr/month. A casual mechanical worker will work for 198 hr/month. A regular or casual electrical worker for preventive maintenance on Machine II will work for 197 hr/month. The expected monthly working hours for a regular mechanical worker and a casual electrical worker were the same (196 hr) during breakdown activity on Machine II. A casual mechanical worker for breakdown activity on Machine II was expected to work for 201 hr/month. The duration which a regular electrical worker will used in carrying out breakdown activity on Machine II was more that of a casual electrical worker (199 hr/month).

Although, the proposed model results show that it has the capacity to generate information on the expected number of machine breakdown, workforce size and service time, it has some limitations. The model does not have the capacity to identify skills of workers that will be schedule for either preventive or breakdown maintenance activity. Another limitation of the model is its inability to generate information on the proportion of time the workers will be busy during at each planning period. This study can be extended by considering the issue of workers' sustainability from the perspective of waste (loss of maintenance time) and skill transfer.

Conclusions

This study has successfully implement a fuzzy goal programming model for workers' allocation problem under a stochastic demand and production rate as well as raw material shortage and maintenance workforce budget. The applicability of the proposed model was carried out using a combination of real and simulated data. Pareto solution for the decision variables was generated using SA and DE algorithms as solution methods. The results obtained showed that SA algorithm has the capacity to generate better result than the DE algorithm.

The proposed model can be used by decision makers in addressing the number of workers required for either preventive and breakdown maintenance activities. Also, the model has the capacity to determine the value of system unavailability and production time efficiency. Beyond being a planning tool for existing maintenance systems, the proposed model can be used to design a maintenance department for start-up manufacturing systems.

References

Ahire, S., G. Greenwood, A. Gupta, and M. Terwilliger. 2000. "Workforce-constrained preventive maintenance scheduling using evolution strategies." *Decision Science Journal* 31(4): 833-859.

Belmokaddem, M., M. Mekidiche. and A. Sahed. (2008). "Application of a fuzzy goal programming approach with different importance and priorities to aggregate production planning." *Journal of Applied Quantitative Methods* 4(3): 317-331.

Caretto, L. 2010. "Notes on Engineering Economic Analysis." Accessed 24 July 2015: http://csun.edu/~lcaretto/me483/engrEcon.pdf.

Ding, F. and Z. Tian. 2012. "Opportunistic maintenance for wind farms considering multi-level imperfect maintenance thresholds." *Renewable Energy* 45: 175-182.

Engelbrecht, A.P. 2007. "*Computational Intelligence: An Introduction*." 2ed. John Wiley & Sons ltd, USA.

Gharbi, A., J-P. Kenne. and M. Beit. 2007. "Optimal safety stock and preventive maintenance periods in unreliable manufacturing systems." *International Journal of Production Economics* 107(2): 422-434.

Kardas, K. 2012. "Evaluation of efficiency of working time of equipment in blast furnace department." *Journal of Achievement in Materials and Manufacturing Engineering* 55(2): 876-880.

Kim, C.S., I. Djamaludin. and D.N.P. Murthy. 2004. "Warranty and discrete preventive maintenance." *Reliability Engineering and System Safety* 84(3): 301-309.

Ighravwe, D.E., S.A. Oke. and K.A. Adebiyi. 2015. "Maintenance workload optimisation with accident occurrence considerations and absenteeism fro work using a genetic algorithm." *International Journal of Management Science and Engineering Management*, DOI: 10.1080/17509653.2015.1065208.

Ighravwe, D.E. and S.A. Oke. 2014. "A nonzero integer non-linear programming model for maintenance workforce sizing." *International Journal of Production Economics* 150: 204-214.

Janiak, A. and M. Lichtenstein. 2011. "Advanced Algorithms in Combinatorial Optimisation." Printpap Lodz, Germany.

Ledesma, S., G. Avina. and R. Sanchez. 2008. "Practical considerations for simulated annealing implementation, In: Simulated Annealing, Tan C.M (Ed.)." Accessed 22 July 2015. http://www.intechopen.com/books/simulated_annealing/practical_considerations_ for_simulated_annealing_implementation.

Li, D., Z. Zhang, Q. Zhong. and Y. Zhai. 2014. "Performance deterioration modelling and optimal preventive maintenance strategy under scheduled servicing subject to mission time." *Chinese Journal of Aeronautics* 47(4): 821-828.

Parida, A. and U. Kumar. 2009. "Maintenance Productivity and Performance Measurement." In: *Handbook of Maintenance Management and Engineering*, edited by M. Ben-Daya, S.O. Duffuaa, A. Raouf, J. Knezevic and Ait-Kadi D.

Rodrigues, M. and K. Hatakeyama. 2006. "Analysis of the fall of TPM in Companies." *Journal of Materials Processing Technology* 179(1-3): 276-279.

Safaei, N., D. Banjevic. and A.K.S. Jardine. 2008. "Multi-objective simulated annealing for a maintenance workforce scheduling problem: A case study." In: *Simulated Annealing*, edited by C.M. Tan: 27-48. I-Tech Education and Publishing, Vienna, Austria.

Sarker, B.R. and T.I. Faiz. 2016. "Minimising maintenance cost for offshore wind turbines following multi-level opportunistic preventive strategy." *Renewable Energy* 85: 104-113.

Shemshadi, A., H. Shirazi, M. Toreihi. and M.J. Tarokh. 2011. "A fuzzy VIKOR method for supplier selection based on entropy measure for objective weighting." *Expert Systems and Application* 38: 12160-12167.

Storn, R. and K. Price. 1997. "Differential evolution: A simple and efficient heuristic for global optimisation over continuous spaces." *Journal of Global Optimisation* 11: 341-359.

Zhou, Y., G. Kou. and D. Ergu. 2013. "Modelling maintenance effect with failure-counting." *Procedia Computer Science* 17: 298-305.

Appendix 1

$$\text{«}_{\bar{g}_{ik}}(Low) = \begin{cases} 1 & \bar{g}_{ik} \leq 0 \\ \dfrac{0.4 - \bar{g}_{ik}}{0.4} & 0 \leq \bar{g}_{ik} \leq 0.40 \end{cases}$$

$$\text{«}_{\bar{g}_{ik}}(Medium) = \begin{cases} 0 & \bar{g}_{ik} \leq 0 \\ \dfrac{0.4}{\bar{g}_{ik}} & 0.1 \leq \bar{g}_{ik} \leq 0.4 \\ 1 & 0.4 \leq \bar{g}_{ik} \leq 0.6 \\ \left(\dfrac{0.9 - \bar{g}_{ik}}{0.3}\right) & 0.6 \leq \bar{g}_{ik} \leq 0.9 \\ 0 & \bar{g}_{ik} \geq 0.9 \end{cases}$$

$$\text{«}_{\bar{g}_{ik}}(High) = \begin{cases} 0 & \bar{g}_{ik} \leq 0.6 \\ \dfrac{1}{\bar{g}_{ik}} & 0.6 \leq \bar{g}_{ik} \leq 1 \end{cases}$$