A Tropical Model for Analyzing Radio Refractivity: Selected Locations in North Central, Nigeria

M. E. Emetere¹, O.A. Akinwumi¹, T.V. Omotosho¹ and J. S. Mandep²
¹Department of Physics, Covenant University, Ota, Nigeria
²Space Science Centre (ANGKASA), University Kebangsaan Malaysia, 43600 UKM Bangi, Selangor Darul Ehsan, Malaysia
moses.emetere@covenantuniversity.edu.ng, akinsayo.akinwumi@covenantuniversity.edu.ng, omotosho@covenantuniversity.edu.ng, mandeep@eng.ukm.my

Abstract—The effect of climatic global change has altered the meteorological factor of the ITU model especially in the lower atmosphere. The refractivity results obtained in an active tropical region of West Africa need to be reappraised. A model known as the Tropic model was derived and tested using NOAA data set. The data set were obtained from Meteorological data obtained from NOAA (USAF) Climatology center. Radiosonde data set was at least 39 years between 1973-2012 for six stations within the North-central of Nigeria. It was observed that the Tropic model detected tropospheric perturbations caused by the extensive influence of aerosol influx. This was confirmed by the hourly refractivity obtained from the ITU model. Hence, the ITU model and the Tropic model are complementary to each other for an accurate terrestrial radio links calculations.

Keywords-ITU model; Tropic model; refractivity; troposphere; aerosol

I. INTRODUCTION

The propagation of electromagnetic waves in the Tropospheric region is subject to various factors within the atmosphere (1). The Tropospheric space of an active tropic region is characterized by massive updraft and downdraft. This occurrence affects the refractivity conditions of the air and by extension the field strength of radio signals. Variations in temperature, pressure and relative humidity (2) have been responsible for the expansion and contraction of the Tropospheric sub-layer i.e. residual layer, surface layer, mixed layer and stable boundary layer. These parameters were analyzed diurnally and seasonally in the tropics. Hence, an accurate precision of the temporal and spatial variations of surface refractivity is important for good planning of terrestrial radio links over a region.

Recently, the effect of global climatic change on planning terrestrial radio links worldwide is a source of concern. The ITU model is vast becoming obsolete in some region (3) due climatic global change. For example, it is believed that surface refractivity generally correlate well at the first one kilometer of height (stable boundary Tropospheric layer) above the surface of the Earth (4), however, recent study of aerosol dispersions (5,6) in the Tropospheric region shows that the stable boundary layer undergoes perturbations. In the lower atmosphere, the radio frequencies above 30 MHz are affected by Refractive index variations of the atmosphere (7).

We propose that the height of the refractivity can be used to determine the range of the radio waves which also depends on the massive contraction or expansion of the Tropospheric sub-layers. From literatures, atmospheric refractive index (n) deviation from unity hinged upon polarization of the constituents of air molecules (8) and Quantum mechanical molecular resonance effect (9). Hence, the terrestrial radio links budget is complex on a mesoscale basis. In this paper, we examine the north central region of Nigeria. The regional climate systems of the research location is characterized by inter-tropical discontinuity, subtropical anticyclones, atmospheric winds, Jet stream, monsoons, sea surface temperature (SST) anomalies etc. Hence there are distinct seasonal shift in the prevailing winds, alternation between winter dry conditions and summer rainy conditions (10). It experiences dry northeasterly winds coming from the Sahara Desert during the winter. A new mathematical model was derived and plotted side by side with the ITU model.

II. METHODOLOGY

Minna is a main city of Niger state (see the Ox-blood colour of figure 1) is located 09° 37N, 06° 32E. Before now it has already been established that refractivity in Minna is about 300–375 N-units while seasonal trend affirmed that refractivity rises from February to April, is almost stable between April and September and gradually decreases from October to a minimum in February (11). It has also been established that maximum refractivity values in Minna occur in the night while minimum values occur towards the evening; and a recurrent trend of higher values exhibited in rainy season and lower values shown in dry season (7).

Abuja is a main city of the Federal Capital Territory (see the green colour of Figure 1) is located 7° 30’E - 8° 48’E. Previous study has shown that refractivity variation for dry and wet season over Abuja over the years are governed by global changes in atmospheric parameters and conversely cause changes in refractive index (12).
Makurdi is located at the boundary of the south-east and north central (see the blue colour of figure 1). Makurdi can be found at latitude 7.74°N, longitude 8.51°E. Its topography is at an elevation that is almost corresponding to Abuja i.e. of about 104 m above sea level. Its closeness to the south east Nigeria have been influential to its meteorology. The temperatures in Markurdi (like other location in north-central) are relatively high during the days of March and April. Along the river valleys, these high temperatures coupled with high relative humidities produce increases the thermal comfort of the location (13).

Jos is located in Plateau State (see the purple colour of figure 1). Its topographical underlay is high plains of about 1,300 m above sea level. Jos is located at latitude 9°52′N, Longitude: 8°54′E. Its topography is an elevation of about 1285m. The variation of refractivity both in the wet and dry seasons is in agreement with the variation of relative humidity. Also, the refractivity variation is inverse to that of temperature (12). Also, the Surface refractivity in Jos has day-night cycle period. The data used in the computations reported in this paper were obtained from Meteorological data obtained from NOAA (USAF) Climatology center. Radiosonde data set was at least 39 years between 1973-2012 for six stations within the North-central of Nigeria.

III. FORMULATION OF THE TROPIC MODEL

Many theorists and bodies have propounded many model of refractivity (N). The formula for N-units/km was confirmed in a resolution (14) as

\[ N = (N - 1) \times 10^6 = \frac{77.624 P_d}{T} + 64.7 \frac{P_w}{T} + 371897 \frac{P_w}{T^2} \]  

[1]

where \( P_d \) is the partial pressure of the dry air, \( P_w \) is the partial pressure and \( T \) is the temperature.

The ITU-R (15) gave another formula in terms of measured meteorological quantities, the refractivity \( N \) can be expressed as:

\[ N = \frac{77.62}{T} + 3.73 \times 10^5 \frac{e}{T^2} = N_{\text{dry}} + N_{\text{wet}} \text{ (N-units)} \]  

[2]

where \( e \) is the water vapour pressure, \( P \) is the atmospheric pressure (hpa) and \( T \) is the absolute temperature (K). The mathematical relationship between relative humidity and water vapour pressure is expressed in the following equation:

\[ e = \frac{RH}{100} a \exp \left( \frac{bT}{T+c} \right) \]  

[3]

Here \( T \) is the temperature in the above equation is given in °C and the coefficients \( a \) and \( b \) and \( c \) takes the following values: \( a = 6.1121, b = 17.502, \) and \( c = 240.97 \). Therefore,

\[ e = 6.1121 \exp \left( \frac{17502T}{(T+240.97)} \right) \]  

[4]

where \( RH = \) relative humidity (%), \( T = \) temperature in degree Celsius °C.

Mathematically, both equation 1& 2 can be expressed in a polymeric format where \( \frac{P_d}{T} = x, \frac{P_w}{T} = y \)

\[ N = ax + by + \frac{90}{P_w} (by)^2 \]  

[5]

or

\[ N = ax + \frac{c}{P_w} y^2 \]  

[6]

where \( a = 77.624, b = 64.7, c = 371897, P_w = e \)
Hence, from equation 4&5, we may conclude that the changing factor i.e. the ratio of the polymeric representation is written as

\[ P_w = 14.5 \exp\left(-\frac{17.5027}{(T + 240.97)}\right) \]  

Hence, applying the Maclaurin series, equation 7 can be written as

\[ P_w = 14.5 - \frac{253.6T}{(T + 240.97)} + \left(\frac{17.5027}{2(T + 240.97)}\right)^2 - \left(\frac{17.5027}{3(T + 240.97)}\right)^3 \]  

The meteorological changes can be accounted for via 2D refractivity approach, hence

\[ \frac{dN}{dx} = a + b \frac{dy}{dx} + \frac{90 d(by)^2}{P_w} \frac{dx}{dx} \]  

\[ \frac{dN}{dy} = a \frac{dx}{dy} + b + \frac{180}{P_w} yb^2 \]  

Equation (9) is differentiated with respect to y and equation (10) with respect to x. Then apply elimination method to obtain

\[ b \frac{d}{dx} - a \frac{d}{dy} = 0 \]  

Hence, the tropical model is written as

\[ N = (N - 1) \times 10^{10} = 155.248 \frac{P_d}{T} + 376748 \frac{P_w}{T^2} \]  

Recall \( P_w \) is calculated via equation [8].

### III RESULTS AND DISCUSSION

The odd-positive terms in equation (8) were adopted for the Tropic model shown for locations-Ilorin, Minna, Abuja, Jos and Markurdi (Figure 2). The ITU model for Ilorin, Minna, Abuja, Jos and Markurdi is shown in Figure 3. The refractivity of the ITU and Tropic model differs in structure especially in Ilorin (figure 4) and Minna (figure 5). It is observed from past research that the hourly refractivity is same in magnitude (see the red circle in figure 6) and structure. This means that the Tropic Model is not inaccurate in Ilorin and Minna. Both cities are closer to the influence of the north-east dust wind. The deviations from March to November are the resultant effect of aerosol influence on the troposphere. The reports in [14] shows that the events within March to November in Ilorin. All characteristic behaviour for each day within March to November elaborates on the accuracy of the Tropic model.
The Tropic model simulations within December to February accurately mimic the ITU model trend. Hence, the ITU model could only sense the Tropospheric perturbations from December to February.

In Figures 7-9, the Tropic model indicates that there were minor perturbations in the troposphere between May to November. Theoretically, this period (May to November) falls within the rainy season and refractivity is expected to be high. However, the influence of aerosols within the troposphere reduces the refractivity because aerosols in the accumulation mode have longest atmospheric lifetime (even within the troposphere), high scattering efficiency of electromagnetic wave, and very active interference with the rainfall rate through a process known as cloud condensation nuclei (CCN) formation (16).

IV CONCLUSION

The Tropic model has accurately shown the effect of aerosol influence in the troposphere. This fact has been supported by hourly refractivity simulations of troposphere data set. The
ITU model and the Tropic model are complementary to each other for an accurate terrestrial radio links calculations. Refractivity within May to November differs from same period in the ITU model. May to November falls within the rainy season and refractivity is expected to be high. However, the aerosols extensive influence on the sub-layer of the troposphere affirms that aerosol within the sub-layer have longest atmospheric lifetime (even within the troposphere), high scattering efficiency of electromagnetic wave, and very active interference with the rainfall rate through a process known as cloud condensation nuclei (CCN) formation.

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