

## Second Law Analysis of a Reactive MHD Couple Stress Fluid Through Porous Medium

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### ABSTRACT

*In this work, effect of magnetic field on the entropy generation rate of a reactive couple stress fluid through porous medium is investigated. The equations governing the fluid flow are formulated, non-dimensionalised and solved using the rapidly convergent semi-analytical Adomian decomposition method (ADM). The obtained velocity and temperature profiles are utilised to compute the entropy generation rate, irreversibility ratio and Bejan number. The effects of pertinent flow parameters on velocity, temperature, entropy generation rate and Bejan number are analyzed graphically.*

**Keywords:** Magnetohydrodynamic, ADM, Entropy generation, Porous medium, Couple stress fluid

**Mathematics Subject Classification:** 76D05, 34L30, 65L10

### 1. INTRODUCTION

Over the past centuries, analysis of magnetohydrodynamics flow has received much attention owing to its applications in nuclear reactor, MHD generators, purifications of metal from non-metal enclosures, plasma studies, geothermal energy extractions, polymer technology and metallurgy. Magnetic fields induce many complex phenomena in an electrically conducting flow regime which includes Hall currents, ion slip effects (at higher strength magnetic fields), Joule (Ohmic heating), Alfvén waves in plasma flows, etc. Cramer et al (1973). Such effects can have a considerable influence on heat transfer and flow dynamics. For example in ionized gases with low density subjected to a strong magnetic field, the electrical conductivity perpendicular to the magnetic field is lowered owing to free spiraling of electrons and ions about the magnetic lines of force prior to collisions; a current is thereby induced which is mutually perpendicular to both electrical and magnetic fields, constituting the Hall current effect. Under very high magnetic fields, in ionized plasmas, the diffusion velocity of ions becomes significant and ion slip effects arise. Hall current effects however tend to be more dominant. In magnetic material fabrication applications, porous media are frequently used to regulate flow regimes Beg et al. (2009).

The main origin of MHD dates back to pioneering discoveries of Northrup, Hartmann, Alfvén, and others in the first half of the twentieth century Molokov et al.(2007). A considerable number of studies on electrically conducting fluid has been reported in literature, these include: Batchelor (1949) who investigated the spontaneous magnetic field in a conducting liquid in turbulent motion, Moffatt (1970)

considered the Turbulent dynamo action at low magnetic Reynolds number while Hunt et al.(1971) investigated magnetohydrodynamics at high Hartmann number. Some extremum principles for pipe flow in magnetohydrodynamics was presented by Smith, he argued that with appropriate choices for the extrema, an asymptotic expansion for the mass-flow rate at large Hartman number can be constructed, later Sloan (1973) extended it to extremum principles for magnetohydrodynamic channel flow. Other important work on hydromagnetic fluid flow include Seth et al. (1982), Chandran et al.(1992), Gbadeyan et al. (2005, 2006), Adesanya et al. (2012,2015a) and Khan et al. (2015).

Analysis of a reactive fluid has been carried out by numerous researchers over the past years. According to Hassan et al. (2015), a reacting material which undergoes an exothermic reaction generates heat in accordance with Arrhenius rate law if reactant consumption is neglected. The heat produced in such reactions has attracted the attention of numerous researchers studying reactive hydromagnetic flows; due to its importance in many engineering applications. Significant among such studies include: Makinde (2013) who considered thermal stability of a reactive third-grade fluid in a channel with convective cooling the walls. Adesanya (2013) investigated thermal stability of a reactive hydromagnetic third-grade fluid through a channel with convective cooling. Gbadeyan et al.(2012) reported on the multiplicity of solutions for a reactive variable viscous Couette flow under Arrhenius kinetics. Hassan et al. (2014) investigated thermal stability of a reactive hydromagnetic Poiseuille fluid flow through a channel. Other related studies are Jha et al. (2015, 2013).

Entropy generation determines the optimal performance of thermal systems; researchers over the past years have undertaken various investigations into the causes of entropy generation. Some of the causes include heat transfer, fluid friction, magnetic field, etc. In view of the above, Bejan (1980, 1996) reported that entropy generation minimisation should be taken into consideration in the following different situations; when there is thermodynamic irreversibility, heat transfer through finite temperature gradient, convective heat transfer characteristics, viscous effects etc. Bejan (1982, 1996) also pioneered the application of second law of thermodynamics in predicting the performance of engineering processes. Thereafter, numerous researchers have investigated entropy generation under various flow configurations. Notable among them are Ajibade et al. (2011) who studied entropy generation under the effect of suction/injection. Makinde et al. (2013) presented effects of convective heating on entropy generation rate in a channel with permeable wall. Adesanya et al. (2015c) considered the effects of couple stresses on entropy generation rate in a porous channel with convective heating. Also, Adesanya (2015b) studied entropy generation analysis for a reactive couple stress fluid flow through a channel saturated with porous material while Opanuga et al. (2016) analysed the effect of thermal radiation on the entropy generation of MHD flow through porous channel.

Motivated by Adesanya et al. (2015b), the objective of this paper is analysis of the effect of magnetic field on the entropy generation of a reactive couple stress fluid, owing to the enormous applications of hydromagnetic couple stress fluid. Furthermore, authors apply the rapidly convergent semi-analytical technique of ADM instead of the regular perturbation technique used in Adesanya et al. (2015b); this is due to the fact that approximate solution by perturbation technique is valid for the small values of the parameters used. However, ADM is easy to apply, highly accurate and rapidly converges to the exact solution, see the following Refs. (Adesanya et al.2013, Opanuga et al. 2015, 2015, 2017). The rest of the paper is organised as follows: section two presents problem formulation and non-dimensionalization, section three gives the solution to the boundary value problems by Adomian

decomposition method. In section four, results are graphically discussed, while section five states the concluding remarks.

## 2. MATHEMATICAL FORMULATION

Consider the steady and thermally developed flow of an incompressible reactive couple stress fluid placed between two parallel impermeable plates with isothermal boundary conditions under the influence of a transverse magnetic field strength  $B_0$ . It is assumed that the fluid motion is induced by applied axial pressure gradient. Neglecting the consumption of the reactant, then the governing equations for the momentum, heat balance and entropy generation rate can be written as Adesanya et al. (2015b)

$$0 = -\frac{dp}{dx'} + \mu \frac{d^2u'}{dy'^2} - \frac{\mu u'}{k} - \eta \frac{d^4u'}{dy'^4} - \frac{\sigma B_0^2 u'}{\rho} \quad (1)$$

$$0 = \frac{dT'}{dy'^2} + \frac{QC_0A}{k} e^{-\frac{E}{RT}} + \frac{\mu}{k} \left(\frac{du'}{dy'}\right)^2 + \frac{\mu u'^2}{kK} + \frac{\eta}{k} \left(\frac{d^2u'}{dy'^2}\right)^2 + \frac{\sigma B_0^2 u'^2}{\rho c_p} \quad (2)$$

$$E_G = \frac{k}{T_0^2} \left(\frac{dT'}{dy'}\right)^2 + \frac{\mu}{T_0} \left(\frac{du'}{dy'}\right)^2 + \frac{\mu u'^2}{T_0 K} + \frac{\eta}{T_0} \left(\frac{d^2u'}{dy'^2}\right)^2 + \frac{\sigma B_0^2 u'^2}{T_0} \quad (3)$$

$$u(0) = u''(0) = u(h) = u''(h) = 0$$

$$\theta(0) = \theta(h) = 0 \quad (4)$$

In the equations above,  $u'$  is the axial velocity,  $\rho$  is the fluid density,  $c_p$  is the specific heat at constant pressure,  $\sigma$  is the electrical conductivity of the fluid,  $P$  is the fluid pressure,  $T'$  is the absolute temperature,  $T_0$  is the geometry wall temperature,  $k$  is the thermal conductivity of the fluid,  $K$  is the porous permeability of the medium,  $Q$  is the heat of reaction,  $A$  is the rate constant,  $E$  is the activation energy,  $R$  is the universal gas constant,  $c_0$  is the initial concentration of the reactant species,  $h$  is the channel width,  $(x, y)$  is the distance measured in the axial and normal directions, respectively,  $\mu$  is the combustible material dynamic viscosity coefficient,  $\eta$  is the fluid particle size effect due to couple stresses,  $B_0$  is the uniform transverse magnetic field,  $E_G$  is the local volumetric entropy generation rate.

$$y = \frac{y'}{h}, u = \frac{u'}{UM}, \theta = \frac{E(T - T_0)}{RT_0^2}, \lambda = \frac{QEAC_0 h^2 e^{-\frac{E}{RT}}}{RT_0^2 k}, M = -\frac{h^2}{\mu U} \frac{dP}{dx}, H^2 = \frac{\sigma B_0^2 h^2}{\rho} \quad (5)$$

$$\epsilon = \frac{RT_0}{E}, \delta = \frac{U^2 \mu M^2 e^{-\frac{E}{RT}}}{QAC_0 h^2}, \gamma = \frac{h}{l}, l = \sqrt{\frac{\eta}{\mu}}, \beta = \sqrt{\frac{1}{Da}}, Da = \frac{k}{h^2}, N_s = \frac{h^2 E^2 E_G}{kR^2 T_0^2},$$

using equation (5) in equations (1-3), we obtain the boundary value problems and the dimensionless entropy generation expression

$$0 = 1 + \frac{d^2u}{dy^2} - \beta^2u - \frac{1}{\gamma} \frac{d^4u}{dy^4} - H^2u \tag{6}$$

$$0 = \frac{d^2\theta}{dy^2} + \lambda \left\{ e^{\frac{\theta}{1+\epsilon\theta}} + \delta \left( \frac{du}{dy} \right) + \frac{\delta}{\gamma} \left( \frac{d^2u}{dy^2} \right)^2 + \delta\beta^2u^2 + \delta H^2u^2 \right\} \tag{7}$$

$$Ns = \left( \frac{d\theta}{dy} \right)^2 + \frac{\delta\lambda}{\epsilon} \left( \left( \frac{du}{dy} \right)^2 + \frac{1}{\gamma} \left( \frac{d^2u}{dy^2} \right)^2 + \beta^2u^2 + H^2u^2 \right) \tag{8}$$

and the boundary conditions are

$$\begin{aligned} u''(0) = u(0) = u''(1) = u(1) = 0 \\ \theta(0) = 0, \theta(1) = 0 \end{aligned} \tag{9}$$

where  $u$  is the dimensionless velocity,  $\lambda$  is the Frankkameneskii parameter,  $\epsilon$  is the activation energy parameter,  $\delta$  is the viscous heating parameter,  $\beta$  is the porous permeability parameter,  $Da$  is the Darcy number,  $\gamma$  is the couple stress inverse parameter,  $l$  is a function of molecular dimension of the fluid,  $H$  is the magnetic field parameter,  $Ns$  is the dimensionless entropy generation rate,  $\theta$  is the dimensionless temperature  $Be$  and  $Bi_{1,2}$  are the Bejan number and Biot numbers respectively and  $G$  is the axial pressure gradient.

### 3. METHOD OF SOLUTION

#### 3.1 Solution via ADM

Writing equation (6) in the integral form, we obtain

$$u(y) = f_1y + \frac{f_2}{3!}y^3 + \gamma \int_0^y \int_0^y \int_0^y \int_0^y \left\{ 1 + \frac{d^2u}{dY^2} - \beta^2u - H^2u \right\} dYdYdYdY \tag{10}$$

where  $f_1, f_2$  are the parameters to be determined later.

The infinite series solution of ADM is of the form

$$u(y) = \sum_{n=0}^{\infty} u_n(y), \theta(y) = \sum_{n=0}^{\infty} \theta_n(y) \tag{11}$$

Applying (11) in (10) yields

$$\sum_{n=0}^{\infty} u_n(y) = f_1y + \frac{f_2}{3!}y^3 + \gamma \int_0^y \int_0^y \int_0^y \int_0^y \left\{ 1 + \sum_{n=0}^{\infty} \frac{d^2u_n}{dY^2} - \beta^2 \sum_{n=0}^{\infty} u_n - H^2 \sum_{n=0}^{\infty} u_n \right\} dYdYdYdY \tag{12}$$

From equation (12), the zeroth order term can be written as

$$\sum_{n=0}^{\infty} u_0(y) = f_1 y + \frac{f_2}{3!} y^3 + \gamma \int_0^y \int_0^y \int_0^y \int_0^y 1 \, dY dY dY dY \quad (13)$$

while other terms can be determined using the recurrence relations below

$$\sum_{n=0}^{\infty} u_{n+1}(y) = \gamma \int_0^y \int_0^y \int_0^y \int_0^y \left\{ 1 + \sum_{n=0}^{\infty} \frac{d^2 u_n}{dY^2} - \beta^2 \sum_{n=0}^{\infty} u_n - H^2 \sum_{n=0}^{\infty} u_n \right\} dY dY dY dY \quad (14)$$

Also, equation (7) in the integral form can be written as

$$\theta(y) = f_3 - \lambda \int_0^y \int_0^y \left\{ e^{\frac{\theta}{1+\epsilon\theta}} - \left( \gamma \left( \frac{du}{dY} \right)^2 - \frac{1}{\gamma} \left( \frac{d^2 u}{dY^2} \right)^2 + \beta^2 u^2 + H^2 u^2 \right) \right\} dY dY \quad (15)$$

where  $f_3$  is to be determined later. Applying (11) in (15) gives

$$\sum_{n=0}^{\infty} \theta_n(y) = f_3 - \lambda \int_0^y \int_0^y \left\{ e^{\frac{\sum_{n=0}^{\infty} \theta_n}{1+\epsilon \sum_{n=0}^{\infty} \theta_n}} - \left( \gamma \left( \sum_{n=0}^{\infty} \frac{du}{dY} \right)^2 - \frac{1}{\gamma} \left( \sum_{n=0}^{\infty} \frac{d^2 u}{dY^2} \right)^2 + \beta^2 \sum_{n=0}^{\infty} u^2 + H^2 \sum_{n=0}^{\infty} u^2 \right) \right\} dY dY \quad (16)$$

and the zeroth order term can be written as

$$\sum_{n=0}^{\infty} \theta_0(y) = f_3 \quad (17)$$

other terms can be determined using the recurrence relations

$$\sum_{n=0}^{\infty} \theta_{n+1}(y) = -\lambda \int_0^y \int_0^y \left\{ e^{\frac{\sum_{n=0}^{\infty} \theta_n}{1+\epsilon \sum_{n=0}^{\infty} \theta_n}} - \left( \gamma \left( \sum_{n=0}^{\infty} \frac{du}{dY} \right)^2 - \frac{1}{\gamma} \left( \sum_{n=0}^{\infty} \frac{d^2 u}{dY^2} \right)^2 + \beta^2 \sum_{n=0}^{\infty} u^2 + H^2 \sum_{n=0}^{\infty} u^2 \right) \right\} dY dY \quad (18)$$

and the non-linear term in equation (18) is written as

$$\sum_{n=0}^{\infty} A_n(y) = e^{\frac{\sum_{n=0}^{\infty} \theta_n(y)}{1+\epsilon \sum_{n=0}^{\infty} \theta_n(y)}} \quad (19)$$

Some of the Adomian polynomials  $A_n$ 's from (19) are given as

$$A_0 = e^{\frac{\theta_0(y)}{1+\delta\theta_0(y)}}, A_1 = \frac{e^{\frac{\theta_0(y)}{1+\delta\theta_0(y)}} \theta_1(y)}{[1+\delta\theta_0(y)]^2} \tag{20}$$

$$A_2 = \frac{e^{\frac{\theta_0(y)}{1+\delta\theta_0(y)}} \left[ (1-2\delta-2\delta^2\theta_0(y))\theta_1(y)^2 + 2(1+\delta\theta_0(y))^2\theta_2(y) \right]}{2[1+\delta\theta_0(y)]^2} \tag{21}$$

Using equations (20-21) in equation (18), we obtain

$$\sum_{n=0}^{\infty} \theta_{n+1}(y) = -\lambda \int_0^y \int_0^y \left\{ A_n - \left( \gamma \left( \sum_{n=0}^{\infty} \frac{du}{dY} \right)^2 - \frac{1}{\gamma} \left( \sum_{n=0}^{\infty} \frac{d^2u}{dY^2} \right)^2 + \beta^2 \sum_{n=0}^{\infty} u^2 + H^2 \sum_{n=0}^{\infty} u^2 \right) \right\} dYdY \tag{22}$$

All calculations associated with equations (13, 14, 17 and 22) are carried out by coding the equations in an algebra symbolic package-Mathematica. We present only the graphical results in Figures 1-15 due to large size of the computational solution. To verify the accuracy of these computations, the approximate solution of the velocity profile obtained via ADM is compared with exact solution in Table 1.

Table 1: Computation showing convergence of solution when  $\gamma = H = 1, \beta = 0.1$

| $u(y)$ | $U_{EXACT}$ | $\sum_{n=0}^{14} U_{ADM}$ | Abs. error  |
|--------|-------------|---------------------------|-------------|
| 0      | 0           | 0                         | 0           |
| 0.1    | 0.003678164 | 0.003678164               | 2.10448E-12 |
| 0.2    | 0.006955122 | 0.006955122               | 4.08814E-12 |
| 0.3    | 0.009517005 | 0.009517005               | 5.82758E-12 |
| 0.4    | 0.011142114 | 0.011142114               | 7.19439E-12 |
| 0.5    | 0.011698431 | 0.011698431               | 8.05267E-12 |
| 0.6    | 0.011142114 | 0.011142114               | 8.25621E-12 |
| 0.7    | 0.009517005 | 0.009517005               | 7.64618E-12 |
| 0.8    | 0.006955122 | 0.006955122               | 6.04804E-12 |
| 0.9    | 0.003678164 | 0.003678164               | 3.26987E-12 |
| 1      | 3.45E-17    | -9.0182E-13               | 9.01855E-13 |

### 3.2 ENTROPY GENERATION

The expression for entropy generation in equation (3) suggests five sources of entropy production. The first term is irreversibility due to heat transfer, the second, third, fourth and fifth terms are irreversibility due to fluid friction, porosity, couple stresses and the effect of magnetic field.

$$E_G = \frac{k}{T_o^2} \left( \frac{dT'}{dy'} \right)^2 + \frac{\mu}{T_o} \left( \frac{du'}{dy'} \right)^2 + \frac{\mu u'^2}{T_o K} + \frac{\eta}{T_o} \left( \frac{d^2u'}{dy'^2} \right)^2 + \frac{\sigma B_o^2 u'^2}{T_o}$$

The dimensionless form in equation (8) can be written

$$E_G = \left( \frac{d\theta}{dy} \right)^2 + \frac{\delta\lambda}{\epsilon} \left\{ \left( \frac{du}{dy} \right)^2 + \frac{1}{\gamma} \left( \frac{d^2u}{dy^2} \right)^2 + \beta^2 u^2 + H^2 u^2 \right\}$$

We now set

$$N_1 = \left( \frac{d\theta}{dy} \right)^2 \quad \text{and} \quad N_2 = \frac{\delta\lambda}{\epsilon} \left\{ \left( \frac{du}{dy} \right)^2 + \frac{1}{\gamma} \left( \frac{d^2u}{dy^2} \right)^2 + \beta^2 u^2 + H^2 u^2 \right\} \quad (23)$$

where  $N_1$  is irreversibility due to heat transfer and  $N_2$  represents fluid friction irreversibility with magnetic field. To describe the contribution of heat transfer irreversibility to the overall entropy generation, the Bejan number  $Be$  is employed. It gives the ratio of heat transfer and viscous dissipation with magnetic field within the channel as

$$Be = \frac{N_1}{N_s} = \frac{1}{1 + \Phi} \quad (24)$$

where  $\Phi = \frac{N_2}{N_1}$  is the irreversibility ratio.

Equation (24) indicates that Bejan number takes values between 0 and 1. i.e  $0 \leq Be \leq 1$ . The value  $Be = 1$  signifies the limit at which heat transfer dominates entropy generation,  $Be = 0$  gives the limit at which viscous dissipation and magnetic field dominates and  $Be = 0.5$  represents equal contribution of both heat transfer and viscous dissipation to entropy production.

#### 4. RESULTS AND DISCUSSION

In this present study, effect of magnetic field on entropy generation rate of a reactive couple stress fluid flow through porous medium has been investigated using the rapidly convergent Adomian decomposition method. The velocity and temperature profiles are obtained and utilized to compute the entropy generation rate. The effects of various flow parameters on velocity, temperature, entropy generation and Bejan number are discussed to provide insight to the problems, and the results are graphically displayed in Figs. 1-15.

##### 4.1 Effect of Parameters Variation on Velocity Profile

In Figs. 1-3 we present the variations of velocity for different flow parameters. Figure 1 displays the effect of variations in magnetic field intensity on fluid velocity. The Figures indicates that velocity of the fluid reduces with increasing value of magnetic field due to the retarding effect of Lorentz force present in the flow. Figure 2 is the plot of porous medium permeability parameter variations on fluid velocity; it is shown from the plot that increase in porous medium term corresponds to reduction in fluid velocity. This is due to the fact that increase in porous medium parameter always results to reduction in porous medium permeability of the fluid. Figure 3 shows the graph of couple stress inverse parameter on fluid

temperature. From the plot, we observed that increase in couple stress inverse increases fluid velocity, meaning that couple stresses will retard fluid velocity due to increased fluid thickness.

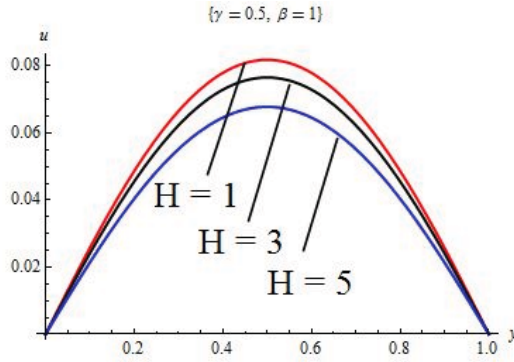


Figure 1: Effect of magnetic field on fluid velocity

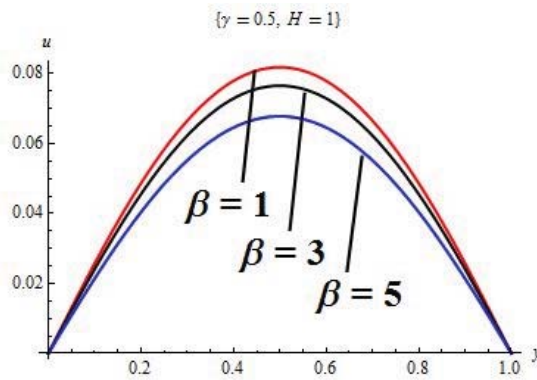


Figure 2: Effect of porous medium parameter on fluid velocity

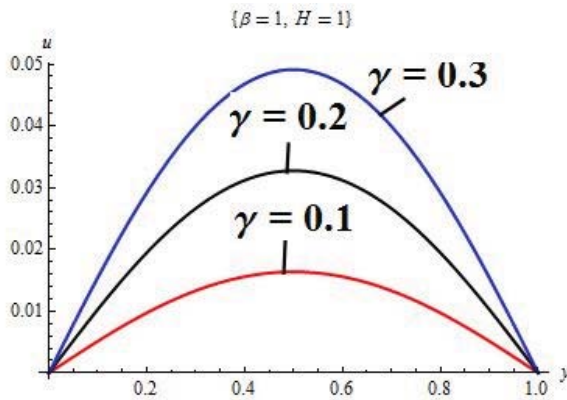


Figure 3: Effect of couple stress inverse parameter on fluid velocity



4.2 Effect of Parameters Variation on Temperature Profile

The influences of different governing parameters on temperature are presented in Figs. 4-7. The effect of magnetic field parameter on fluid temperature is shown in Fig. 4. The plot shows a rise in fluid temperature as magnetic field parameter increases. The Lorentz heating effect is attributed to the rise in fluid temperature. In Figure 5, effect of viscous heating parameter on temperature is depicted. Fluid temperature is enhanced by increasing viscous heating parameter due to the additional heat generated by the conversion of fluid kinetic energy to internal energy. Also, Figure 6 displays the effect of variation in Frank-Kameneskii parameter ( $\lambda$ ) on fluid temperature. It is observed from the plot that there is a rise in fluid temperature with increase in Frank-Kameneskii parameter, this is caused by increase in the initial concentration of the reactant species ( $c_0$ ) within the flow channel. From Figure 7, we observed an increase in fluid temperature with increase in couple stress inverse parameter. This indicates that fluid temperature will decrease with increase in couple stress parameter because of the rise in fluid dynamic viscosity which results in the drop of fluid temperature.

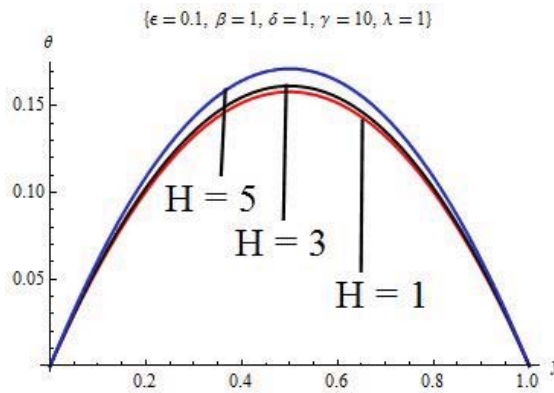


Figure 4: Effect of magnetic field parameter on fluid temperature

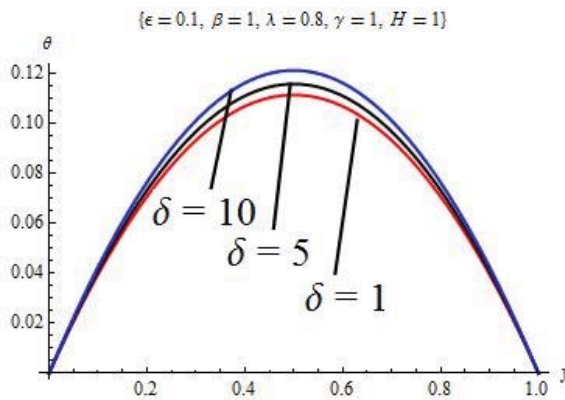


Figure 5: Effect of viscous heating parameter on fluid temperature

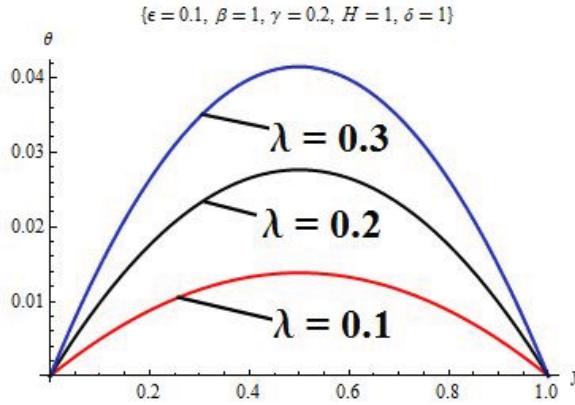


Figure 6: Effect of Frank-Kamenetskii parameter on fluid temperature

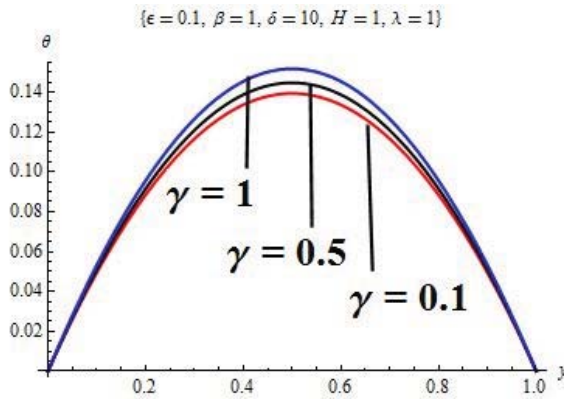


Figure 7: Effect of couple stress inverse parameter on fluid temperature

### 4.3 Effect of Parameters Variation on Entropy generation rate

In Figures 8-11, we present the effects of variation of some governing parameters on entropy generation rate. Figure 8 indicates the effect of magnetic field parameter on entropy generation. We found that entropy production registers an increase as magnetic field parameter increases. This is caused by increased temperature (as indicated in Fig.4) which increases the rate of disorderliness of the flow. Also in Figure 9 we observed that an increase in viscous heating parameter results in higher entropy production due to increased heat generated by the conversion of fluid kinetic energy to internal energy (see Figure 5). Furthermore, entropy generation increases significantly at the walls of the channel with increase in Frank-Kamenetskii parameter as shown in Figure 10. This is due to increase in heat generated by the rise in concentration of the reactant species. In Figure 11, the effect of variation in couple stress inverse parameter is displayed, the plot indicates that increase in couple stress inverse corresponds to a significant rise in entropy generation at the center of the channel, meaning that couple stresses will lead to reduction in entropy generation.

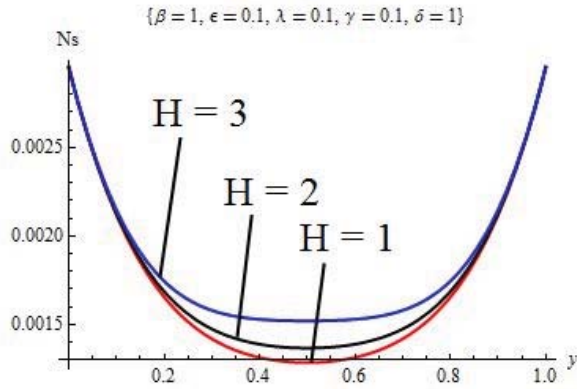


Figure 8: Effect of magnetic field on entropy generation

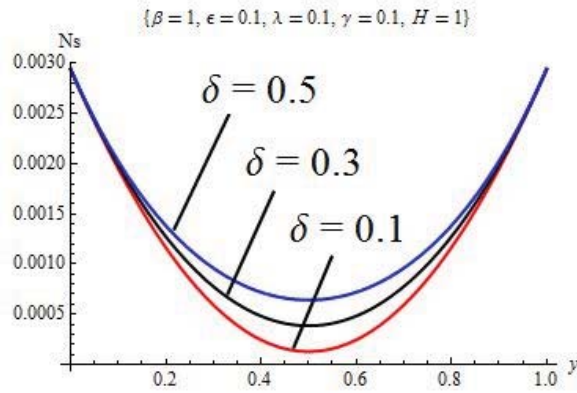


Figure 9: Effect of viscous heating on entropy generation

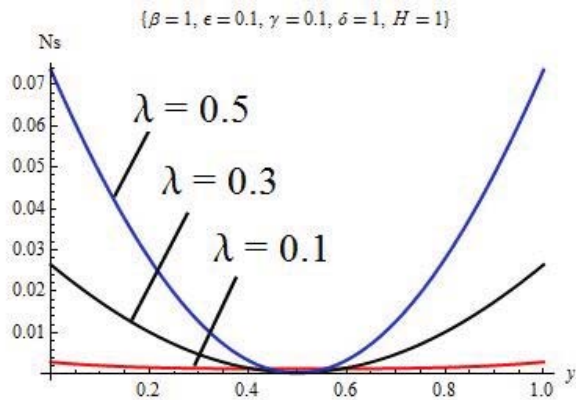


Figure 10: Effect of Frank-Kamenetskii parameter on entropy generation

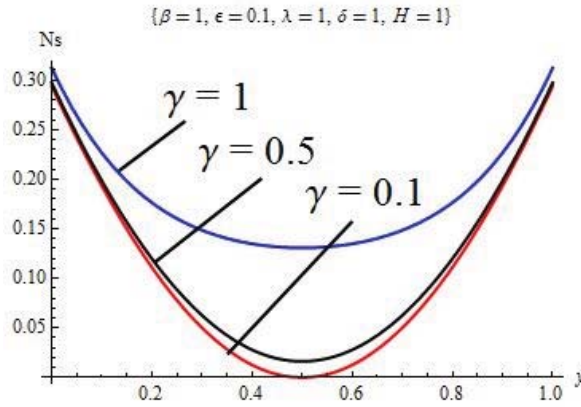


Figure 11: Effect of couple stress inverse on entropy generation

#### 4.4 Effect of Parameters Variation on Bejan number

We present the influence of parameters variation on Bejan number in Figs.12-15. Figures 12 and 13 depict the effect of magnetic field intensity parameter and viscous heating parameter on Bejan number. The plots show a decrease in Bejan number as magnetic field and viscous heating parameters varied. This is an indication that irreversibility due to viscous dissipation is the dominant contributor to entropy generation. Finally Figures 14 and 15 present the plots of Frank-Kameneskii and couple stress parameters on Bejan number. The plots display an increase in Bejan number as Frank-Kameneskii and couple stress parameters increase. They show that irreversibility due to heat transfer dominates over viscous dissipation.

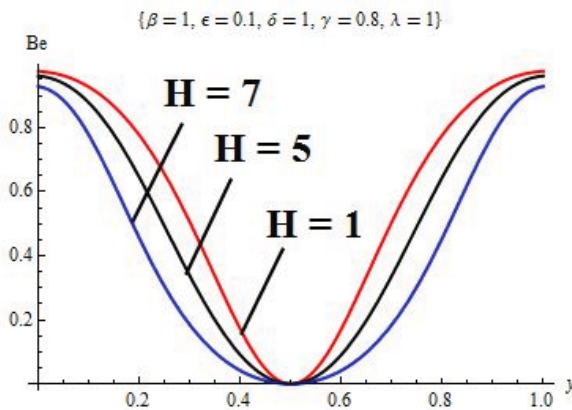


Figure 12: Effect of magnetic field intensity on entropy generation

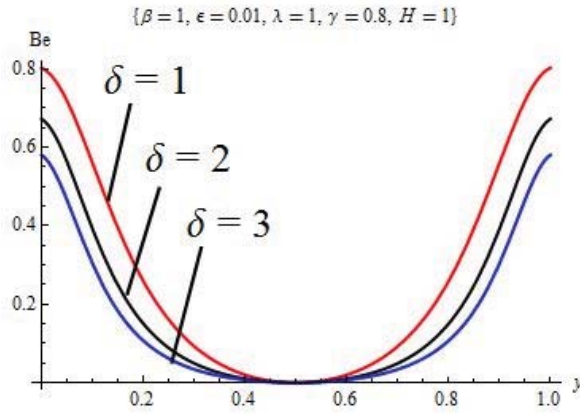


Figure 13: Effect of viscous heating parameter on Bejan number

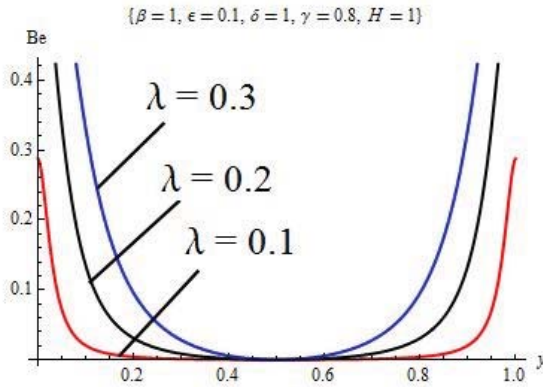


Figure 14: Effect of Frank-Kamenetskii parameter on Bejan number

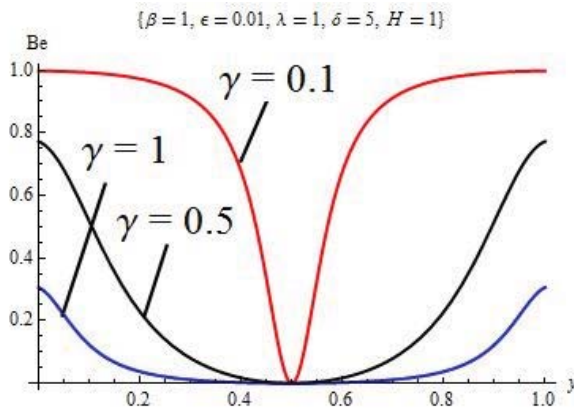


Figure 15: Effect of couple inverse parameter on entropy generation

## 5. CONCLUSIONS

In this work, we have investigated entropy generation of hydromagnetic reactive couple stress fluid through a channel filled with porous medium. The non-linear governing equations of momentum and energy are solved numerically by Adomian decomposition method. The results are used to compute the non-dimensional entropy generation and Bejan number. Conclusions of the study are as follows:

- Increase in magnetic field reduces fluid velocity but increases the temperature and entropy generation rate
- Increase in viscous heating parameter increases both the temperature and entropy generation of the fluid
- Increase in Frank-Kamenetskii parameter increases fluid temperature and entropy generation
- Increase in couple stresses reduce fluid velocity, temperature and entropy generation

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