Forecasting Nigeria Foreign Exchange Risk with Extreme Value Theory

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ABSTRACT

Foreign exchange forecasting is important in all forms of foreign investments and transactions, the skills which compliments the field of finance and related disciplines. This paper uses extreme value theory to estimate the Value-at-Risk (VaR) and Expected Shortfall (ES), by fitting negative log returns of Nigeria Naira (NGN) against nine other regional and world currencies into the Generalized Pareto Model. In this paper VaR is being used to determine daily foreign exchange risk on investments and the ES is used to determine the average risk over a period of time. Excess distribution and the tail of the underlying distribution were obtained over a required threshold. Empirical analysis shows that parameter estimate of the underlying distribution can be used to describe the performance of the VaR and ES. The findings contributes to the knowledge on foreign exchange forecasting and helps investors and policy makers in Nigeria to measure daily and possible risk over a period of time on certain investments.

Keywords: Extreme Value Theory, Value-at-Risk, Expected Shortfall, Foreign exchange rate, Nigeria Naira, GPD model

INTRODUCTION

The need to forecast foreign exchange risk is of utmost importance to international investors or firm, central banks of countries, financial institutions. Forecasting currency exchange rate rates is an important financial problem that has recorded a great deal of attention especially because of intrinsic difficulty and practical applications, Hamadu and Adeleke (2009).

Nigeria as a developing nation, the price of foreign exchange play significant roles in ability of the economy to attain optimal productivity capacity (Ogiogio, 1996). Nigeria experienced several exchange rate regimes in the past four decades starting from exchange rate system between 1980 and 1986, to a flexible exchange rate system from 1986-1995 (Oyelami & Edo 2012). In Nigeria, the design and implementation of monetary policy is carried out by Central Bank of Nigeria (CBN).

The Nigerian foreign exchange has witnessed tremendous changes over the years; in fact the evolution up to its present state was influenced by a number of factors such as the international trade dynamic, military intervention in politics, institutional changes in the economy and structural shifts in production, Hamadu and Adeleke (2009).

In reforming the exchange rate management system in Nigeria, various policies have been and are still put in place by CBN to stabilize exchange rate of naira. In 1995, the Autonomous Foreign Exchange Market (AFEM) was introduced and later followed by the inter-bank Foreign Exchange Market (IFEM) in October 1999. In July 2002 the Wholesale Dutch Auction System (WDAS) was introduced and adopted but later replaced by the Inter-Bank Foreign Exchange market (IFEM), Oyelami & Edo (2012).

It is observed that foreign exchange rate is not normally distributed hence the introduction of Extreme Value Theory with generalized Pareto distribution to fit the tail area of the distribution over a relatively high threshold.

Extreme value theory explains occasion where there is likelihood of rare or damaging events occurring. Extreme value distribution arises as limiting distributions for maximums or minimums of some identically and independently distributed random variables, as sample sizes increases, Yang Hu (2013).

EVT deals with stochastic behavior of the extreme values in process. For a single process, the behavior of maxima can be described by three extreme value distribution- Gumbel, Frechet and negative Weibull as suggested by Fisher-Tippet (1928). Extreme event risk is present in all areas of financial risk management, whether we are concerned with market, credit, operational or insurance risk, one of the greatest challenges to the risk managers is to implement models which allows for rare but damaging events and permit measurement of their consequences (McNeil, A. J 1999).

Extreme event occur when a risk takes values from the tail of its distribution, extreme value theory is a reliable tool which attempts to provide us with the best possible estimate of the tail area of the distribution, Wainnaina, & Waititu, (2014).

Traditionally portfolio financial returns, especially exchange rate and interest rate returns are not normally distributed. Over the past three decades, global financial crisis events occurred frequently, almost three to five years as stated by Lan Y, Chokethaworn. K (2014).

Merger and acquisition has been pronounced in the past one decade in Nigerian financial institutions, especially in the banking sector; due to lack of strong financial base in Nigeria. The financial misfortune can be traced to inadequate risk management.

Banks and other institutions are expected to communicate their daily VaR forecasts to monetary authorities and investors must pay attention to the risk that they incur as well as the expected return from their activities (Cesar C. R and Emmanuel G. 2011).

The Value at Risk (VaR) and Expected shortfall (ES) are measures which attempt to describe the tail of a loss distribution. Extreme value is most naturally developed as a theory of large losses rather than a theory of small profits. Standard VaR models face series of problems because it have no tool for extreme events. Therefore, an alternative risk measurement; Expected shortfall (ES) has been put in place as a supplement to VaR.

Exchange rate risk is mainly due to the imperfect correlations in the movement of currency prices and fluctuations in international interest rates. Foreign exchange volatility can make a firm lose the return from expensive investments, and at the same time place a competitive disadvantage compared with its foreign competitors (Lan, Chokethaworn. 2014; Embretch, Resnick & Samorodnitsky 1998) outlined a fully parametric models based on the generalized Pareto distribution (GPD) through POT class of models which is observed to obtain a simple parametric formulae for measures of extreme risk for which it is relatively easy to give estimates of statistical error using the techniques of maximum likelihood Inference.

The block maxima models was used to model Extreme values with large observations, but a more modern group models are peaks-over-threshold (POT) models; these are models for all large observations which exceed a high threshold. The POT models are generally considered to be the most useful for practical applications due to their more efficient use of the (often limited) data on extreme values (McNeil, A. J 1999).

The GPD model was fitted with R statistical software with fextreme package, considering the central exchange rate which gives values that lies between the buying and the selling rate. We consider official quoted daily exchange rates of NGN against nine observed higher and/ or leading world currencies exchange rates (the US dollar, the European Euro, the British Pound, Denmark Danish Krone, Swiss Franc, Special Drawing right SDR, West African WAUA, Saudi Riyal and Chinese Yuan).

Empirical analysis results have revealed that Value-at-Risk is able to help in forecasting exchange rate, and in occasion where it is ineffective Expected Shortfall remedies its deficiencies. We expected that the findings in this research contribute immeasurably to foreign exchange modelling and forecasting.

DATA DESCRIPTION

In this paper, data of foreign exchange were collected from Central Bank of Nigeria (CBN) which spans for fifteen (15) years, October, 2001 to April 2015 of nine currency exchange rates against Nigeria Naira. The data are ‘time series’ data which are special case of panel data. Nine foreign exchange rates were considered against the NGN which include US dollars, pounds sterling, Euro, Chinese Yuan, Franc, Danish Krone, SDR, WAUA, Saudi RIYAL. Website:*www.cenbank.org/ExchangeRateByCurrency.asp.*

Statistical tests of normality of Anderson Darling and Kolmogorov-Smirnov are implemented on the data as seen in table 2. Quantile-Quantile (Q-Q) plot was also used to inspect the normality of the data as seen in the appendix. Both graphical and statistical methods proved that the data is far from being normally distributed.

According to Reiss & Thomas (2002), if S_t be the exchange rate of USD against NGN and let $r_t = \log\left(\frac{S_t}{S_{t-1}}\right)$, then the exchange rate of NGN against USD is $\frac{1}{S_t}$. This will yield the log returns $-r_t = \log\left(\frac{1/S_t}{1/S_{t-1}}\right)$.

Generalized Extreme Value Distribution

If there exist sequences of constants $\{a_n > 0\}$ and $\{b_n\}$, such that $\Pr\{(M_n - b_n)/a_n \le x\} \to H_\gamma(x)$ where is a non-degenerate distribution function, then H_γ is a member of GEV family:

$$H_\gamma(x) = \begin{cases} \exp\{-(1-\gamma x)^{-1/\gamma}, if\, \gamma \neq 0 \\ \\ \exp\{-\exp\{-x\}\}, if\, \gamma = 0 \end{cases} \tag{1}$$

Where $1+\gamma x > 0$. Hence the support of H_γ corresponds to

$x > -\gamma^{-1}$ if $\gamma > 0$

$x < -\gamma^{-1}$ if $\gamma < 0$

$x \in \mathbb{R}$ if $\gamma = 0$

H_γ is called the standard generalized extreme value distribution (GEV).

The three types of distributions used in extreme Value distributions; Frechet, Weibull and Gumbel earlier mentioned can be combined into one and expressed as:

$$G(x) = \exp\left[-\left(1+\xi\left(\frac{x-\mu}{\sigma}\right)\right)^{-1/\xi}\right] \tag{2}$$

Thus the Frechet and Weibull types corresponds to $\xi > 0\left(\xi = 1/\alpha\right)$ and $\xi < 0\left(\xi = -1/\alpha\right)$, respectively. The case $\xi = 0$ is interpreted as limit $\xi \to 0$ and thus $G(x)$ reduces to the Gumbel type:

$$G(x) = \exp\left[-\exp\left(-\frac{x-\mu}{\sigma}\right)\right], -\infty < x < \infty \tag{3}$$

It is clearly seen that the value of ξ dictates the tail behavior of G, thus we refer to ξ as the shape parameter. μ and σ are referred to the location and scale parameters, respectively.

The Generalized Pareto Distribution (GPD)

The GEV distribution is found to waste data when the complete dataset (or at least all extreme values) are available (Yang Hu 2013). One of the ways to overcome this challenge is to model all the data above some sufficiently high thresholds, which is referred to as the peaks over threshold or threshold excess model.

Coles (2001) explains that let $X_1,.....X_n$ be a sequence of iid random variables, the excess $X-u$ over a suitable u can be approximated by Generalized Pareto distribution. The cumulative density function (cdf) of a two-parameter GPD distribution is as follows:

$$G_{\xi,\beta} = 1-\left(1+\xi\frac{x}{\beta}\right)^{\frac{-1}{\xi}}, \xi \neq 0$$
$$G_{\xi,\beta} = 1-\exp(-x/\beta), \xi = 0 \tag{4}$$

Where $\beta > 0, x \geq 0$ when $\xi \geq 0$ and $0 \leq x \leq -\frac{\beta}{\xi}$ when $\xi < 0$, ξ is the important shape parameter of the distribution and β is an additional scaling parameter. If $\xi > 0$, the $G_{\xi,\beta}$ is re-parametrized version of the ordinary Pareto distribution use for large losses, $\xi = 0$ corresponds to the exponential distribution and $\xi < 0$ is known as a Pareto of type II distribution.

The case where $\xi > 0$ is the most relevant for risk management. Whereas normal distribution has moments of all orders, a heavy-tailed distribution does not possess a complete set of moments.

Fitting a GPD Model

In the case of the GPD with $\xi > 0$ we observe that $E(X^k)$ is infinite for $k \geq \frac{1}{\xi}$ when $\xi = \frac{1}{2}$, the GPD is a second moment distribution, when $\xi = \frac{1}{4}$, the GPD has an infinite fourth moment .

With $G_{\xi,\beta}$ for the density of the GPD, the log-likelihood may be calculated to be:

$$L\left(\xi,\beta;Y_j\right) = -N_u In\beta + \left(\frac{1}{\xi}-1\right)\sum_{j=1}^{N_u}\left(1-\xi\frac{Y_j}{\beta}\right),\ \xi \neq 0 \tag{5}$$

$$L\left(\xi,\beta;Y_j\right) = -N_u In\beta - \frac{1}{\beta}\sum_{j=1}^{N_u} Y_j,\quad \xi = 0 \tag{6}$$

The range of β is $\beta > 0$, for $\xi \leq 0$ and $\beta > \xi Y_j$ for $\xi > 0$ if $\xi > 1$, there is no maximum likelihood estimate, since for any $\xi > 1$

$$\lim_{\frac{\beta}{\xi}\to Y_j^+} L\left(\xi,\beta;Y_j\right) = \infty ,$$

To obtain a maximum likelihood, the constraint $\xi \le 1$ is imposed. Let us consider the special case of $\xi = 0$. As stated by Scott (1993) pp.191 and adapted in this paper accordingly; the gradient vector at $\xi = 0$, has elements:

$$\frac{\partial L}{\partial \xi}\left(\xi,\beta;Y_j\right) = \frac{N_u}{\xi}\left(\frac{1}{\xi}-1\right) - \frac{1}{\xi^2}\sum_{j=1}^{N_u} In\left(1-\xi\frac{Y_j}{\beta}\right) - \frac{1}{\xi}\left(\frac{1}{\xi}-1\right)\sum_{j=1}^{N_u}\left(1-\xi\frac{Y_j}{\beta}\right)^{-1} \quad (7)$$

$$\frac{\partial L}{\partial \xi}\left(\xi,\beta;Y_j\right) = -\frac{N_u}{\xi\beta} + \frac{1}{\beta}\left(\frac{1}{\xi}-1\right)\sum_{j=1}^{N_u}\left(1-\xi\frac{Y_j}{\beta}\right)^{-1} \quad (8)$$

$$\lim_{\xi\to 0}\frac{\partial L}{\partial \xi}\left(\xi,\beta;Y_j\right) = \sum_{j=1}^{N_u}\frac{Y_j^2}{2\beta^2} - \sum_{j=1}^{N_u}\frac{Y_j}{\beta} \quad (9)$$

From (6) we have;

$$\lim_{\xi\to 0}\frac{\partial L}{\partial \xi}\left(\xi,\beta;Y_j\right) = -\frac{1}{\beta}n + \frac{1}{\beta^2}\sum_{i=1}^{n}X_i$$

$$= \frac{1}{\beta^2}\sum_{i=1}^{n}X_i - \frac{1}{\beta}n$$

$$\lim_{\xi\to 0}\frac{\partial L}{\partial \xi}\left(\xi,\beta;Y_j\right) = \frac{1}{\beta}\left(\sum_{j=1}^{N_u}\frac{Y_j}{\beta} - N_u\right) \quad (10)$$

Which are equal to zero if and only if $\frac{1}{N_u}\sum_{j=1}^{N_u} Y_j^2 = 2\bar{Y}^2$. Therefore, if this condition is not satisfied, then the case $\xi = 0$ can be eliminated from the consideration. Therefore, Computing the GPD maximum likelihood estimates is an optimization on the constraints space $\Im = \{\xi < 0, \alpha > 0\} \cup \left\{\xi \le 1, \frac{\beta}{\xi} > Y_{j(n)}\right\}$ Where value of (ξ,β) must be investigated to compute the GPD maximum likelihood estimate. We investigate with the local maximum of the log-likelihood on the space $\Im$.

Local Maximum on $\Im$

To compute the local maximum on the space $\Im$, we consider equations (7) and (8), the solution to the simultaneous equations may be simplified as follows

$$\frac{\partial L}{\partial \xi}\left(\xi,\beta;Y_j\right) = 0$$

$$\Rightarrow \left(\hat{\xi}-1\right) - \sum_{j=1}^{N_u} In\left(1-\hat{\xi}\frac{Y_j}{\beta}\right) + \left(\hat{\xi}-1\right)\sum_{j=1}^{N_u}\left(1-\hat{\xi}\frac{Y_j}{\beta}\right)^{-1} \quad (11)$$

$$N_u = \left(\hat{\xi} - 1\right) \sum_{j=1}^{N_u} \left(1 - \hat{\xi}\frac{Y_j}{\beta}\right)^{-1} \tag{12}$$

$$\left[1 + \left(\frac{1}{N_u}\right) \sum_{j=1}^{N_u} In\left(1 - \hat{\xi}\frac{Y_j}{\hat{\beta}}\right)\right] . \left[\left(\frac{1}{N_u}\right) \sum_{j=1}^{N_u} In\left(1 + \hat{\xi}\frac{Y_j}{\hat{\beta}}\right)^{-1}\right] = 1 \tag{13}$$

$$\hat{\xi} = -\left(\frac{1}{N_u}\right) \sum_{j=1}^{N_u} In\left(1 - \hat{\xi}\frac{Y_j}{\hat{\beta}}\right)^{-1} \tag{14}$$

The bivariate search for zeros of the gradient vector over $\Im$ can be replaced to a univariate search because the equation (14) Is closed- form representation of $\hat{\xi}$, given the ration $\hat{\xi}/\hat{\beta}$, and equation 14 depends on $\hat{\xi}/\hat{\beta}$. To compute the maximum likelihood estimate, we consider re-parametrization (β, ξ) to (θ, ξ) , where $\theta = \frac{\xi}{\beta} \left(\theta < \frac{1}{Y_{i(n)}}\right)$which is one to one in space $\Im$. We then substitute

$$\hat{\xi}_{MLE} = -\left(\frac{1}{N_u}\right) \sum_{j=1}^{N_u} In\left(1 - \hat{\theta}_{MLE} Y_j\right) \tag{15}$$

and $$\hat{\beta}_{MLE} = \frac{\hat{\xi}_{MLE}}{\hat{\theta}_{MLE}} \tag{16}$$

An explicit function of the data and θ, into the GPD log-likelihood for (ξ, β) so that log-likelihood depends on θ. This is called the profile log-likelihood of θ. The value of θ which corresponds to local maximum of this profile log-likelihood, is given by

$$L(\theta; Y_j) = -N_u - \sum_{j=1}^{N_u} In\left(1 - \theta Y_j\right) - nIn\left[\frac{1}{N_u} \sum_{j=1}^{N_u} In\left(1 - \theta Y_j\right)\right] \tag{17}$$

The first derivative of the profile log-likelihood for θ is given by

$$\frac{\partial L(\theta; Y_j)}{\partial \theta} = \frac{1}{\theta} \sum_{j=1}^{N_u} In\left(1 - \theta Y_j\right)^{-1} - \frac{N_u}{\theta}\left[\frac{1}{\sum_{j=1}^{N_u} In\left(1 - \theta Y_j\right)}\right] . \left[N_u - \sum_{j=1}^{N_u} In\left(1 - \theta Y_j\right)^{-1}\right] \tag{18}$$

It is important to emphasize that the local maximum of the profile log-likelihood corresponds to the local maximum of the GPD corresponds to the local maximum of the GPD log-likelihood.

Distribution of Exceedances

From the theorem by ((Pickands, J (1975), Balkema and de Haan (1974)), for a large class of underlying F the conditional functions F the conditional excess distribution function $F_u(y)$, for u large, is well approximated by
$F_u(y) \approx G_{\xi,\sigma}(y),\ u \to \infty,$

We consider modelling excess distribution; an approach for distribution of exceedances is the peak over threshold (POT) method. Considering an unknown distribution function F of a random variable X. We are interested in estimating the distribution function F_u of values of x above certain threshold u. The distribution F_u is called the conditional excess distribution function and is defined as :

$$F_u(y) = P(X - u \le y \mid X > u), \quad 0 \le y \le x_F - u \tag{19}$$

Where X is a random variable, u is a given threshold, $y = x - u$ are the right endpoint of F. We verify that F_u can be terms as F,

$$F_u(y) = \frac{F(u+y) - F(u)}{1 - F(u)} = \frac{F(x) - F(u)}{1 - F(u)} \tag{20}$$

$$0 \le X \le u$$

From GPD, the tail index ξ gives an indication of heaviness or lightness of the tail, the larger ξ the heavier the tail. Only distributions with tail parameter $\xi \ge 0$ are suitable to model financial returns.

Value-at-Risk

Value at Risk is a threshold loss value, such that the probability that the loss on a given portfolio over a given time horizon exceeds an estimated value.
Value at Risk is the capital sufficient to cover, in most instances, loss from a portfolio over a holding period of a fixed number of days.
The Value at Risk of a random variable X with continuous distribution function F that models losses or negative return on a certain financial instrument over a certain time horizon is given by

$$VaR_p = F^{-1}(1 - p) \tag{21}$$

Where VaR_p is the p^{th} quantile of the distribution F. Where F^{-1} is the so called quantile function, and is the inverse of the distribution function F.

Estimating Value at Risk

The VaR_p and Expected shortfall (ES_p) for the tail distribution of a GPD function can be defined as a function of GPD parameters. As stated by Manfred & Evis (2006), the conditional excess distribution in (19) was extended to derive the VAR and ES. Expected shortfall (ES) is defined as the excess of a loss that exceeds VaR_p.

Isolating $F(x)$ from (20).

$$F(x) = (1 - F(u))F_u(y) + F(u) \tag{22}$$

And replacing F_u by the GPD and $F(u)$ by the estimate $\left(n - N_u / n \right)$, where n is the total number of observations and N_u the number of observations above the threshold u, we obtain.

$$\hat{F}(x) = \frac{N_u}{n}(1 - (1 + \frac{\hat{\xi}}{\hat{\beta}}(x - u)^{-\frac{1}{\xi}} + \left(1 - \frac{N_u}{n}\right) \tag{23}$$

Simplifying we have:

$$\hat{F}(x) = 1 - \frac{N_u}{n}((1 + \frac{\hat{\xi}}{\hat{\beta}}(x - u)^{-\frac{1}{\hat{\xi}}} \tag{24}$$

Inverting (24) for a given probability

$$V\hat{a}R_p = u + \frac{\hat{\beta}}{\hat{\xi}}\left(\left(\frac{n}{N_u}p\right)^{\hat{\xi}} - 1\right) \tag{25}$$

Expected shortfall can be written as:

$$ES_p = V\hat{a}R_p + E(X - V\hat{a}R_p \mid X > V\hat{a}R_p)$$

Where the second term on the right is the expected value of the exceedances over VaR_p. The excess function for the GPD with parameter $\xi < 1$ is:

$$e(w) = E(X - w \mid X > w) = \frac{\sigma + \xi w}{1 - \xi}, \tag{26}$$

$$\sigma + \xi w > 0$$

The function gives average of the excesses of X over varying values of threshold w.

Similarly, for $z = VaR_p - u$ and X representing the excesses y over u we obtain

$$\hat{E}S_p = V\hat{a}R_p + \frac{\hat{\sigma} + \hat{\xi}(V\hat{a}R_p - u)}{1 - \hat{\xi}} = \frac{V\hat{a}R_p}{1 - \hat{\xi}} + \frac{\hat{\sigma} - \hat{\xi}}{1 - \hat{\xi}} \tag{27}$$

Table 1: Summary Statistics of Exchange Rates and Returns

	Exchange Rate				Returns			
	MEAN	**STDEV**	**SKEW**	**KURT**	**MEAN**	**STDEV**	**SKEW**	**KURT**
DAN/NGN	0.03728977	0.003258	-0.145018	22.92653	0.00009782	0.426735	0.022701	432.52
USD/NGN	0.00728542	0.0008311	0.2913582	3.411472	0.00007213	0.060664	-0.07056	1294.954
PDS/NGN	0.00433575	0.0005139	1.33247	6.14288	0.0043263	0.0005115	1.327568	6.165194
FRA/NGN	0.00741155	0.0016866	0.4882045	-1.40485	0.0001046	0.014545	-3.289164	149.894
EUR/NGN	0.005790	0.0011907	1.632834	3.217446	0.0000299	0.219131	0.0315421	1593.771
SDR/NGN	0.0044711	0.0016543	19.0968	387.685	0.0000848	0.148959	-0.110314	212.4753
RIY/NGN	0.02678711	0.00301277	0.799839	4.065309	0.0264303	0.000086	0.0222068	608.9605
YUA/NGN	0.03932956	0.00262732	-8.148127	101.8718	0.0003268	0.473721	-0.000495	355.9507
WAU/NGN	0.006688342	0.09935003	57.01523	3250.155	0.0000427	0.353158	-0.027712	511.5745

Data Source: CBN website: www.cenbank.org/ExchangeRateByCurrency

Plots of Exchange Rate of NGN against other Foreign Currencies

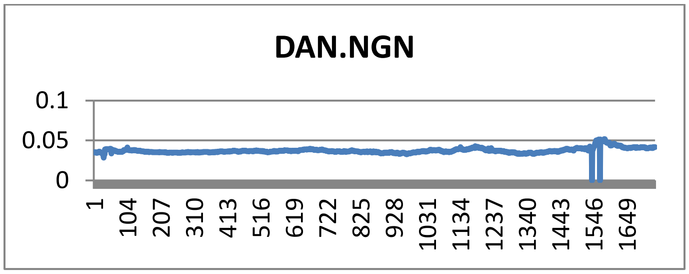

Fig 1 Line graph of DAN/NGN

The graph displays foreign exchange of Danish Krone against Naira from 3 July, 2008 to 9 April 2015. Where one (1) is the latest date and 1736 is the earliest date.

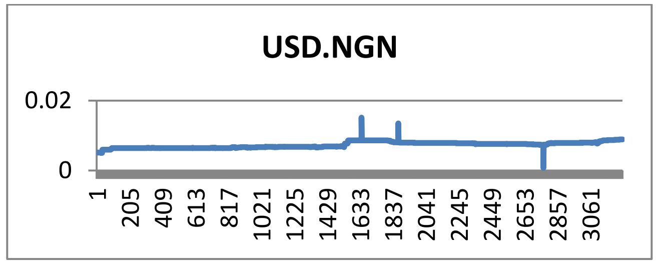

Fig 2 Line graph of USD/NGN

The graph displays foreign exchange of US Dollars against Naira from 12 October, 2001 to 9 April 2015. Where one (1) is the latest date and 3261 is the earliest date

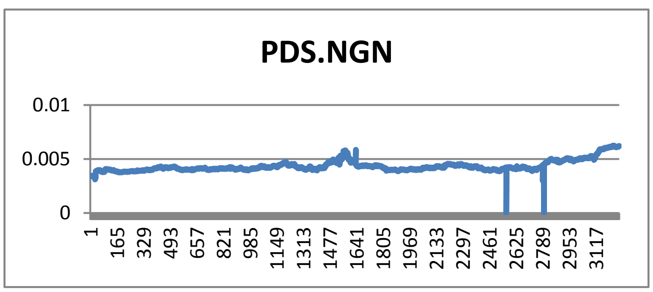

Fig 3 Line graph of PDS/NGN

The graph displays foreign exchange of Pounds Sterling against Naira from 12 October, 2001 to 9 April 2015. Where one (1) is the latest date and 3261 is the earliest date

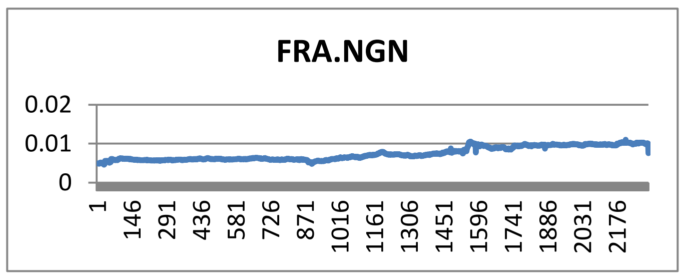

Fig 4 Line graph of FRA/NGN

The graph displays foreign exchange of FRANC against Naira from 21 July, 2005 to 9 April 2015. Where one (1) is the latest date and 2310 is the earliest date

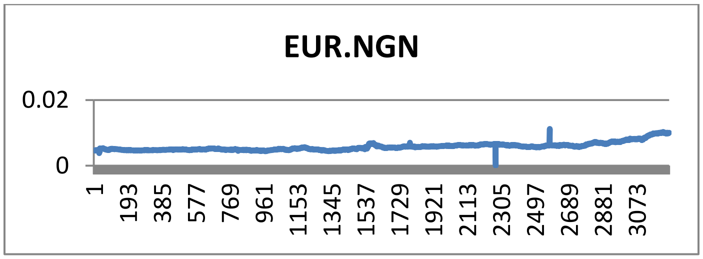

Fig 5 Line graph of EUR/NGN

The graph displays foreign exchange of EURO against Naira from 12 October, 2001 to 9 April 2015. Where one (1) is the latest date and 3257 is the earliest date.

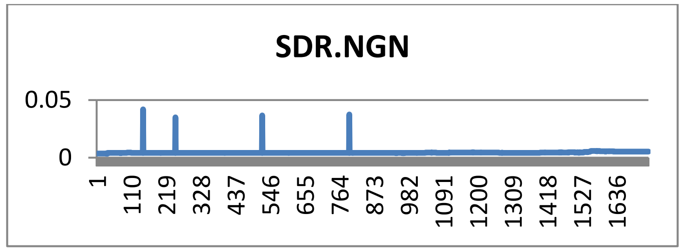

Fig 6 Line graph of SDR/NGN

The graph displays foreign exchange of SDR against Naira from 13 July, 2006 to 9 April 2015. Where one (1) is the latest date and 1736 is the earliest date.

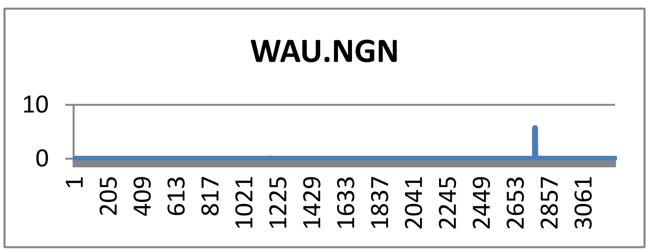

Fig 7 Line graph of WAU/NGN

The graph displays foreign exchange of WAUA against Naira from 12 October, 2008 to 9 April 2015. Where one (1) is the latest date and 3257 is the earliest date

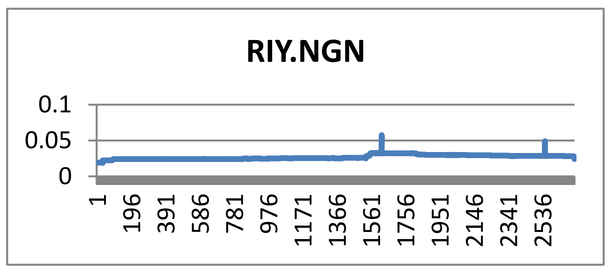

Fig 8 Line graph of RIY/NGN

The graph displays foreign exchange of RIY against Naira from 24 April, 2004 to 9 April 2015. Where one (1) is the latest date and 2714 is the earliest date.

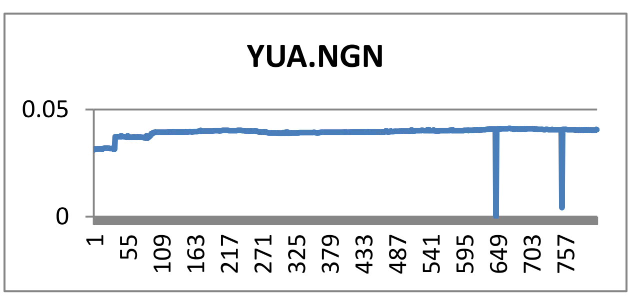

Fig 9 Line graph of YUA/NGN

The graph displays foreign exchange of YUAN against Naira from 23 December, 2011 to 9 April 2015. Where one (1) is the latest date and 809 is the earliest date

From Table 1, only two (2) out of nine (9) *exchange rate* are negatively skewed, but eight (8) exhibits fat tails with kurtosis greater than that of normal distribution while one (1) exhibit thin tail with kurtosis less than that of normal distribution. Four (4) out of nine (9) *returns* are negatively skewed, and all exhibits fat tails with kurtosis greater than that of normal distribution.

Table 2 Normality tests of Returns (H_0: Normal): significance at 1%

	Anderson Darling	Craver-Von Misses
DAN/NGN	636.821**	136.384**
USD/NGN	1211.90**	260.602**
PDS/NGN	206.275**	37.856**
FRA/NGN	234.3015**	42.1386**
EUR/NGN	1159.866**	237.3343**
SDR/NGN	617.0543**	131.756**
RIY/NGN	968.3148**	2070.33**
YUA/NGN	121.063**	65.4319**
WAU/NGN	1215.457**	261.427**
p-value	2.2×10^{-16}	7.37×10^{-16}

Table 3: The Result Of Parameters Estimation And Return Level (2015-2035)

	u	ξ	β	5 years (2020)	10 years (2025)	20 years (2035)
DAN/NGN	0.051	1.935024	0.0314536	0.05170247	0.05170405	0.05170419
95%	-	-	-	(0.0520992 0.05130573)	(0.05210143 0.0513066)	(0.05210162 0.05130675)
PDS/NGN	0.0061	-0.513406	8.610e-05	0.006237441	0.006247	0.0062453422
95%	-	-	-	(0.0062411 0.0062337)	(0.006253772 0.00624037	(0.00626816 0.006244027)
USD/NGN	0.010	1.467953	0.0253465	0.0109018	0.01318511	0.01420689
95%	-	-	-	(0.01086774 0.0109358)	(0.01313501, 0.0132352)	(0.01426707 0.014167)
FRA/NGN	0.0097	-0.1882	0.004655	0.01073781	0.081080795	0.01086579
95%	-	-	-	(0.01073976 0.01073586)	(0.01080393 0.01081193)	(0.01087209 0.0108595)

EUR/NGN 95%	0.010	0.752169 -	0.00365455	0.01027714 (0.0103816 0.0101727)	0.01045636 (0.01015594 0.01075678)	0.01069721 (0.01003158 0.01136285
SDR/NGN 95%	0.005 -	1.32127 -	0.0024523 -	0.01138652 (0.0113583 0.01141473)	0.01136361 (0.01356005 0.01371215)	0.0164112 (0.01625325 0.01656914)
WAU/NGN 95%	0.006 -	1.361658 -	0.0265433 -	0.01535296 (0.01536136 0.01540456)	0.01971988 0.01957108 0.01986867)	0.02560734 (0.02527683 0.02593785)
RIY/NGN 95%	0.022	0.987809	0.052724	0.0463574 (0.04635148 0.04636333)	0.44764806 (0.0476351 0.04766046)	0.04875847 (0.04873829 0.04877864)
YUA/NGN 95%	0.0001 -	0.73866 -	0.0006065 -	- -	- -	- -

Table 2 shows a statistical test for normality with Anderson Darling and Craver-Von Misses test approach. The data is found not to be normally distributed. Table 3 displays results of parameters estimation and return level from exchange loss return series of NGN based on GPD model methods in the next twenty years (2015—2035). Since we study log returns for exchange rate loss return series, the loss risk will increase gradually during (2015-2035).

In 2020, the loss returns for DAN/NGN is 0.51702247 % (0.0520992%, 0.05130573%) at the confidence level of 95%, for PDS/NGN is 0.00623% (0.0062411%, 0.0062337%), for USD/NGN is 0.0109018% (0.01086774%, 0.0109358%) and it follows in that order.

In 2035 DAN/NGN is 0.05170419% (0.05210162%, 0.05130675%) at the confidence level of 95%, for PDS/NGN is 0.0062453422% (0.00626816%, 0.006244027%), for PDS/NGN is 0.0062453422% (0.00626816%, 0.006244027%), for USD/NGN is 0.01420689% (0.01426707%, 0.014167%) and others is as follows in table 3.

Table 4: Results of VaR and ES calculation based on GPD model

	95%		99%	
	VaR	**ES**	**VaR**	**ES**
DAN/NGN	0.042474	0.0212362	0.0502248	0.0251124
USD/NGN	0.008615	0.0043075	0.008795	0.0043975
PDS/NGN	0.005436	0.002718	0.006138	0.003069
FRA/NGN	0.010032	0.005016	0.010297	0.005485
EUR/NGN	0.008159	0.0040795	0.01005	0.0050025
SDR/NGN	0.005376	0.002688	0.00579	0.002895
WAU/NGN	0.0063051	0.00315225	0.0071019	0.00355095
RIY/NGN	0.032283	0.0161415	0.032311	0.011555
YAU/NGN	0.045954	0.020477	0.041031	0.0205155

CONCLUSION

The final results of calculation of VaR and ES are shown in table 4 with probability of 95%, the loss risk of DAN/NGN, USD/NGN, PDS/NGN, FRA/NGN, EUR/NGN, SDR/NGN, WAU/NGN, RIY/NGN, YAU/NGN will be less than 0.042474, 0.008615, 0.005436, 0.010032, 0.008159, 0.005376, 0.0063051, 0.032283, 0.045954. It means that extreme value in tomorrow's loss for DAN/NGN, USD/NGN, PDS/NGN, FRA/NGN, EUR/NGN, SDR/NGN, WAU/NGN, RIY/NGN, YAU/NGN.

It means that the extreme value in tomorrow's loss for DAN/NGN will be 0.042474% at a significant level of 95%, for USD/NGN 0.008615%, for PDS/NGN 0.005436%, for FRA/NGN 0.010032%, for EUR/NGN 0.008159%, for SDR/NGN 0.005376%, for WAU/NGN 0.0063051%, for RIY/NGN 0.032283%, and for YAU/NGN 0.045954%.

If we invest $ 1 million in USD/NGN, we are 95% confident that our daily loss will not exceed $ 86.15 on one day trade. Then assuming the loss happens, the mean loss over a given period will not exceed 0.0212362, 0.0043075, 0.002718, 0.005016, 0.0040795, 0.002688, 0.00315225, 0.020477, and 0.0161415. It means that the average of DAN/USD loss will be 0.0212362% at a significant level of 95%, for USD/NGN 0.0043075%, for PDS/NGN 0.002718%, for FRA/NGN0.005016%, for EUR/NGN loss will be 0.0040795%, for SDR/NGN 0.002688 %, for WAU/NGN0.00315225%, for RIY/NGN 0.020477%, and for YAU/NGN0.0161415%. The same follows 99% significant level as displayed in the table above.

In addition, under the confidence level of 95% and 99%, the ξ estimation of USD/NGN is more than PDS/NGN also the VaR and ES of USD/NGN are greater than PDS/NGN. Also, the ξ estimation of DAN/NGN is less than EUR/NGN; also VaR and ES of EUR/NGN are less than DAN/NGN. These imply that parameter estimates are good predictor of VaR and ES.

RECOMMENDATIONS

From above results, the following are hereby recommended:

(i) Investors (private and public) should endeavor to engage experts so as to avoid possible financial risk on certain investments.

(ii) Risk measurement should not be solely based on Value at Risk, but Expected Shortfall should also be used, as VaR may be insufficient to measure tail risk.

(iii) Policy makers in financial institutions in Nigeria should be well educated on the need to take risk financial risk measurement and management as a priority.

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APPENDIX

1. Danish Krone Plots

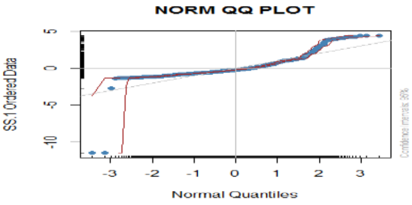

Fig 1 DAN/NGN Q-Q plot

The plot shows that the data is not normally distributed.

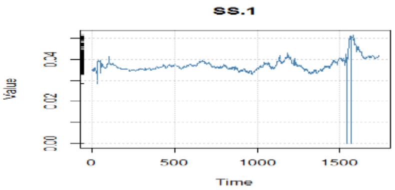

Fig 2 DAN/NGN time series plot

The plot shows that the data follows a time-series.

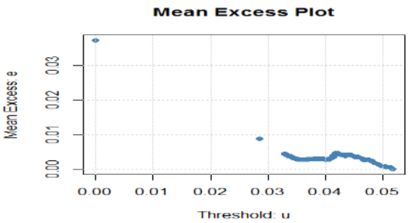

Fig 3 DAN/NGN mean excess plot

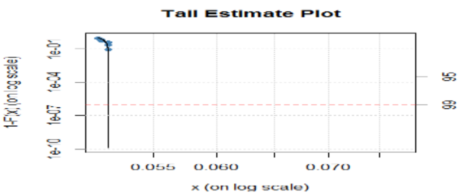

Fig 4 DAN/NGN Tail plot:

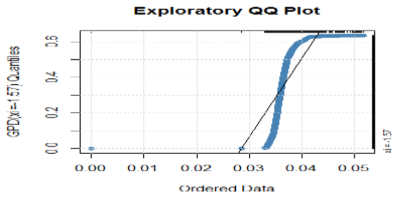

Fig 5 DAN/NGN Exploratory Q-Q plot:

The plot shows a smooth curve; therefore parametric data fit the data well.

2. US DOLLARS PLOTS

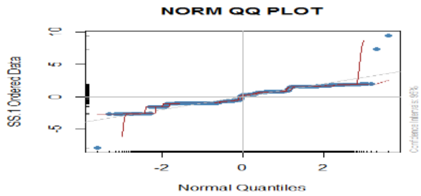

Fig 6 USD/NGN Q-Q plot:

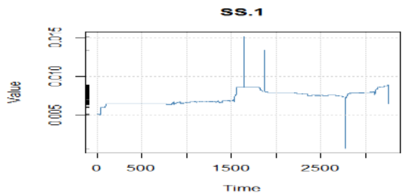

Fig 7 USD/NGN Time-series plot

The plot shows that the data follows time series

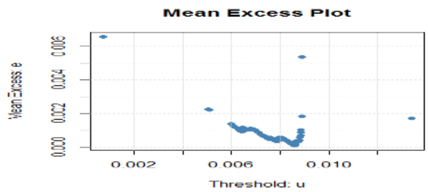

Fig 8 USD/NGN Mean excess plot

The plot shows a fat tail and threshold of 0.010 was chosen.

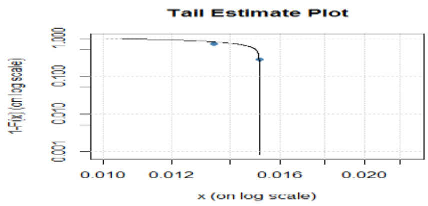

Fig 9 USD/NGN tail plot

The plot shows a smooth curve; therefore parametric model fits the data well.

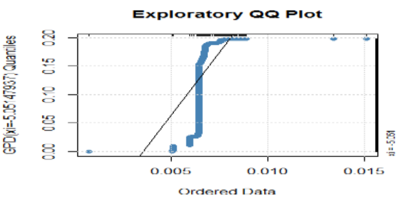

Fig 10 USD/NGN exploratory Q-Q plot

The plot shows a straight line therefore, parametric model fits the data well.

3. Pounds Sterling Plots

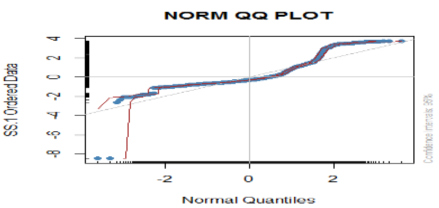

Fig 11 PDS/NGN Normal Q-Q plot

The plot shows that the data does not normally distributed

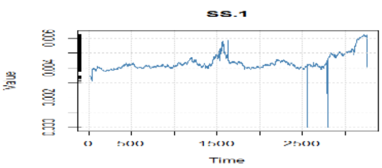

Fig 12 PDS/NGN time series plot

The plot shows that the data follows time series

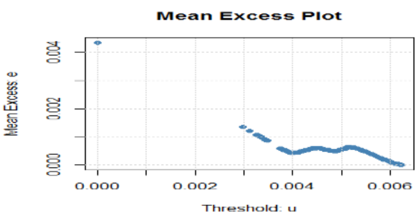

Fig 13 PDS/NGN mean excess plot

The plot a fat tailed distribution, a threshold of 0.061was chosen.

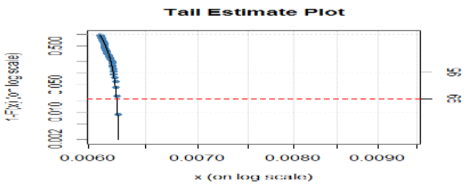

Fig 14 PDS/NGN tail plot

The plot shows a smooth curve, therefore parametric model fits the data well.

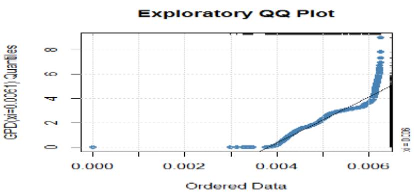

Fig 15 PDS/NGN exploratory Q-Q plot

The plot shows a straight line therefore, parametric model fits the data well.

4. SWISS FRANC PLOTS

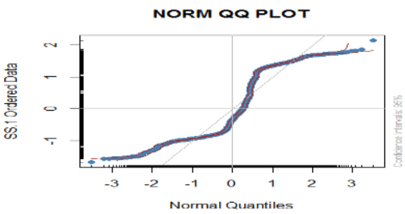

Fig16 FRA/NGN Normal Q-Q plot

The plot shows the data is not normally distributed.

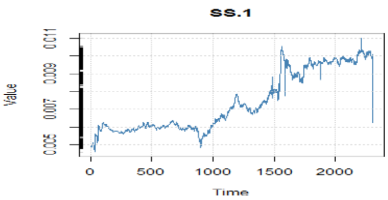

Fig 17 FRA/NGN time series plot

The plot shows the data follows a time series.

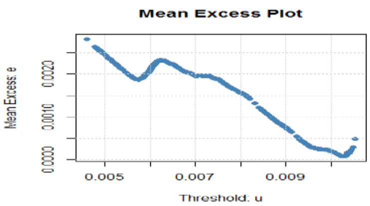

Fig 18 FRA/NGN mean excess plot

The plot shows a fat tailed distribution, and a threshold of 0.0097 is chosen.

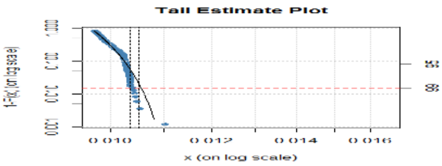

Fig 19 FRA/NGN tail plot

The plot shows a smooth curve; therefore parametric model fit the data well.

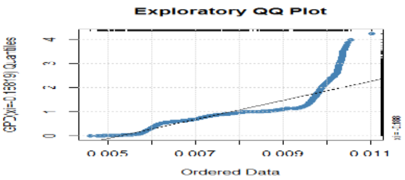

Fig 20 FRA/NGN exploratory Q-Q plot

The plot forms a straight line; therefore parametric model fits the data well.

5. EURO PLOTS

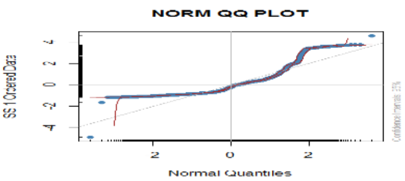

Fig 21 EUR/NGN Normal Q-Q plot

The plot shows the data is not normally distributed

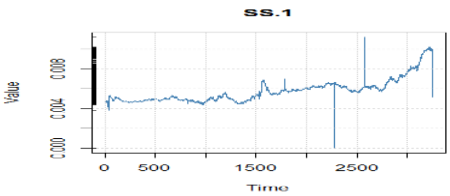

Fig 22 EUR/NGN time-series plot

The plot shows the data follows a time-series

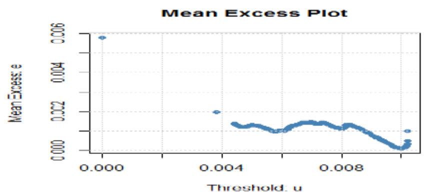

Fig 23 EUR/NGN mean excess plot

The plot a fat tailed distribution, threshold of 0.010 is chosen.

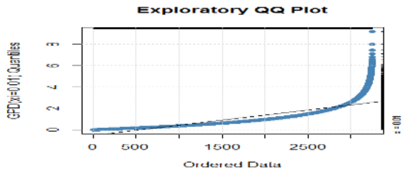

Fig 24 EUR/NGN exploratory Q-Q plot

The plot shows that parametric model fits the data well.

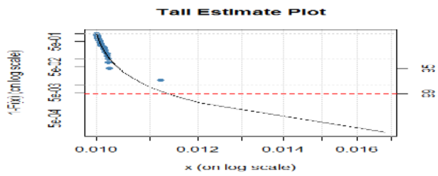

Fig 25 EUR/NGN tail plot

The plot shows a smooth curve, therefore parametric model fits the data well.

6. SDR (SPECIAL DRAWING RIGHT) PLOTS

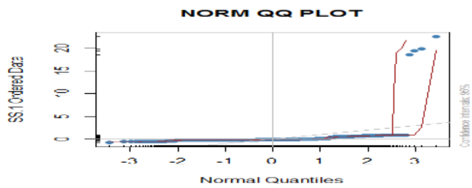

Fig 26 SDR/NGN Normal Q-Q plot

The plot shows the data is not normally distributed.

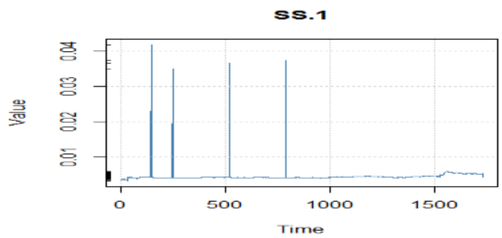

Fig 27 SDR/NGN time-series plot

The plot shows the data follows a time series.

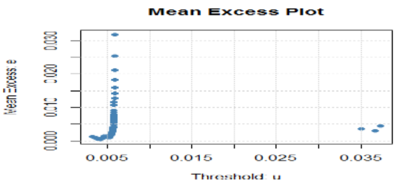

Fig 28 SDR/NGN mean excess plot

The plot shows a fat tailed distribution, a threshold 0.005 is chosen.

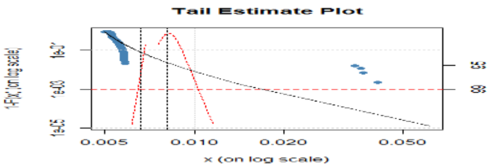

Fig 29 SDR/NGN tail plot

The plot shows a smooth curve, therefor parametric model fits the data well.

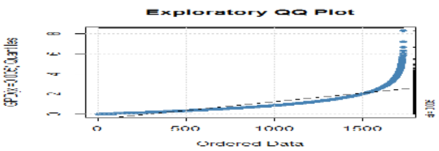

Fig 30 SDR/NGN exploratory Q-Q plot

The plot shows a straight graph; therefore parametric model fits the data well.

7. West African WAUA PLOTS

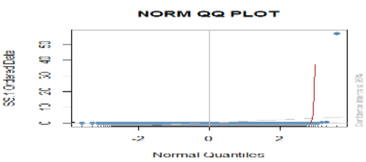

Fig 31 WAU/NGN Normal Q-Q plot

The plot shows that the data is not normally distributed.

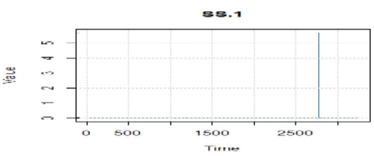

Fig 32 WAU/NGN time series plot

The plot shows that the data a time series but peaks at a particular point.

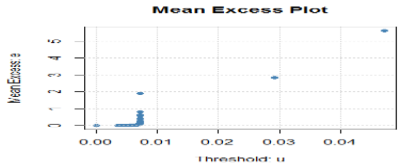

Fig 33 WAU/NGN mean excess plot

The plot shows a fat tailed distribution and threshold of 0.006 is chosen.

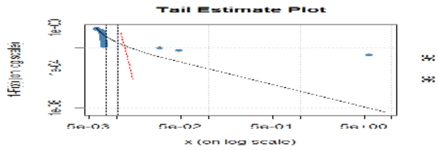

Fig 34 WAU/NGN tail plot

The plot shows a smooth curve therefore parametric model fits the data well.

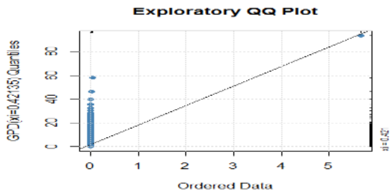

Fig 35 WAU/NGN exploratory Q-Q plot

The plot shows that parametric data fits the data well.

8. SAUDI RIYAL

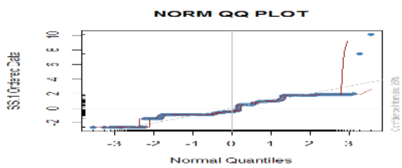

Fig 36 RIY/NGN Normal Q-Q plot

The plot shows that the data is not normally distributed.

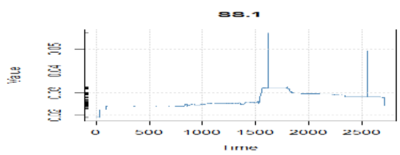

Fig 37 RIY/NGN Normal Q-Q plot

The plot shows that the data follows a time series

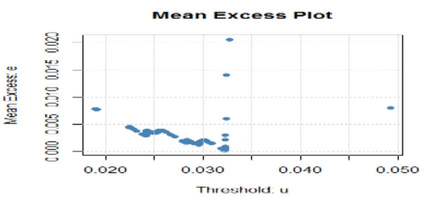

Fig 38 RIY/NGN mean excess plot

The plot shows a fat tailed distribution, and threshold of 0.0022 was chosen.

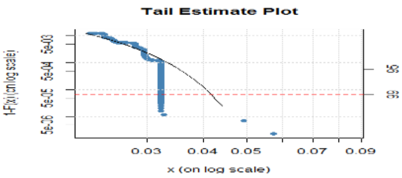

Fig 39 RIY/NGN tail plot.

The plot shows a smooth curve; therefore parametric model fits the data well.

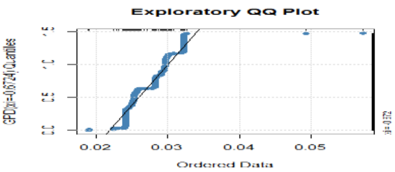

Fig 40 RIY/NGN exploratory plot

The plot shows a straight graph; therefore parametric model fits the data well.

9. CHINESE YUAN

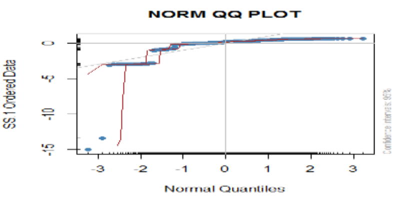

Fig 41 YUA/NGN normal Q-Q plot

The plot shows that the data is not normally distributed.

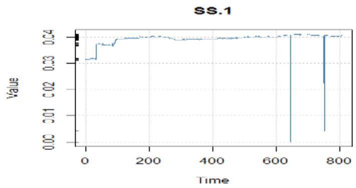

Fig 42 YUA/NGN time series plot

The plot shows that the data follows a normal distribution.

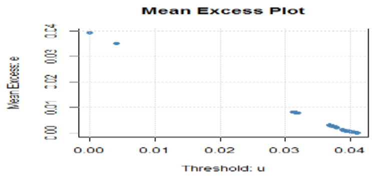

Fig 43 YUA/NGN mean excess plot

The plot shows a thin tailed distribution and threshold of 0.00001 is chosen.

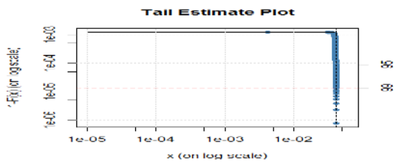

Fig 44 YUA/NGN tail plot

The plot shows a smooth curve, therefore, parametric model fits the data well.

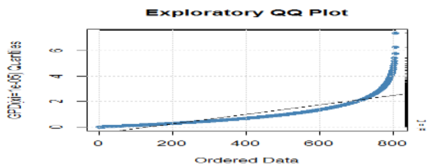

Fig 45 YUA/NGN exploratory Q-Q plot

The plot shows a straight line, therefore parametric model fits the data well.