Free Vibration Analysis of Euler-Bernoulli beam using differential transformation method

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Abstract
Using differential transformation method, the analysis of the free vibration of a prismatic Euler-Bernoulli beam under various supporting conditions is carried out in this study. Some numerical examples are presented to demonstrate the efficiency and reliability of the method. The results obtained are in good agreement with the results in available literature obtained using different approaches. These results show that the technique introduced here is accurate and easy to apply.
Key words: Differential transform, Vibration, Natural frequency, Mode shape

1. Introduction

Vibration problems for uniform Euler-Bernoulli beams have attracted considerable attention of a number of researchers. It is noted that several solution procedures have been employed. Chun (1972) studied the free vibration of a Bernoulli-Euler beam hinged at one end by a rotational spring with constant spring stiffness and with the other end free. Wang and Lin (1996) utilized the Fourier series to investigate the dynamic analysis of beams having arbitrary boundary conditions. Yeih *et al.* (1999) employed a dual multiple reciprocity method (MRM) to determine the natural frequencies and natural modes of an Euler-Bernoulli beam. Kim and Kim (2001) used Fourier series to obtain frequency expressions for uniform beams with generally restrained boundary conditions. Mauarizi *et al.* (1976) studied the problem of free vibration of a uniform beam hinged at one end by a rotational spring and subjected to the restraining action of a translational spring at the other end using exact expression of trigonometric and hyperbolic functions. The transverse vibration of uniform Euler-Bernoulli beams under linearly varying axial force was presented by Naguleswaran (2004). He solved for the mode shape using the method of Frobenius. In a study conducted by Lai *et al.* (2008), a technique based on the Adomian decomposition method was applied to solve free vibration problems of Euler-Bernoulli beams with various elastically supported conditions. Recently, Liu *et al.* (2009) used He's variational iteration method to calculate the natural frequencies and mode shapes of an Euler-Bernoulii beam under various supporting conditions.

In the present work, a technique called differential transformation is applied in analyzing vibration problem of Euler-Bernoulli beam under various supporting conditions. The concept of differential transformation was first introduced by Zhou (1986) to solve linear and nonlinear initial value problems in electric circuit analysis. This approach makes it possible to obtain highly accurate results or exact solutions for differential equations.

The organization of this paper is as follows: In the next section, the principle of differential transformation is reviewed. In section 3, the method is applied to analyze the free vibration problem. In the penultimate section, numerical results are given and compared with those obtained by Adomian decomposition and He's variational iteration methods. Conclusion is given in the last section.

2. The principle of Differential Transform Method

The differential transformation technique, which was first proposed by Zhou (1986), is one of the numerical methods for solving ordinary and partial differential equations with fast convergence rate and small calculation error. It uses a polynomial form that is sufficiently differentiable as the approximation to the exact solution. The technique is based on Taylor series expansion. The main difference between Taylor series method and the differential transformation method is that the former requires computations of higher order derivatives that are quite often formidable, while the latter involves iterative procedures instead. Applying the differential transformation technique in solving free vibration problems generally involves two transformations, namely, differential transformation and inverse differential transformation.

The basic definitions and operations of differential transformation are introduced as follows (available in the literature):

The differential transformation of the k^{th} derivative of function *Y(x)* is defined as follows:

$$\overline{Y}(k) = \frac{1}{k!}\left[\frac{d^k Y(x)}{dx^k}\right]_{x=x_0} \qquad (1)$$

and the differential inverse transformation of Y(k) is defined as follows:

$$Y(x) = \sum_{k=0}^{\infty} \overline{Y}(k)(x - x_0)^k \qquad (2)$$

Combining Eqs. (1) and (2), we have

$$Y(x) = \sum_{k=0}^{\infty} \frac{(x - x_0)^k}{k!}\left[\frac{d^k Y(x)}{dx^k}\right]_{x=x_0} \qquad (3)$$

which is the Taylor series of *Y(x)* at $x = x_0$. Eq. 3 implies that the concept of differential transformation is derived from the Taylor series expansion. In practical applications, the function Y(x) is expressed by a finite series and the inverse differential transform is written as

$$Y(x) = \sum_{k=0}^{n} \overline{Y}(k)(x - x_0)^k \qquad (4)$$

Eq. (4) implies that $\sum_{k=n+1}^{\infty} \overline{Y}(k)(x - x_0)^k$ is negligibly small. In this study, the convergence of the natural frequencies determines the value of *n*.

Theorems that are frequently used in the transformation of the equations of motion and the boundary conditions are listed in Tables I and II, respectively.

Table I Basic theorems of DTM for equations of motion

Original function	Transformed function
$w(x)=y(x)\pm z(x)$	$W(k)=Y(k)\pm Z(k)$
$w(x)=\lambda y(x)$	$W(k)=\lambda Y(k)$
$w(x)=\dfrac{d^n y(x)}{dx^n}$	$W(k)=(k+1)(k+2)...(k+n)Y(k+n)$
$w(x)=y(x)z(x)$	$W(k)=\sum_{l=0}^{k} Y(l)Z(k-l)$
$w(x)=x^m$	$W(k)=\delta(k-m)=\begin{cases}1, & k=m\\ 0, & k\neq m\end{cases}$

Table II DTM theorems for boundary conditions

x = 0		x = 1	
Original BC	transformed BC	Original BC	Transformed BC
$w(0)=0$	$W(0)=0$	$w(1)=0$	$\sum_{k=0}^{\infty} W(k)=0$
$\dfrac{dw(0)}{dx}=0$	$W(1)=0$	$\dfrac{dw(1)}{dx}=0$	$\sum_{k=0}^{\infty} kW(k)=0$
$\dfrac{d^2w(0)}{dx^2}=0$	W(2) = 0	$\dfrac{d^2w(1)}{dx^2}=0$	$\sum_{k=0}^{\infty} k(k-1)W(k)=0$
$\dfrac{d^3w(0)}{dx^3}=0$	W(3) = 0	$\dfrac{d^3w(1)}{dx^3}=0$	$\sum_{k=0}^{\infty} k(k-1)(k-2)W(k)=0$

3. Application of Differential Transform Method (DTM) to solve free vibration problem of uniform beam

The equation of motion for lateral vibrations of a uniform Euler-Bernoulli beam of finite length L ignoring shear deformation and rotary inertia effects can be written as ( )

$$EI\frac{\partial^4 y(x,t)}{\partial x^4}+\rho A\frac{\partial^2 y(x,t)}{\partial t^2}=0,\ 0<x<L \tag{5}$$

where $y(x,t)$ is the lateral deflection at distance x along the length of the beam and time t, EI is the flexural rigidity of the beam, ρ is the mass per unit volume of the beam, and A is the cross-sectional area of the beam.

The beam is subjected to the homogeneous boundary conditions described as follows:

at x = 0,

$$c_{r3}\frac{\partial^3 y(x,t)}{\partial x^3}+c_{r2}\frac{\partial^2 y(x,t)}{\partial x^2}+c_{r1}\frac{\partial y(x,t)}{\partial x}+c_{r0}y(x,t)=0,\quad r=1,\,2 \tag{6}$$

and
at x = L,

$$d_{r3}\frac{\partial^3 y(x,t)}{\partial x^3}+d_{r2}\frac{\partial^2 y(x,t)}{\partial x^2}+d_{r1}\frac{\partial y(x,t)}{\partial x}+d_{r0}y(x,t)=0,\quad r=1,\,2 \tag{7}$$

c_{r0}, c_{r1}, c_{r2}, c_{r3}, d_{r0}, d_{r1}, d_{r2}, d_{r3} are constants coming from different boundary conditions for Euler-Bernoulli beam, where $r=1$ and 2.

To obtain the natural frequencies and mode shapes, one can assume:

$$y(x,t)=Y(x)e^{i\omega t} \tag{8}$$

where $Y(x)$ is the modal deflection and ω denotes the natural frequency of the flexural beam
Substituting Eq. (8) into Eq. (5), we have

$$EI\frac{d^4Y(x)}{dx^4}-\rho A\omega^2 Y(x)=0,\quad 0<x<L \tag{9}$$

Eq. (9) simplifies in the dimensionless form as follows:

$$\frac{d^4Y(X)}{dX^4}-\lambda Y(X)=0,\quad 0<X<1 \tag{10}$$

upon introducing the following dimensionless quantities:

$$X=\frac{x}{L},\quad Y(X)=\frac{Y(x)}{L},\quad \lambda=\Omega^2=\frac{\rho A\omega^2L^4}{EI} \tag{11}$$

X is the dimensionless space co – ordinate, Y(X) the dimensionless deflection, $\Omega=\omega\sqrt{\frac{\rho AL^4}{EI}}$ is the dimensionless natural frequency of the beam.

The boundary conditions of Eqs. (6) and (7) also are given by the following dimensionless forms:
at X = 0,

$$\alpha_{r3}\frac{d^3Y(X)}{dX^3}+\alpha_{r2}\frac{d^2Y(X)}{dX^2}+\alpha_{r1}\frac{dY(X)}{dX}+\alpha_{r0}Y(X)=0,\quad r=1,2 \tag{12}$$

at X = 1,

$$\beta_{r3}\frac{d^3Y(X)}{dX^3}+\beta_{r2}\frac{d^2Y(X)}{dX^2}+\beta_{r1}\frac{dY(X)}{dX}+\beta_{r0}Y(X)=0,\quad r=1,2 \tag{13}$$

or

$$\sum_{j=0}^{3}\alpha_{rj}Y^{(j)}(0)=0,\qquad r=1,\,2 \tag{14}$$

$$\sum_{j=0}^{3}\beta_{rj}Y^{(j)}(1)=0,\qquad r=1,\,2 \tag{15}$$

where the 16 constants, α_{rj}, β_{rj} *(r=1,2; j=0,1,2,3)*, are dimensionless and $Y^{(j)}(X)$ denotes the jth-order derivative with respect to *X*, and sets $Y(X)=Y^{(0)}(X)$.

From the definition and properties of differential transformation given in Table 1, the differential transform of the equation of motion (10) is found as:

$$(k+1)(k+2)(k+3)(k+4)\overline{Y}(k+4)-\lambda\overline{Y}(k)=0 \tag{16}$$

Rearranging Eq. (16), one has the following recurrence relation

$$\overline{Y}(k+4)=\frac{\lambda\overline{Y}(k)}{(k+1)(k+2)(k+3)(k+4)} \tag{17}$$

Combining Eq. (17) and the appropriate boundary conditions, solutions to the free vibration problem can be obtained. Now, the solution procedure of the differential transformation method will be shown for clamped-free and hinged-free uniform beams with each of them having elastic spring restraints at the right end *x = L*.

Case1:

Let us first consider a clamped-free uniform beam whose right end *x = L* is connected by translational spring and rotational spring as shown in Fig. 2. Since the deflection and slope are zero at *X = 0*, then the boundary conditions at *x = 0* and *x = L* are given as

$$Y(0)=0,\ \left.\frac{dY(x)}{dx}\right|_{x=0}=0 \tag{18}$$

and

$$\left.\left[EI\frac{d^2Y(x)}{dx^2}+k_{RR}\frac{dY(x)}{dx}\right]\right|_{x=L}=0 \tag{19}$$

$$\left.\left[EI\frac{d^3Y(x)}{dx^3}-k_{TR}Y(x)\right]\right|_{x=L}=0 \tag{20}$$

where k_{RR} and k_{TR} are the rotational and translational spring constants respectively.

The dimensionless boundary conditions are:

$$\left.Y(X)\right|_{X=0}=0;\ \left.\frac{dY(X)}{dX}\right|_{X=0} \tag{21}$$

and

$$\left.\left[\frac{d^2Y(X)}{dX^2}+\beta_{RR}\frac{dY(X)}{dX}\right]\right|_{X=1}=0 \tag{22}$$

$$\left.\left[\frac{d^3Y(X)}{dX^3}-\beta_{TR}Y(X)\right]\right|_{X=1}=0 \tag{23}$$

where

$$\beta_{RR} = \frac{k_{RR}L}{EI}, \ \beta_{TR} = \frac{k_{TR}L^3}{EI} \tag{24}$$

Performing differential transform to the boundary conditions at X = 0, one has

$$\overline{Y}(0) = 0, \quad \overline{Y}(1) = 0 \tag{25}$$

At the right end, that is, at X = 1, we have

$$\sum_{k=0}^{n} k(k-1)\overline{Y}(k) + \beta_{RR}\sum_{k=0}^{n} k\overline{Y}(k) = 0 \tag{26}$$

$$\sum_{k=0}^{n} k(k-1)(k-2)\overline{Y}(k) - \beta_{TR}\sum_{k=0}^{n} \overline{Y}(k) = 0 \tag{27}$$

The values of $\overline{Y}(2)$ and $\overline{Y}(3)$ are set as unknowns such as

$$\overline{Y}(2) = s, \quad \overline{Y}(3) = z \tag{28}$$

Substituting Eqs. (25) and (28) into Eq. (17) setting k = 0, 1, 2, … , it can be seen that $\overline{Y}(k)$ is a linear function of $\overline{Y}(2)$ and $\overline{Y}(3)$.

By substituting $\overline{Y}(0)$ to $\overline{Y}(n)$ into Eqs. (26) and (27) and writing the resulting equations in matrix form, we obtain

$$\begin{bmatrix} \sum_{k=0}^{n}\lambda^k\left[\frac{1}{(4k)!}+\frac{\beta_{RR}}{(4k+1)!}\right] & \sum_{k=0}^{n}\lambda^k\left[\frac{1}{(4k+1)!}+\frac{\beta_{RR}}{(4k+2)!}\right] \\ -\frac{\beta_{TR}}{2}+\sum_{k=0}^{n}\lambda^k\left[\frac{1}{(4k-1)!}-\frac{\beta_{TR}}{(4k+2)!}\right] & \sum_{k=0}^{n}\lambda^k\left[\frac{1}{(4k)!}-\frac{\beta_{TR}}{(4k+3)!}\right] \end{bmatrix} \begin{bmatrix} 2!s \\ 3!z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{29}$$

Setting the determinant of the coefficient matrix of Eq. (29) to zero gives the characteristic (eigenvalue) equation of the structure, that is,

$$\begin{vmatrix} \sum_{k=0}^{n}\lambda^k\left[\frac{1}{(4k)!}+\frac{\beta_{RR}}{(4k+1)!}\right] & \sum_{k=0}^{n}\lambda^k\left[\frac{1}{(4k+1)!}+\frac{\beta_{RR}}{(4k+2)!}\right] \\ -\frac{\beta_{TR}}{2}+\sum_{k=0}^{n}\lambda^k\left[\frac{1}{(4k-1)!}-\frac{\beta_{TR}}{(4k+2)!}\right] & \sum_{k=0}^{n}\lambda^k\left[\frac{1}{(4k)!}-\frac{\beta_{TR}}{(4k+3)!}\right] \end{vmatrix} = 0 \tag{30}$$

Solving Eq. (30), we get $\lambda_i^{[n]}$ as the ith estimated eigenvalue which corresponds to n where n is decided by the relation

$$\left|\lambda_i^{[n]} - \lambda_i^{[n-1]}\right| \le \varepsilon \tag{31}$$

(31)

where ε is a preset small positive value.

The mode shapes are found via inverse differential transform at the corresponding natural frequencies, that is

$$\overline{Y}(X)=\sum_{k=0}^{n} X^{k} Y(k). \tag{32}$$

Therefore,

$$\overline{Y}(X)=\sum_{k=1}^{n} \lambda^{k-1}\left[\frac{2!}{(4k-2)!} X^{4k-2} s+\frac{3!}{(4k-1)!} X^{4k-1} z\right] \tag{33}$$

Case 2:

Similarly, let us consider the hinged-free beam, whose right end x = L is connected by translational spring and rotational spring as shown in Fig. 3. The deflection and bending moment are zero at X = 0, that is,

$$Y(x)\big|_{x=0}=0, \quad \left.\frac{d^{2}Y(x)}{dx^{2}}\right|_{x=0}=0. \tag{34}$$

having

$$Y(X)\big|_{X=0}=0, \quad \left.\frac{d^{2}Y(X)}{dX^{2}}\right|_{X=0}=0. \tag{35}$$

as their dimensionless form.

The boundary conditions at *x = L* are as specified by Eqs. (19) and (20).

Taking the differential transform of these boundary conditions, we have

At *X = 0*,

$$\overline{Y}(0)=0, \ \overline{Y}(2)=0 \tag{36}$$

At *X = 1*, we have equations same as Eqs. (26) and (27).

By following the same procedure as Case 1, one obtains the frequency equation as

$$\begin{vmatrix} \beta_{RR}+\sum_{k=1}^{n-1}\lambda^{k}\left[\frac{1}{(4k-1)!}+\frac{\beta_{RR}}{(4k)!}\right] & \sum_{k=1}^{n-1}\lambda^{k}\left[\frac{1}{(4k+1)!}+\frac{\beta_{RR}}{(4k+2)!}\right] \\ -\beta_{TR}+\sum_{k=1}^{n-1}\lambda^{k}\left[\frac{1}{(4k-2)!}-\frac{\beta_{TR}}{(4k+1)!}\right] & \sum_{k=1}^{n-1}\lambda^{k}\left[\frac{1}{(4k)!}-\frac{\beta_{TR}}{(4k+3)!}\right] \end{vmatrix}=0 \tag{37}$$

which is solved to yield the ith estimated eigenvalue corresponding to n.

Furthermore, the ith mode shape corresponding to the ith natural frequency is obtained by Eq. (32).

Here,

$$\overline{Y}(X)=\sum_{k=1}^{n} \lambda^{k-1}\left[\frac{X^{4k-3}}{(4k-3)!}\overline{Y}(1)+\frac{3!}{(4k-1)!} X^{4k-1}\overline{Y}(3)\right]$$

4. Numerical Examples

In order to explore the precision and efficiency of DTM in this study, the two examples of the cases in the previous section are discussed as follows. By following the procedures described in the two cases, the ith estimated eigennvalues, natural frequencies and consequently the mode shapes are calculated using the computer package Matlab and analyzed in this section. The results obtained here are compared to those yielded from the Adomian decomposition and He's variational iteration methods and very good agreement is found.

Example 1: Clamped-free beam with elastic spring restraints at *X = 1 (x = L)*
A clamped-free beam, whose right end is connected to a translational and a rotational spring, is considered (as discussed in the first case). The solution of Eq. (29) (taking real part for eigenvalue λ and assuming $\beta_{RR} = 1$, $\beta_{TR} = 1$ and $\varepsilon = 0.0001$) is displayed in Table 1. The result shows that the first estimated eigenvalue λ_1 is obtained in the fourth iteration and the second eigenvalue λ_2 is obtained in the seventh iteration. Other eigenvalues are obtained in a similar fashion. These results show that convergence of eigenvalues with DTM is the fastest when compared to the results of ADM and He's variational iteration method (see Ref. ()). Specifically, the first eigenvalue corresponding to n = 4 is λ_1=21.3305. Thus, the first dimensionless natural frequency denoted by $\overline{\omega}_1$ is obtained using

$$\overline{\omega}_1 = \sqrt{\lambda_1} = 4.6185$$

and the first natural frequency can be written as

$$\omega_1 = 4.6185\sqrt{\frac{\rho A L^4}{EI}}.$$

In Figs. 1 and 2, as the term number, n, increases, the natural frequencies $\varpi_1 - \varpi_6$ converge to 4.6185, 23.7831, 63.4827, 122.7415, 201.7308 and 300.4482 very quickly.
Substituting λ_1=21.3305 into Eq. (33) for n=4, we obtain the first mode shape function as

$$\begin{aligned}\overline{Y}_1(X) = {} & s(X^2 + 0.059251X^6 + 2.507662\times10^{-4}X^{10} + 2.226511\times10^{-7}X^{14} + 6.466855\times10^{-11}X^{18}) \\ & + z(X^3 + 0.02539X^7 + 6.839079\times10^{-5}X^{11} + 4.453021\times10^{-8}X^{15} + 1.021082\times10^{-11}X^{19})\end{aligned}$$

Following the same procedure as shown above, the other natural frequencies and mode shape functions can be obtained.

Table 1: Results of the ith estimated eigenvalues $\lambda_i^{[n]}$ for n = 18 approximate terms for Example 1

n	$\lambda_1^{[n]}$	$\lambda_2^{[n]}$	$\lambda_3^{[n]}$	$\lambda_4^{[n]}$	$\lambda_5^{[n]}$	$\lambda_6^{[n]}$
1	22.3736	231.5189				
2	21.3335	472.0628				
3	21.3305	562.1272				
4	21.3305	565.6123				
5	21.3305	565.6379	4049.8335	9339.5961		
6	21.3305	565.6380	4030.2659	13572.9074		
7	21.3305	565.6380	4030.0576	14998.4559		
8	21.3305	565.6380	4030.0563	15064.5559		
9	21.3305	565.6380	4030.0563	15065.4757	40917.1709	62059.3504
10	21.3305	565.6380	4030.0563	15065.4834	40698.4888	81698.4012
11	21.3305	565.6380	4030.0563	15065.4834	40695.3485	89737.8999
12	21.3305	565.6380	4030.0563	15065.4834	40695.3163	90259.8484
13	21.3305	565.6380	4030.0563	15065.4834	40695.3161	90269.0215
14	21.3305	565.6380	4030.0563	15065.4834	40695.3161	90269.1301
15	21.3305	565.6380	4030.0563	15065.4834	40695.3161	90269.1311
16	21.3305	565.6380	4030.0563	15065.4834	40695.3161	90269.1311
17	21.3305	565.6380	4030.0563	15065.4834	40695.3161	90269.1311
18	21.3305	565.6380	4030.0563	15065.4834	40695.3161	90269.1311

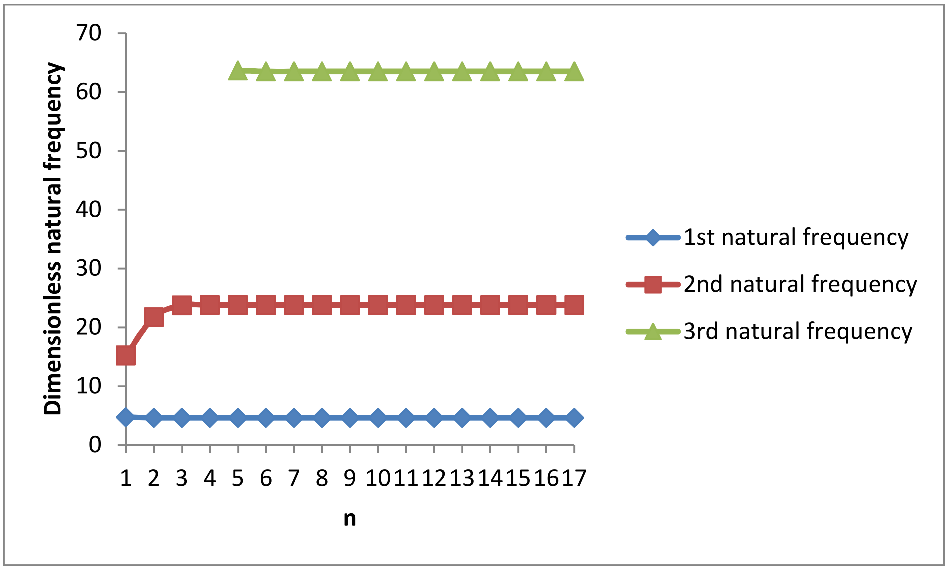

Fig. 1: Convergence of the first three dimensionless natural frequencies

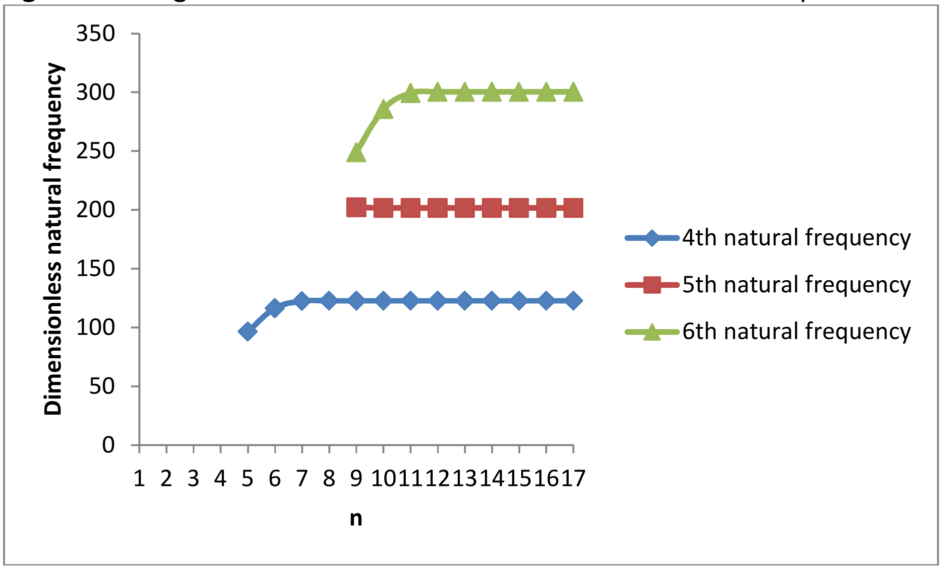

Fig. 2: Convergence of the fourth, fifth and sixth dimensionless natural frequencies

Example 2: Pinned-free beam with elastic spring restraints at *X = 1 (x = L)*

Here, a pinned-free beam as described in the second case, whose right end is connected to a translational spring and a rotational spring, is considered. The solution of Eq. (29) (taking real part for eigenvalue λ and assuming $\beta_{RR} = 0$, $\beta_{TR} = 25$ and $\varepsilon = 0.0001$) is provided in Table 2. The table indicates that the first estimated eigenvalue λ_1 is obtained in the fifth iteration and the second eigenvalue λ_2 is obtained in the sixth iteration. Other eigenvalues are obtained in the same way. In Figs. 3 and 4, as the term number, n, increases, the natural frequencies $\varpi_1 - \varpi_6$ converge to 6.8858, 18.8462, 51.0100, 104.7358, 178.5526 and 272.2156 very quickly.

Consequently, the natural frequencies and mode shapes for the pinned-free beam can be obtained based on these eignevalues.

For instance, the first mode shape function is given by

$$\begin{aligned}\overline{Y}_1(X) = {} & a(X + 0.039511X^5 + 6.195074\times10^{-3} X^9 + 1.711725\times10^{-5} X^{13} + 1.420858\times10^{-8} X^{17}) \\ & + b(X^3 + 0.056445X^7 + 3.379131\times10^{-4} X^{11} + 4.890643\times10^{-7} X^{15} + 2.492733\times10^{-10} X^{19})\end{aligned}$$

where

$$a = \overline{Y}(1);\ b = \overline{Y}(3).$$

Following the same procedure as shown above, the other natural frequencies and mode shape functions can be obtained.

Table 2: Results of the i[th] estimated eigenvalues $\lambda_i^{[n]}$ for n = 18 approximate terms for Example2

n	$\lambda_1^{[n]}$	$\lambda_2^{[n]}$	$\lambda_3^{[n]}$	$\lambda_4^{[n]}$	$\lambda_5^{[n]}$	$\lambda_6^{[n]}$
1	73.5340	92.2059				
2	47.5095	298.9577				
3	47.4140	353.6648				
4	47.4138	355.1710	3130.0422	3437.2527		
5	47.4138	355.1791	2605.7908	7663.6966		
6	47.4138	355.1791	2602.0470	10423.4768		
7	47.4138	355.1791	2602.0176	10955.5542		
8	47.4138	355.1791	2602.0174	10969.4425		
9	47.4138	355.1791	2602.0174	10969.5922	31928.3732	55908.0996
10	47.4138	355.1791	2602.0174	10969.5931	31881.6234	70289.5518
11	47.4138	355.1791	2602.0174	10969.5931	31881.0366	73969.0166
12	47.4138	355.1791	2602.0174	10969.5931	31881.0316	74099.4367
13	47.4138	355.1791	2602.0174	10969.5931	31881.0315	74101.3372
14	47.4138	355.1791	2602.0174	10969.5931	31881.0315	74101.3567
15	47.4138	355.1791	2602.0174	10969.5931	31881.0315	74101.3569
16	47.4138	355.1791	2602.0174	10969.5931	31881.0315	74101.3569
17	47.4138	355.1791	2602.0174	10969.5931	31881.0315	74101.3569

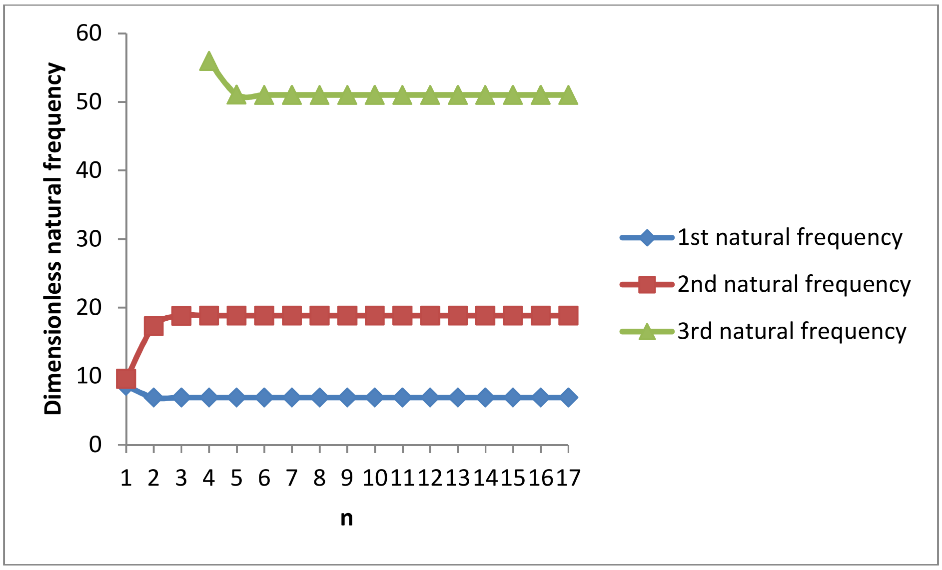

Fig. 3: Convergence of the first three dimensionless natural frequencies

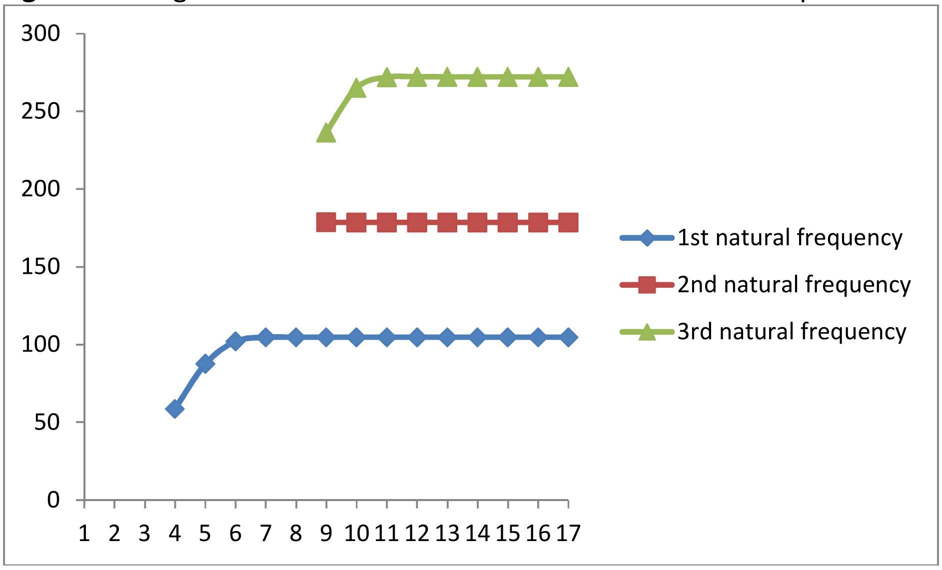

Fig. 4: Convergence of the fourth, fifth and sixth dimensionless natural frequencies.

5. Conclusion

The free vibration problem of uniform beam with various elastically restrained end conditions is studied in this paper. Differential transformation technique is used to solve the pertinent initial-boundary value problem. Natural frequencies are obtained and compared with those predicted by Adomian decomposition and He's variational iteration methods. Excellent agreement is found. It was observed that the precision of natural frequencies becomes higher with increasing value of *n.*

As a matter of fact, this work has demonstrated that the differential transformation method has high precision and computational efficiency is vibration problem of beams.

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