Bidirectional Synchronization of Two Identical Jerk Oscillators with Memristor

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ABSTRACT

In this paper, the case of introducing memristor as a coupling component when synchronizing two identical simple chaotic oscillator (3-D Jerk equations) were observed. Also, the numerical simulation of the phase portraits are in good agreement with the MultiSIM and experimental simulations. Due to the complex chaotic dynamics of this oscillator, the realization of the electronic circuit involving two identical Jerk oscillator with memristor as coupling component were synchronized via bidirectional coupling and further applied to secure communication.

Keywords- complex chaotic dynamics, memristor, synchronized, bidirectional coupling, secure communication.

1. INTRODUCTION

The study of 3-D jerk equations has attracted lots of interest in the past decades. Some of the most elegant circuits were third-order semistate equations of the form $\dddot{x} = J(\ddot{x}, \dot{x}, x)$ whose solutions are chaotic. The nonlinear function is called Jerk, which serves as acceleration in a mechanical system [1-7]. Past Jerk circuits have the nonlinearity in the x term and no simpler case can exist, the quadratic Jerk function with $J = -A\ddot{x} - x \mp \dot{x}^2$ as reported by J.C. Sprott [8] and Z. Fu et. al.[9]. Recently, dynamics in a chaotic Jerk circuit have been reported by Munmuangsaenet. al. [10] with $J = -\ddot{x} - x - f(\dot{x})$ for a wide variety of nonlinear function $f(\dot{x}) = \alpha^2 \exp(\dot{x}/\alpha)$ with $\alpha < 0.27$,except for many mostly periodic windows, where $f(\dot{x})$ can be implemented using ideal diode. Jerk system has useful application in many field of science such as mathematical biology, life science, engineering, and physics.

Chaos synchronization as an important area in nonlinear science due to its widely applications in cryptography [11] and secure communication [12-13].Pecora and Carroll [14] first demonstrated synchronization of two coupled subsystem via unidirectional with perspective of observing new chaotic dynamics. Therefore, the synchronization of 3-D jerk systems just recently attracts increasing attention due to its potential application to secure communication [15]. VinodPatidar and K. k. Sud investigated synchronization of two identical Jerk system using Pecora – Carrol (PC), Feedback (FB) and Active Passive decomposition (APD) technique [16], while A. Sambas et. al. [17-18] also reports the Jerk synchronization via computer simulation and its application to secure communication. J.C. Sprott [19] proposed a new chaotic Jerk circuit in which a particular elegant circuit with a single diode as nonlinear component were implemented.

Recently, the discovery of nanometer-scale electric switch "Memristor" which can remember past information after its power its turned off has generated a great attention in the implementation of nonlinear function and synchronization of complex dynamical systems [20-26]. In this work, we are focusing on synchronization of two oscillators with the introduction of memristor as the coupling component. Due to its complex nonlinearity, we eventually synchronized the two Jerk oscillators via bidirectional memristive coupling and further applied to secure communication.

2. JERK CIRCUIT

2.1 Numerical Simulations

A new chaotic Jerk three dimensional equation reported by J.C. Sprott with nonlinear system can be described by the following system of ordinary differential equations:

$$\dddot{x} + A\ddot{x} + x + 10^{-9}[\exp(y/0.026) - 1] \qquad (1)$$

where A is the only bifurcation and route to chaos parameter. This equation has only one nonlinear term $f(y) = I_o R[\exp(y/\alpha) - 1]$, where $\alpha = 0.026V$, $I_o = 10^{-12}A$, R as a factor due to the fact that time is measured in units of RC and $f(y)$ have units of volts.

In this work, we introduced second damping parameter B into equation (1) which is greater than 1 as shown in equation (2) below:

$$\dddot{x} + A\ddot{x} + Bx + 10^{-9}[\exp(y/0.026) - 1] \qquad (2)$$

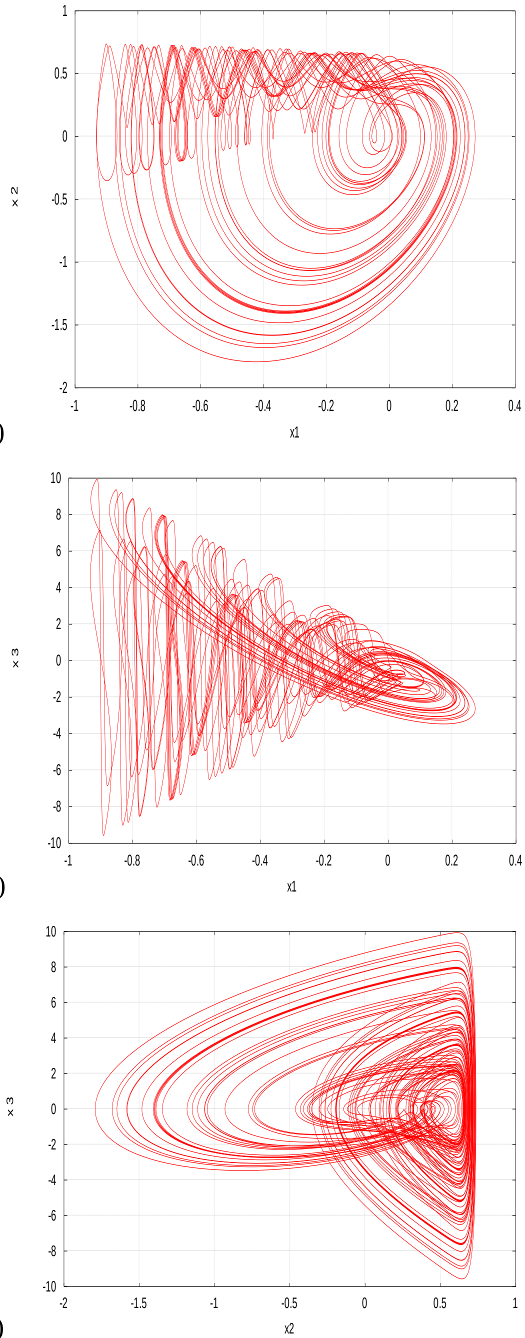
Equation (2) can be resolved into three first order differential form as shown in equation (3) as follows:

$\dot{x} = y$

$\dot{y} = z$ (3)

$\dot{z} = -Az - Bx - 10^{-9}[\exp(y/0.026) - 1]$

With the same initial conditions $(0, 0.4, 0)$ from numerical simulation, Fig.1 shows the results of the phase space plots of y vs x, z vs x and z vs y respectively. In figure 1 with (a) $A = 2$ and $B = 40$, complex dynamical behaviours were observed which can eventually be compared with the electronic simulations.



(a) (b) (c)

Figure 1: Chaotic nature of the phase space of 3-D jerk system for y vs x, z vs x and z vs y respectively $A = 2$ and $B = 40$.

2.2 Jerk and Memristor Simulations Using MultiSIM 12.0

The analog implementation of a new 3-D Jerk circuit (3), is designed below with operational amplifier ($AD711KN$), ideal diode ($D1N4148$), resistors ($R_1 - R_6$), capacitors ($C_1 - C_2$) and power source. The relationship between the variable resistor and fixed resistors along with the damping parameter A fixing B as used in the circuit below as:

$$R_A = \frac{R}{A}$$
$$R_B = \frac{R}{B} \qquad (4)$$

The numerical results in Fig. 1 with the electronic dynamics observed in Fig 3 and 4 above, a close agreement in dynamics indicates chaotic results can be achieved in electronic platform if carefully chosen correct damping parameters, where $R_6 = R_A$ and $R_4 = R_B$.

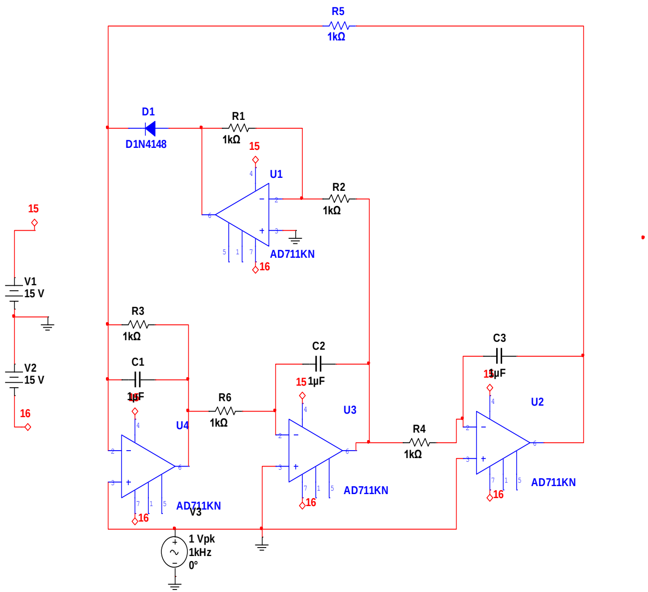
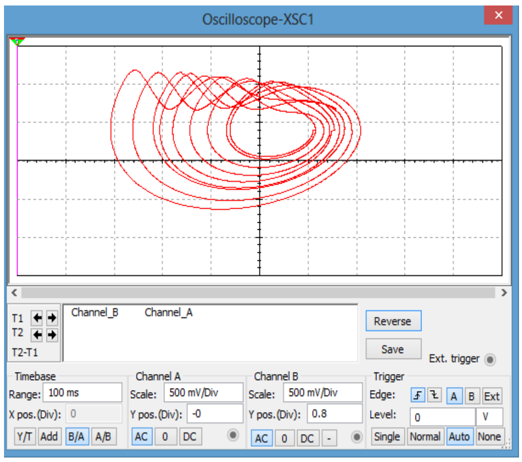


Figure 2: Analog Implementation of Jerk Circuit



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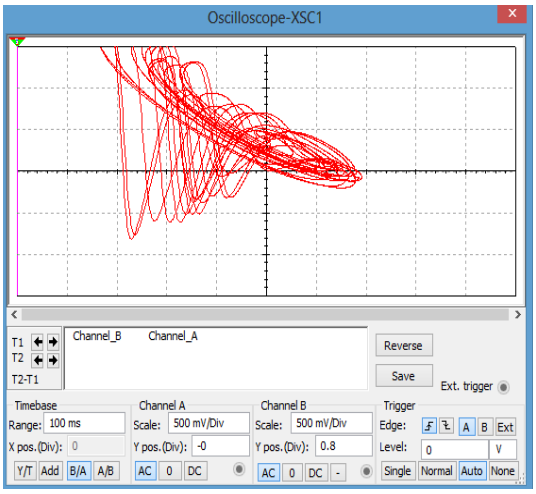

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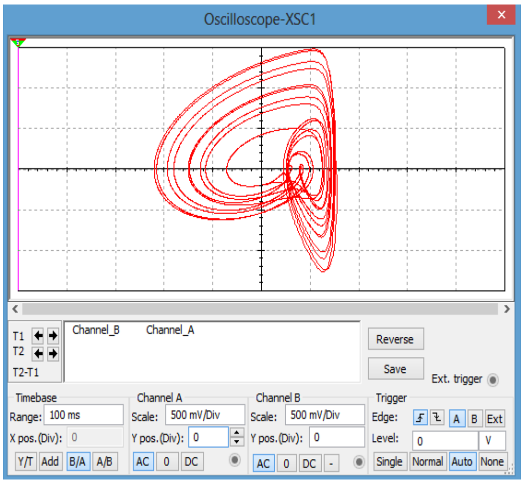

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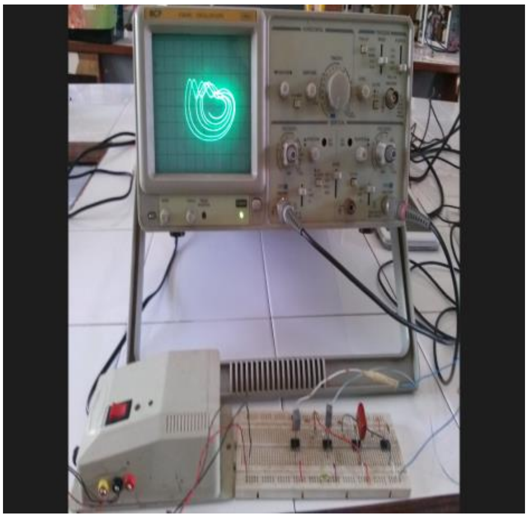
Figure 3: Various projections of phase portraits and time series for (a) y vs x, (b) z vs x and (c) z vs y respectively.



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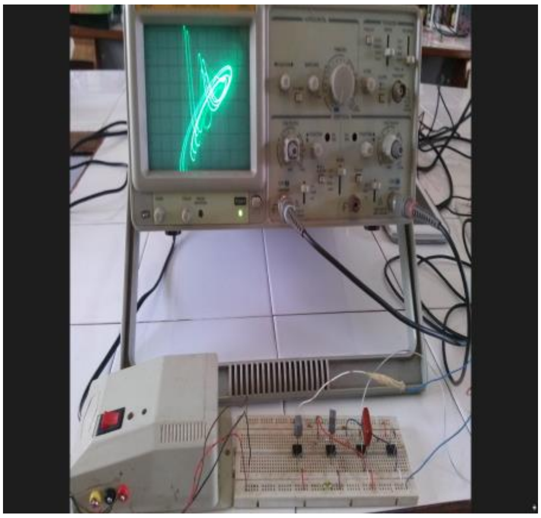


(b)

(c)

Figure 4: Experimental set-up of 3-D jerk circuit and various projections of phase portraits for (a) y vs x, (b) z vs x and (c) y vs z respectively.

3. BIDIRECTIONAL SYNCHRONIZATION OF TWO IDENTICAL JERK CIRCUIT

Synchronization between the master and slave systems is said to be achieved if $\|e(t)\| \to 0$ as $t \to \infty$. A mutual coupling of identical 3-D jerk circuit Fig. 6 (a) and 7 (a) is designed by setting one circuit as slave and the other as master system.

Chosen a cubic nonlinearity function as the coupling component for the $q-\phi$ characteristic as:

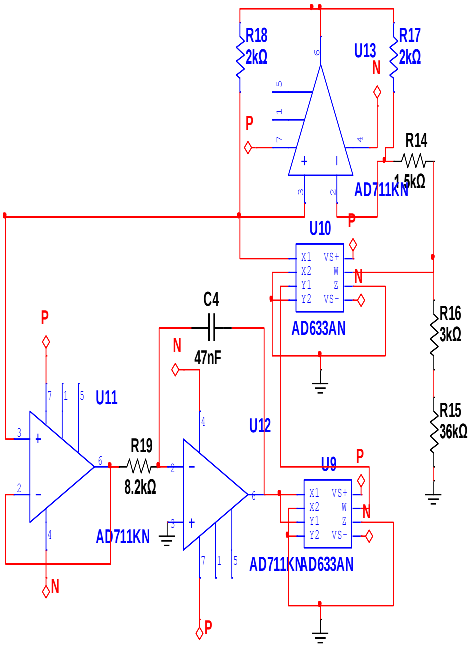
$$q(\phi) = \alpha\phi + \beta\phi^3 \tag{5}$$

Also, the memductive function $W(\phi)$ is given by

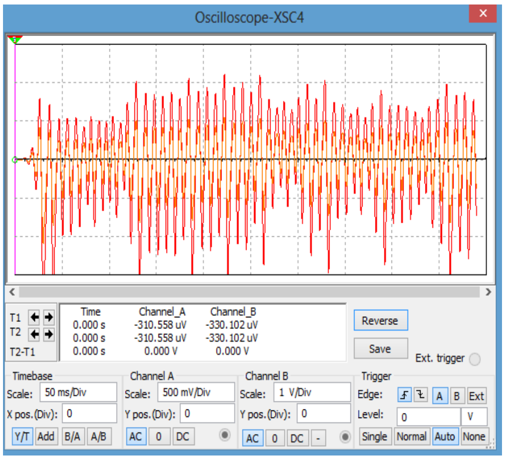
$$W(\phi) = \frac{dq}{d\phi} = \alpha + 3\beta\phi^2 \tag{6}$$

where $\alpha = -0.667\times10^{-3}$ and $\beta = 0.029\times10^{-3}$. The formuler relating α and β to the memristor circuit parameters [23-27] are:

$$\alpha = \frac{-1}{R_{14}}$$
$$\beta = \frac{1}{3}\left(\frac{R_{15}+R_{16}}{R_{14}.R_{15}.R_{16}}\right) \quad (7)$$



(a)



(b)

Figure 5: The realization of (a) Memristor as a cubic nonlinearity component and,(b) its corresponding time series.

Chaotic synchronization occurs for a coupling strength $R_c = R_{14} \leq 18k\Omega$ as shown in Fig. 6 (i) below, but for coupling strength $R_c = R_{14} > 1k\Omega$, the synchronization cannot occur as shown in Fig. 6 (ii).

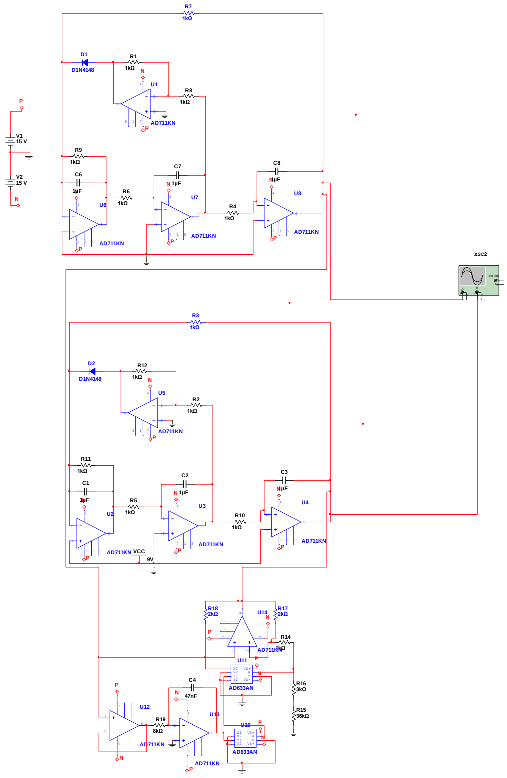
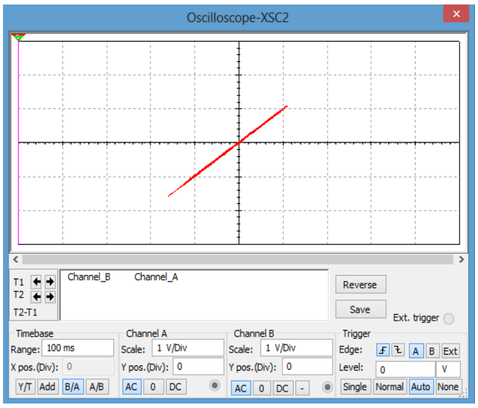
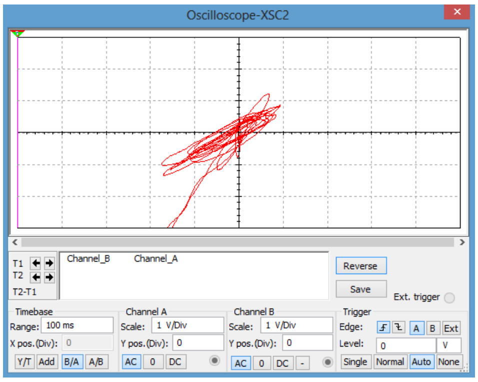


Figure 6 (a): Analog implementation of two mutual coupled 3-D Jerk circuits.



(a)



(b)

Figure 6 (b): Master-Slave phase space of y_2 vs y_1 for (i) complete synchronization at $R_c = 18k\Omega$ and (ii) non - synchronization at $R_c = 1k\Omega$.

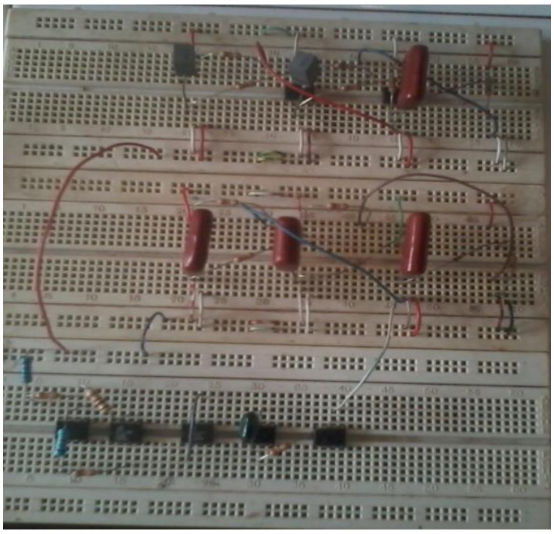


Figure 7 (a): Laboratory Implementation of two Jerk Circuits with Memristor

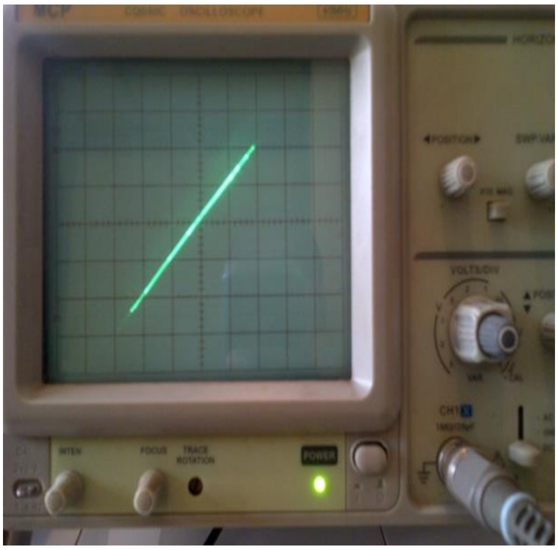


Figure 7 (b): Oscilloscope showing the Synchronization Portrait of the two Coupled Jerk Circuits with Memristor.

4. APPLICATION TO SECURE COMMUNI- CATION

In security set-up, synchronization of the chaos-based systems play a major role when securing communication informations. Chaos synchronization which involved two identical systems, "master or transmitter" oscillator and the other as "slave or receiver" oscillator. In this section, electronic implementation via bidirectional coupling of two 3-D Jerk systems were applied to secure communication after the systems has been synchronized (8), which is a key issue in tracking of information. The sinusoidal wave signal from retriever of amplitude 1V and frequency 10 KHz is added to the generated chaotic signal x and the $s(t) = x + i(t)$ is fed into the receiver. Communication circuit in Fig. 8 shows three stages involves in chaos masking, the transmitter, receiver and retriever, with chaotic masking occuring at $200\,Hz$ as shown in Figure 9.

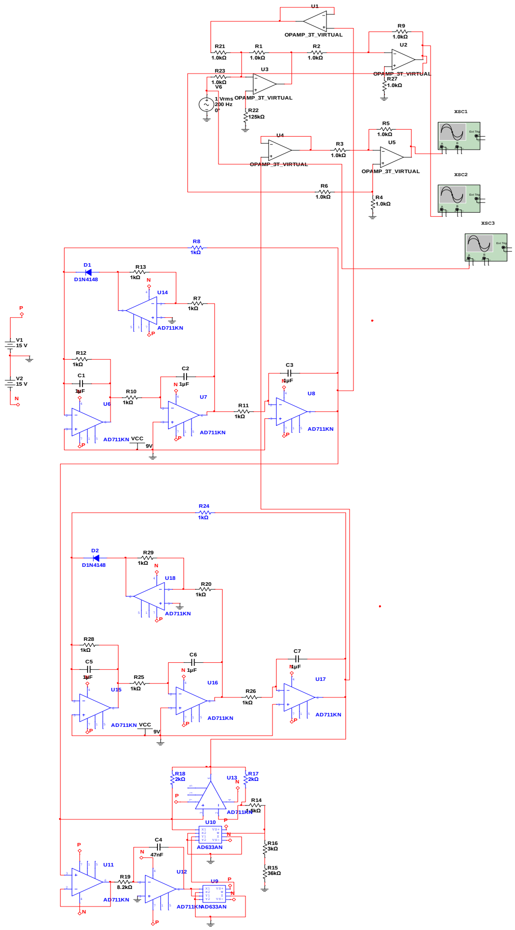
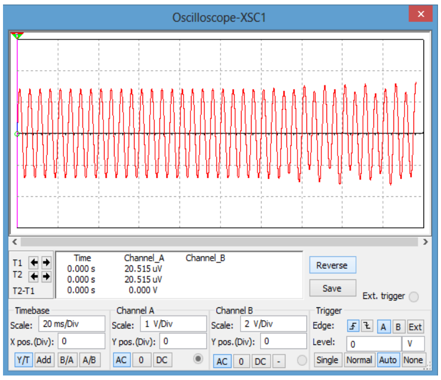
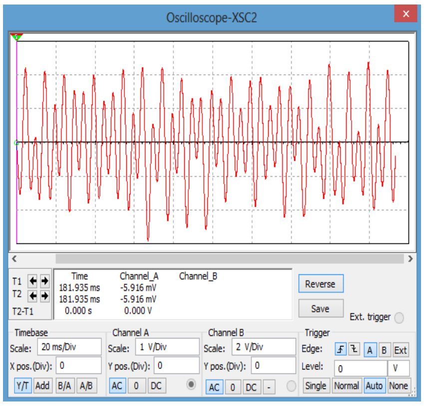


Figure 8: Analog Implementation of Synchronized Identical Jerk Circuits



(a)



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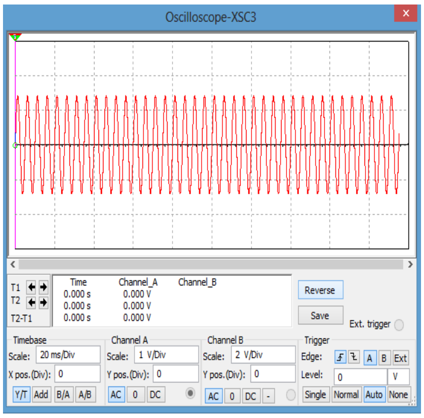

(c)

Figure 9: Results of Jerk masking communication electronic circuits at $200\,Hz$ and amplitude 1V:where, (a) Information signal, (b) Chaotic masking and (c) Retreived signal.

5. CONCLUSION

In this work, the complex dynamical behaviour from a new 3-D Jerk system reported by J.C. Sprott for three phase space has been observed numerically with the introduction of second damping parameter.We also compared these results which is in good agreement with the electronic simulation results. Due to succesfull implementation of this system, we therefore synchronized the two identical circuits via bidirectional coupling at $R_c = 1m\Omega$, and further extend it to secure communication at $f = 200\,Hz$, which demonstrate the effectiveness of the proposed scheme.

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