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Application of Kumaraswamy Inverse Exponential Distribution to Real Lifetime Data

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ABSTRACT

In this research, the Kumaraswamy Inverse Exponential distribution being a generalization of the Inverse Exponential distribution was applied to six real lifetime datasets. The idea is to assess its flexibility and superiority over its sub-models. Some other properties of the Kumaraswamy Inverse Exponential distribution were investigated in minute details. It was demonstrated and confirmed that the Kumaraswamy Inverse Exponential distribution performed better than the competing probability models except for data sets with variances far above the means. The performance was judged based on the log-likelihood and Akaike Information Criteria (AIC) values posed by the distributions.

Keywords: Analysis, Generalized models, Kumaraswamy Inverse Exponential distribution, Lifetime.

Mathematics Subject Classification: 46N30, 60E05, 62E10

Computing Classification System : G.3

1. INTRODUCTION

The Kumaraswamy generalized family of distributions was introduced by Cordeiro and de Castro [1] as an alternative to the Beta generalized family of distributions due to Eugene et al., [2] because of its mild algebraic properties.

The cumulative density function (cdf) and the probability density function (pdf) of the Kumaraswamy Generalized family of distributions are given by;

$$F(x) = 1 - \left\{1 - G(x)^{a}\right\}^{b} \quad (1)$$

and ;

$$f(x) = abg(x)G(x)^{a-1} 1 - \left\{1 - G(x)^{a}\right\}^{b-1} \quad (2)$$

respectively.

For $x > 0, a > 0, b > 0.$

where;

a and b are additional shape parameters

The rich ideas contained in the works of [1, 2] have led to the development of generalized models like the Kumaraswamy Normal distribution, Kumarawamy Weibull distribution, Kumaraswamy Gamma distribution, Kumaraswamy Gumbel distribution and Kumaraswamy Inverse Gaussian distribution; Cordeiro and de Castro [1], Kumaraswamy GP distribution; Nadarajah and Eljabri [3], Kumaraswamy Inverse Weibull distribution; Shahbaz et al., [4], Kumaraswamy Inverse Exponential distribution; Oguntunde et al., [5], Kumaraswamy Exponentiated Lomax distribution; El-Batal and Kareem [6], Kumaraswamy-Transmuted Exponential Modified Weibull distribution; Al-Babtain et al., [7], Kumaraswamy Linear Exponential distribution; Merovci and Elbatal [8], Kumaraswamy Power distribution; Oguntunde et al., [9], Kumaraswamy Transmuted Modified Weibull distribution; Mansour et al., [10] and many more.

Excerpt from these works demonstrated their applications in several fields of science including biology, reliability engineering and medicine but the application of the Kumaraswamy Inverse Exponential (KIE) distribution using real life data sets has not been considered. This article is therefore aimed at demonstrating the usefulness of the KIE distribution with respect to lifetime data and to assess its superiority over its sub-models. The R-code for the analysis can be made available on request.

The rest of this article is structured as follows, the KIE distribution is described in Section 2, real life application is provided in Section 3, followed by a concluding remark.

2. MATERIAL AND METHODS

2.1. The Kumaraswamy Inverse Exponential (KIE) distribution: Existing and more results

In this section, the KIE distribution is defined (as available in [5]); its further statistical properties like the quantile function and distribution of order statistics are also derived.

Let X denotes a non-negative continuous random variable, the cdf and pdf of the KIE distribution are given by;

$$F(x) = 1 - \left[1 - \left\{\exp\left(-\frac{\theta}{x}\right)\right\}^{a}\right]^{b} \quad ; \quad x > 0, a > 0, b > 0, \theta > 0 \qquad (3)$$

And;

$$f(x) = ab\frac{\theta}{x^{2}}\left\{\exp\left(-\frac{\theta}{x}\right)\right\}^{a}\left[1 - \left\{\exp\left(-\frac{\theta}{x}\right)\right\}^{a}\right]^{b-1} \quad ; \; x > 0, a > 0, b > 0, \theta > 0 \qquad (4)$$

respectively.

Where ;

a and b are shape parameters whose roles are to vary tail weight

θ is a scale parameter

For brevity purpose, some possible plots for the pdf of the KIE distribution at various parameter values are shown in Figures 1 to 4.

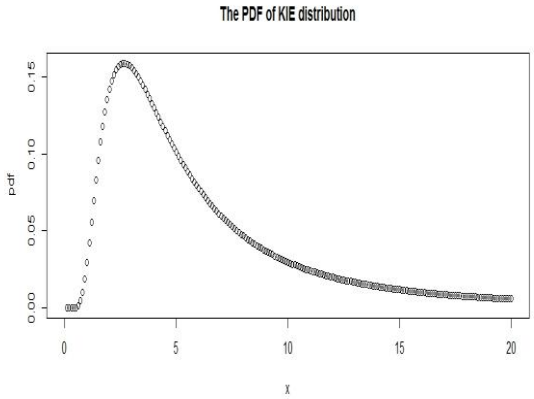

Fig. 1: Plot for PDF of KIE distribution at

$$a = 2, b = 2, \theta = 3$$

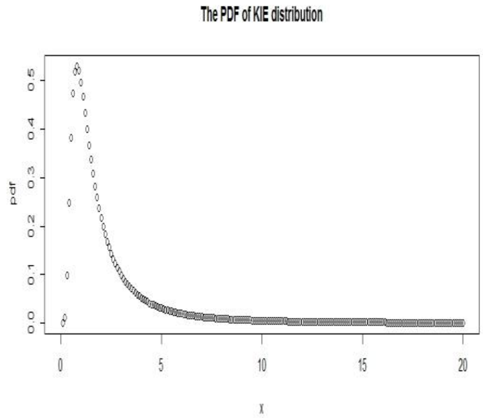

Fig. 3: Plot for PDF of KIE distribution at

$$a = 0.6, b = 2, \theta = 3$$

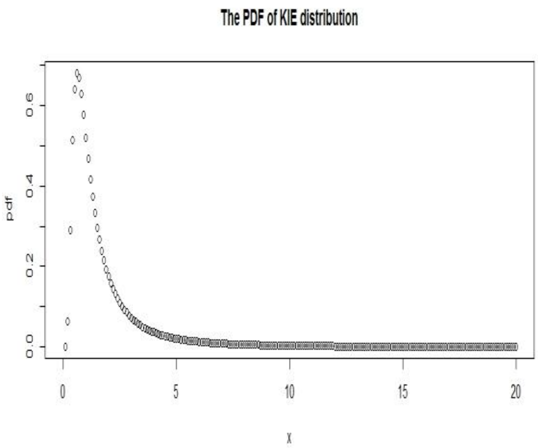

Fig. 2: Plot for PDF of KIE distribution at

$$a = 2, b = 2, \theta = 0.7$$

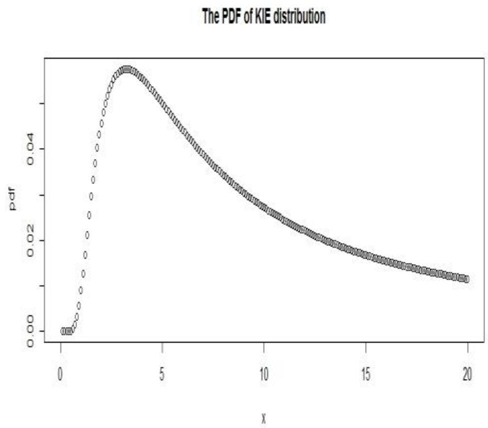

Fig. 4: Plot for PDF of KIE distribution at

$$a = 2, b = 0.6, \theta = 3$$

The shape of the pdf as shown is unimodal, the model is also positively skewed. Meanwhile Figure 5, shows that the shape of the model could be decreasing;

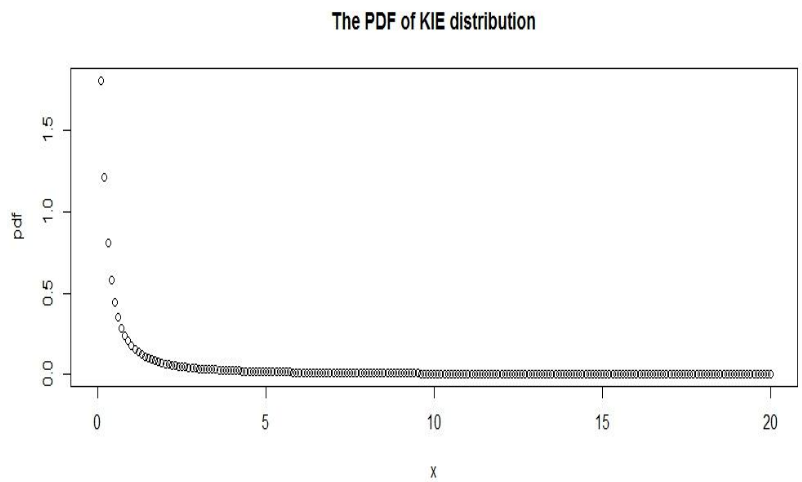

Fig. 5: Plot for PDF of KIE distribution at $a = 0.2, b = 0.5, \theta = 0.8$

A plot for the cdf of the KIE distribution is as shown in Figure 6 ;

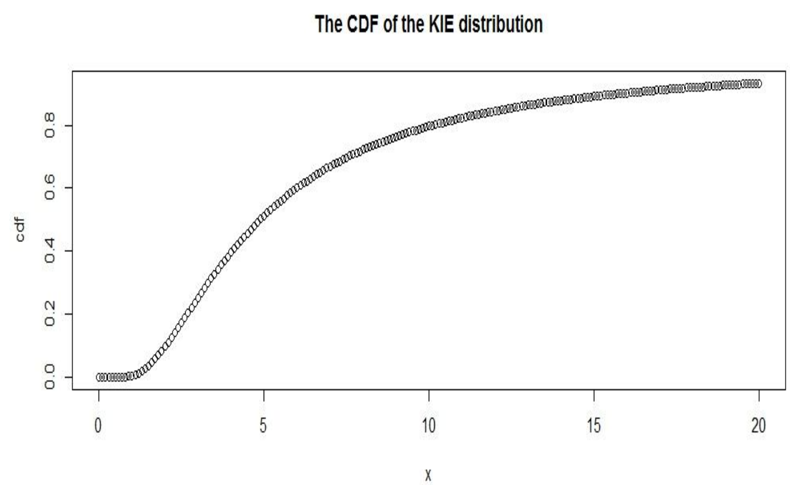

Fig. 6: Plot for CDF of KIE distribution at $a = 2, b = 2, \theta = 3$

Special Cases:

Some existing distributions are found to be sub-models of the KIE distribution. For instance ;

1. For a=1, the KIE distribution reduces to give the Generalized Inverse Exponential (GIE) distribution proposed by Abouammoh and Alshingiti [11].
2. For a=b=1, the KIE distribution reduces to give the Inverse Exponential (IE) distribution proposed by Keller and Kamath [12] ; which is the baseline distribution.

2.2. Quantile Function

The quantile function is defined as the inverse of the cdf and it is given by; $Q(u)=F^{-1}(u)$.

$$F(x)=1-\left[1-\left\{\exp\left(-\frac{\theta}{x}\right)\right\}^{a}\right]^{b}$$

Set $F(x)=u$;

$$u=1-\left[1-\left\{\exp\left(-\frac{\theta}{x}\right)\right\}^{a}\right]^{b}$$

After simple calculations,

$$x=\theta\left[-\log\left(1-(1-u)^{1/b}\right)^{1/a}\right]^{-1} \quad (5)$$

Therefore, the quantile function of the KIE distribution is given by;

$$Q(u)=\theta\left[-\log\left(1-(1-u)^{1/b}\right)^{1/a}\right]^{-1} \quad (6)$$

Random numbers from the KIE distribution can be simulated using the expression in Equation (5) where $u \sim Uniform(0,1)$.

In particular, the median of the KIE distribution can be derived by substituting 'u=0.5' in Equation (6) as follows ;

$$Median=\theta\left[-\log\left(1-(1-0.5)^{1/b}\right)^{1/a}\right]^{-1}$$

Therefore ;

$$Median=\theta\left[-\log\left(1-0.5^{1/b}\right)^{1/a}\right]^{-1} \quad (7)$$

Other quantiles can be derived easily from Equation (6) when the appropriate value of 'u' is substituted.

2.3. Order Statistics

Let $x_1, x_2, ..., x_n$ denote a random sample from a cdf $F(x)$ and an associated pdf $f(x)$ as defined in Equations (3) and (4) respectively, then the pdf of ith order statistics of the KIE distribution is derived as from;

$$f_{i:n}(x)=\frac{n!}{(i-1)!(n-i)!}f(x)F(x)^{i-1}\left[1-F(x)\right]^{n-i} \tag{8}$$

Now, substituting Equations (3) and (4) into (8) gives ;

$$f_{i:n}(x)=n\times\left\{ab\frac{\theta}{x^2}\left\{\exp\left(-\frac{\theta}{x}\right)\right\}^a\left[1-\left\{\exp\left(-\frac{\theta}{x}\right)^a\right\}\right]^{b-1}\right\}\times\left\{1-\left[1-\left\{\exp\left(-\frac{\theta}{x}\right)\right\}^a\right]^b\right\}^{i-1}\times\left\{\left[1-\left\{\exp\left(-\frac{\theta}{x}\right)\right\}^a\right]^b\right\}^{n-i} \tag{9}$$

Therefore, the distribution of minimum order statistics is given by;

$$f_{1:n}(x)=n\times\left\{ab\frac{\theta}{x^2}\left\{\exp\left(-\frac{\theta}{x}\right)\right\}^a\left[1-\left\{\exp\left(-\frac{\theta}{x}\right)^a\right\}\right]^{b-1}\right\}\times\left\{\left[1-\left\{\exp\left(-\frac{\theta}{x}\right)\right\}^a\right]^b\right\}^{n-1} \tag{10}$$

In the same way, the distribution of maximum order statistics for the KIE distribution is given by;

$$f_{n:n}(x)=n\times\left\{ab\frac{\theta}{x^2}\left\{\exp\left(-\frac{\theta}{x}\right)\right\}^a\left[1-\left\{\exp\left(-\frac{\theta}{x}\right)^a\right\}\right]^{b-1}\right\}\times\left\{1-\left[1-\left\{\exp\left(-\frac{\theta}{x}\right)\right\}^a\right]^b\right\}^{n-1} \tag{11}$$

3. RESULTS

In this section, the KIE distribution is applied to six (6) real lifetime data and its potential superiority over its sub-models assessed using the Log-likelihood and Alkaike Information Criteria (AIC). The models compared are; Kumaraswamy Inverse Exponential distribution, Generalized Inverse Exponential distribution and the Inverse Exponential distribution. The analysis in this research work is performed with the aid of R software.

The pdf of the competing models are given in Table 1;

Table 1: The pdfs of competing models

Models	pdf
Kumaraswamy Inverse Exponential	$f(x)=ab\frac{\theta}{x^2}\left\{\exp\left(-\frac{\theta}{x}\right)\right\}^a\left[1-\left\{\exp\left(-\frac{\theta}{x}\right)^a\right\}\right]^{b-1}$
Generalized Inverse Exponential	$f(x)=b\frac{\theta}{x^2}\exp\left(-\frac{\theta}{x}\right)\left[1-\left\{\exp\left(-\frac{\theta}{x}\right)\right\}\right]^{b-1}$
Inverse Exponential	$f(x)=\frac{\theta}{x^2}\exp\left(-\frac{\theta}{x}\right)$

DATA I: The first data represents the times of failures and running times for samples of devices from an eld-tracking study of a larger system. The data has been previously studied by Meeker and Escobar [13] and Merovci and Elbatal [14]. The data has 30 observations and it is as follows;

2.75, 0.13, 1.47, 0.23, 1.81, 0.30, 0.65, 0.10, 3.00, 1.73, 1.06, 3.00, 3.00, 2.12, 3.00, 3.00, 3.00, 0.02, 2.61, 2.93, 0.88, 2.47, 0.28, 1.43, 3.00, 0.23, 3.00, 0.80, 2.45, 2.66

The summary of the data is shown in Table 2 ;

Table 2: Summary of data on failure and running times of devices

n	Min.	Max.	Q_1	Q_2	Q_3	Mean	Variance	Skewness	Kurtosis
30	0.0200	3.0000	0.6875	1.9650	2.9820	1.7700	1.3223	-0.2699	1.4537

It can be noticed from Table 2 that the data is slightly negatively skewed with the coefficient of skewness being -0.2699 and a variance of 1.3223.

The performance of the KIE distribution with respect to the GIE distribution and the IE distribution using the observations in DATA I is as shown in Table 3;

Table 3: Performance Ratings Using DATA I (Standard Error in Parenthesis)

Distributions	$\hat{a}$	$\hat{b}$	$\hat{\theta}$	Log-likelihood	AIC	Rating
KIE	44.5386(12.5828)	0.4953(0.1112)	0.0040(0.0017)	-65.0736	136.1471	1
GIE	-	34.2239(7.6674)	0.0091(0.0026)	-70.6309	145.2617	3
IE	-	-	0.3122(0.0570)	-70.6309	143.2617	2

NOTE: The distribution that corresponds to the lowest AIC or highest log-likelihood is considered the best fit.

The variance covariance matrix for the KIE distribution with respect to DATA I is given by ;

$$A = \begin{pmatrix} 158.3276 & -3.5751\times10^{-2} & -1.4479\times10^{-2} \\ -0.0358 & 1.2383\times10^{-2} & 8.4052\times10^{-5} \\ -0.0144 & 8.4052\times10^{-5} & 2.8776\times10^{-6} \end{pmatrix}$$

DATA II : The second data represents the lifetime data relating to relief times (in minutes) of patients receiving an analgesic. The data was reported by Gross and Clark [15] and it has twenty (20) observations as follows :

1.1, 1.4, 1.3, 1.7, 1.9, 1.8, 1.6, 2.2, 1.7, 2.7, 4.1, 1.8, 1.5, 1.2, 1.4, 3, 1.7, 2.3, 1.6, 2

The summary of the data is given in Table 4;

Table 4: Summary of data on relief times of patients receiving an analgesic

n	Min.	Max.	Q_1	Q_2	Q_3	Mean	Variance	Skewness	Kurtosis
20	1.100	4.100	1.475	1.700	2.050	1.900	0.4958	1.5924	5.9241

It can be noticed from Table 4 that the data is positively skewed with the coefficient of skewness being 1.5924 and a variance of 0.4958.

The performance of the KIE distribution with respect to the GIE distribution and the IE distribution using the observations in DATA II is shown in Table 5;

Table 5: Performance Ratings Using DATA II (Standard Error in Parenthesis)

Distributions	$\hat{a}$	$\hat{b}$	$\hat{\theta}$	Log-likelihood	AIC	Rating
KIE	96.8835(22.3128)	20.7662(7.6817)	0.0637(0.0189)	-17.1046	40.2091	1
GIE	-	0.0031(0.0008)	66.7625(6.4642)	-57.0997	118.1993	3
IE	-	-	1.7247(0.3853)	-32.6687	67.3373	2

The variance covariance matrix for the KIE distribution with respect to DATA II is given by ;

$$A = \begin{pmatrix} 497.8619 & -54.3956 & -0.3779 \\ -54.3956 & 59.0079 & 0.0905 \\ -0.3779 & 0.0905 & 0.0004 \end{pmatrix}$$

DATA III : The third data represents the death times (in weeks) of patients with cancer of tongue with aneuploidy DNA profile. The data has been previously used by Sickle-Santanello et al., [16], Klein and Moeschberger [17]. The data consists of 52 observations out of which 21 are censored observations. The data is as follows;

1, 3, 3, 4, 10, 13, 13, 16, 16, 24, 26, 27, 28, 30, 30, 32, 41, 51, 61*, 65, 67, 70, 72, 73, 74*, 77, 79*, 80*, 81*, 87*, 87*, 88*, 89*, 91, 93*, 96, 97*, 100, 101*, 104, 104*, 108*, 109*, 120*, 131*, 150*, 157, 167, 231*, 240*, 400*

NOTE: * denote censored observations

The summary of the data is given in Table 6;

Table 6: Summary of data on death times of patients with cancer of the tongue

n	Min.	Max.	Q_1	Q_2	Q_3	Mean	Variance	Skewness	Kurtosis
52	1.00	400.00	29.00	77.00	100.50	80.73	4925.205	2.1702	10.0714

It can be noticed from Table 6 that the data is positively skewed with the coefficient of skewness being 2.1702 and a variance of 4,925.205.

The performance of the KIE distribution with respect to the GIE distribution and the IE distribution using the observations in DATA III is shown in Table 7;

Table 7: Performance Ratings Using DATA III (Standard Error in Parenthesis)

Distributions	$\hat{a}$	$\hat{b}$	$\hat{\theta}$	LL	AIC	Rating
KIE	0.5138(0.1333)	0.5771(0.1023)	22.0303(2.5159)	-300.856	607.712	1
GIE	-	0.0244(0.0034)	713.67(2.9671)	-306.1066	616.2133	3
IE	-	-	17.3795(2.4216)	-306.1066	614.2133	2

The variance covariance matrix for the KIE distribution with respect to DATA III is given by ;

$$A = \begin{pmatrix} 0.0178 & 0.0075 & -0.1598 \\ 0.0075 & 0.0105 & -0.0079 \\ -0.1598 & -0.0079 & 6.3302 \end{pmatrix}$$

DATA IV: The fourth data represents the failure times of the air conditioning system of an airplane. The data was given by Linhart and Zucchini [18]. It has thirty (30) observations as follows;

23, 261, 87, 7, 120, 14, 62, 47, 225, 71, 246, 21, 42, 20, 5, 12, 120, 11, 3, 14, 71, 11, 14, 11, 16, 90, 1, 16, 52, 95

The summary of the data is given in Table 8.

Table 8: Summary of data on failure times of the air conditioning system of an airplane

n	Min.	Max.	Q_1	Q_2	Q_3	Mean	Variance	Skewness	Kurtosis
30	1.0	261.0	12.5	22.0	83.0	59.6	5167.421	1.609	4.967

It can be noticed from Table 8 that the data is positively skewed with the coefficient of skewness being 1.609 and a variance of 5,167.421.

The performance of the KIE distribution with respect to the GIE distribution and the IE distribution using the observations in DATA IV is shown in Table 9;

Table 9: Performance Ratings Using DATA IV (Standard Error in Parenthesis)

Distributions	$\hat{a}$	$\hat{b}$	$\hat{\theta}$	Log-likelihood	AIC	Rating
KIE	0.2162(0.0792)	0.6493(0.1506)	37.5806(8.5679)	-159.1929	320.3859	3
GIE	-	0.0093(0.0027)	33.6974(7.8579)	-70.6309	145.2617	1
IE	-	-	11.1800(2.0970)	-159.0620	320.1239	2

The variance covariance matrix for the KIE distribution with respect to DATA IV is given by ;

$$A = \begin{pmatrix} 0.0063 & 0.0057 & -0.4325 \\ 0.0057 & 0.0227 & -0.0123 \\ -0.4325 & -0.0123 & 73.4104 \end{pmatrix}$$

DATA V: The fifth data represents the survival times (in days) of 72 guinea pigs infected with virulent tubercle bacilli. It has been previously used by Bjerkedal [19] and it as given below;

12, 15, 22, 24, 24, 32, 32, 33, 34, 38, 38, 43, 44, 48, 52, 53, 54, 54, 55, 56, 57, 58, 58, 59, 60, 60, 60, 60, 61, 62, 63, 65, 65, 67, 68, 70, 70, 72, 73, 75, 76, 76, 81, 83, 84, 85, 87, 91, 95, 96, 98, 99, 109, 110, 121, 127, 129, 131, 143, 146, 146, 175, 175, 211, 233, 258, 258, 263, 297, 341, 341, 376

The summary of the data is given in Table 10.

Table 10: Summary of data on death times of patients with cancer of the tongue

n	Min.	Max.	Q_1	Q_2	Q_3	Mean	Variance	Skewness	Kurtosis
72	12.00	376.00	54.75	70.00	112.80	99.82	6580.122	1.7589	5.6144

It can be noticed from Table 10 that the data is positively skewed with the coefficient of skewness being 1.7589 and a variance of 6,580.122.

The performance of the KIE distribution with respect to the GIE distribution and the IE distribution using the observations in DATA V is shown in Table 11;

Table 11: Performance Ratings Using DATA V (Standard Error in Parenthesis)

Distributions	$\hat{a}$	$\hat{b}$	$\hat{\theta}$	Log-likelihood	AIC	Rating
KIE	8.0442(3.0699)	2.5424(0.5352)	12.7589(4.8635)	-391.5948	788.1897	2
GIE	-	0.0249(0.0119)	29.9943(13.9093)	-98.4873	200.9745	1
IE	-	-	60.0980(2.9660)	-402.6718	807.3437	3

The variance covariance matrix for the KIE distribution with respect to DATA V is given by ;

$$A = \begin{pmatrix} 9.4244 & 0.3141 & -13.8224 \\ 0.3141 & 0.2864 & 0.3326 \\ -13.8224 & 0.3326 & 23.6534 \end{pmatrix}$$

DATA VI: The sixth data represents the survival times of a group of patients suffering from Head and Neck cancer diseases and treated using a combination of radiotherapy and chemotherapy (RT+CT). The data has been previously used by Efron [20] and Shanker et al., [21]. It has fourty-four (44) observations and it is as given below;

12.20, 23.56, 23.74, 25.87, 31.98, 37, 41.35, 47.38, 55.46, 58.36, 63.47, 68.46, 78.26, 74.47, 81.43, 84, 92, 94, 110, 112, 119, 127, 130, 133, 140, 146, 155, 159, 173, 179, 194, 195, 209, 249, 281, 319, 339, 432, 469, 519, 633, 725, 817, 1776

The summary is given in Table 12.

Table 12: Summary of data on survival times of patients suffering from head and neck cancer

n	Min.	Max.	Q_1	Q_2	Q_3	Mean	Variance	Skewness	Kurtosis
44	12.20	1776.00	67.21	128.50	219.00	223.50	93,286.41	3.2691	16.5596

It can be noticed from Table 12 that the data is positively skewed with the coefficient of skewness being 3.2691 and a variance of 93,286.41.

The performance of the KIE distribution with respect to the GIE distribution and the IE distribution using the observations in DATA VI is shown in Table 13;

Table 13: Performance Ratings Using DATA VI (Standard Error in Parenthesis)

Distributions	$\hat{a}$	$\hat{b}$	$\hat{\theta}$	Log-likelihood	AIC	Rating
KIE	57.1140(5.0256)	1.1679(0.2434)	1.4856(0.3250)	-279.8058	564.6116	3
GIE	-	0.1613(0.0041)	28.3982(5.7082)	-84.0759	172.1519	1
IE	-	-	76.7000(4.1940)	-279.5773	561.1546	2

The variance covariance matrix for the KIE distribution with respect to DATA VI is given by ;

$$A = \begin{pmatrix} 25.2568 & -0.0829 & -0.7571 \\ -0.0829 & 0.0592 & 0.0506 \\ -0.7571 & 0.0506 & 0.1056 \end{pmatrix}$$

4. CONCLUSION

The real life application of the KIE distribution has been demonstrated successfully while statistical properties like the quantile function and distribution of order statistics were also provided. The KIE distribution has the lowest AIC and the highest log-likelihood value compared to the other models considered for (DATA I, II and III). This means that the KIE distribution is more flexible and better than the GIE distribution and the IE distribution for the data considered. On the contrary, the KIE distribution performed poorly when applied to DATA IV, V and VI. It was further observed that the variances of DATA IV, V and VI are very large, especially for DATA VI with a variance of 93,286.41. Therefore, one can confidently say that the KIE distribution may not be suitable for data sets that are over-dispersed.

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