Addendum: On Convergence and Stability of the Generalized Noor Iterations for a General Class of Operators

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1 Introduction

An error was pointed out by Prof. C. E. Chidume in the statements of our Theorems and Corollaries in [1]. The proof of all the Theorems and Corollaries are correct but the statements have flaws. The correct statements of the results are hereby stated.

We define the multistep iteration as:

Let \( E \) be a Banach space and \( T : E \rightarrow E \) a self map of \( E \). For \( x_0 \in E \), the multistep iterative scheme \( \{x_n\}_{n=0}^{\infty} \) is defined by

\[
x_{n+1} = (1 - \alpha_n)x_n + \alpha_n Ty_n^1
\]
\[
y_n^i = (1 - \beta_n^i)x_n + \beta_n^i Ty_{n+1}^i, \quad i = 1, 2, ..., k - 2,
\]
\[
y_{k-1}^{k-1} = (1 - \beta_n^{k-1})x_n + \beta_n^{k-1}Tx_n, \quad k \geq 2
\]

where \( \{\alpha_n\}_{n=0}^{\infty}, \{\beta_n^i\}, i = 1, 2, ..., k - 1 \) (with \( k \geq 2 \)) are real sequences in \([0,1)\) such that \( \sum_{n=0}^{\infty} \alpha_n = \infty \).

2.1. Some Strong Convergence Results in Banach Spaces

**Theorem 2.1.1.** Let \( (E, ||.||) \) be a Banach space, \( T : E \rightarrow E \) be a self map of \( E \) with a fixed point \( p \) satisfying the condition

\[
\|p - Ty\| \leq a\|p - y\|. \quad (2.1)
\]

for each \( y \in E \) and \( 0 \leq a < 1 \). For \( x_0 \in E \), let \( \{x_n\}_{n=0}^{\infty} \) be the multistep iterative scheme defined by (1.1). Then \( \{x_n\}_{n=0}^{\infty} \) converges strongly to \( p \).

**Corollary 2.1.3.** Let \( (E, ||.||) \) be a Banach space, \( T : E \rightarrow E \) be a self map of \( E \) with a fixed point \( p \) satisfying the condition

\[
\|p - Ty\| \leq a\|p - y\|. \quad (2.2)
\]

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for each $y \in E$ and $0 \leq a < 1$. For $x_0 \in E$, let $\{x_n\}_{n=0}^{\infty}$ be the Noor iterative scheme defined by (1.4) in [1]. Then the Noor iterative scheme converges to $p$.

**Corollary 2.1.5.** Let $(E, ||.||)$ be a Banach space, $T : E \to E$ be a selfmap of $E$ with a fixed point $p$ satisfying the condition

$$||p - Ty|| \leq a||p - y||,$$

(2.3)

for each $y \in E$ and $0 \leq a < 1$. For $x_0 \in E$, let $\{x_n\}_{n=0}^{\infty}$ be the Ishikawa iterative scheme defined by (1.3) in [1]. Then the Ishikawa iterative scheme converges to $p$.

**Corollary 2.1.6.** Let $(E, ||.||)$ be a Banach space, $T : E \to E$ be a selfmap of $E$ with a fixed point $p$ satisfying the condition

$$||p - Ty|| \leq a||p - y||,$$

(2.4)

for each $y \in E$ and $0 \leq a < 1$. For $x_0 \in E$, let $\{x_n\}_{n=0}^{\infty}$ be the Mann iterative scheme defined by (1.2) in [1]. Then the Mann iterative scheme converges to $p$.

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**References**


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