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THE STABILITY OF A MODIFIED JUNGCK-MANN HYBRID FIXED POINT ITERATION PROCEDURE

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Abstract

In this paper, we prove some stability results for sequences of nonself mappings using a modified Jungck-Mann hybrid iterative procedure in a Banach space by employing a class of generalized contractive-like definition. As corollaries, some stability results of Jungck (pair of maps) and Picard (single map) iterative procedures are also established. Our stability results generalize and extend several related results involving pair and single maps in the literature.

Keywords: Jungck-Mann hybrid iterative procedure, generalized contractive-like mapping, stability result, Banach space.

1 Introduction and Preliminary Definitions

Let (X, d) be a complete metric space and $T : X \to X$ be a self map of X. Suppose that $F_T = \{p \in X : Tp = p\}$ is the set of all fixed point of T. There are several iterative procedures in the literature for which fixed point of mappings have been approximated over the years. In a complete metric space, for $x_0 \in X$ the Picard iterative sequence $\{x_n\}_{n=1}^{\infty}$ defined by

$$x_{n+1} \quad = \quad Tx_n, \quad n \geq 0, \tag{1.1}$$

has been employed to approximate the fixed points of the mapping satisfying the inequality $d(Tx, Ty) \leq ad(x, y)$, for all $x, y \in X$ and $0 \leq a < 1$ [see 15, 16 for details].

In a Banach space setting, we shall need the following iterative procedures which appear in [13] and [10] to explain our stability results.

Let E be a Banach space and $T : E \to E$ a selfmap of E. For $x_0 \in E$, the sequence $\{x_n\}_{n=1}^{\infty}$

$$x_{n+1} = (1 - \alpha_n)x_n + \alpha_n Tx_n, \quad n \geq 0, \tag{1.2}$$

where $\{\alpha_n\}_{n=0}^{\infty}$ is a real sequence in [0,1] such that $\sum_{n=0}^{\infty} \alpha_n = \infty$ is called the

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Mann iterative scheme [13].

If $\alpha_n = 1$ in (1.2), we have the Picard iterative scheme (1.1).

For $x_0 \in E$, the sequence $\{x_n\}_{n=0}^{\infty}$ defined by

$$\begin{aligned} x_{n+1} &= (1-\alpha_n)x_n + \alpha_n T y_n \\ y_n &= (1-\beta_n)x_n + \beta_n T x_n, \quad n \geq 0, \end{aligned} \tag{1.3}$$

where $\{\alpha_n\}_{n=0}^{\infty}, \{\beta_n\}_{n=0}^{\infty}$ are real sequences in [0,1] such that $\sum_{n=0}^{\infty} \alpha_n = \infty$ is called Ishikawa iterative scheme [10].

Observe that if $\beta_n = 0$ for each n, then the Ishikawa iterative scheme (1.3) reduces to the Mann iterative scheme (1.2).

In [7], Berinde showed that Picard iteration defined in (1.1) is faster than Mann iteration in (1.2) for quasicontractive operators. In [23], Qing and Rhoades by taking example, showed that Ishikawa iteration (1.3) is faster than Mann iteration (1.2) for a certain quasi-contractive operator.

Several generalizations of the Banach fixed point theorem have been proved to date, one of the most commonly studied generalization hitherto is the one proved by Zamfirescu [26] in 1972, which is stated as thus:
Theorem 1.1. Let X be a complete metric space and $T : X \to X$ a Zamfirescu operator satisfying

$$d(Tx, Ty) \leq h \max\{d(x, y), \frac{1}{2}[d(x, Tx) + d(y, Ty)], \frac{1}{2}[d(x, Ty) + d(y, Tx)]\}, \tag{1.4}$$

where $0 \leq h < 1$. Then, T has a unique fixed point and the Picard iteration (1.1) converges to p for any $x_0 \in X$.
Observe that in a Banach space setting, condition (1.4) implies

$$\|Tx - Ty\| \leq \delta\|x - y\| + 2\delta\|x - Tx\|, \tag{1.5}$$

where $0 \leq \delta < 1$ and $\delta = \max\{h, \frac{h}{2-h}\}$.

Several papers have been written on the Zamfirescu operators (1.5), for example (see [5], [6], [7] and [26]). The most commonly used methods of approximating the fixed points of the Zamfirescu operators are Picard, Mann [13] and Ishikawa [10].

The first important result on T-stable mappings was established by Ostrowski [22] for Picard iteration. Berinde [5], also gave the following remarkable explaination on the stability of iteration procedures.
Throughout this study, X shall denote metric space and E a Banach space.
Let $\{x_n\}_{n=0}^{\infty}$ be the sequence generated by an iteration procedure involving the

operator T

$$x_{n+1} = f(T, x_n), \tag{1.6}$$

$n = 0, 1, 2, ...,$ where $x_0 \in X$ is the initial approximation and f is some function. For example, the Picard iteration (1.1) is obtained from (1.6) for $f(T, x_n) = Tx_n$, while the Mann iteration (1.2) is obtained for $f(T, x_n) = (1-\alpha_n)x_n + \alpha_n Tx_n$, with $\{\alpha_n\}_{n=0}^{\infty}$ is a sequence in [0,1]. Suppose $\{x_n\}_{n=0}^{\infty}$ converges to a fixed point p of T. When calculating $\{x_n\}_{n=0}^{\infty}$, then we cover the following steps:
1. We chose the initial approximation $x_0 \in X$;
2. Then we compute $x_1 = f(T, x_0)$, but due to various errors (rounding errors, numerical approximations of functions, derivatives or integrals), we do not get the exact value of x_1, but a different one z_1, which is very close to x_1;
3. Consequently, when computing $x_2 = f(T, x_1)$ we shall have actually $x_2 = f(T, z_1)$ and instead of the theoretical value x_2, we shall obtain a closed value and so on. In this way, instead of the theoretical sequence $\{x_n\}_{n=0}^{\infty}$ generated by the iterative method, we get an approximant sequence $\{z_n\}_{n=0}^{\infty}$. We say the iteration method is stable if and only if for z_n closed enough to x_n, $\{z_n\}_{n=0}^{\infty}$ still converges to the fixed point p of T. Following this idea, Harder and Hicks [8] introduced the following concept of stability.

Definition 1.2. Let (X, d) be a complete metric space and $T : X \to X$ a self map, $x_0 \in X$ and the iteration procedure defined by (1.1) such that the generated sequence $\{x_n\}_{n=0}^{\infty}$ converges to a fixed point p of T. Let $\{z_n\}_{n=0}^{\infty}$ be arbitrary sequence in X, and set $\epsilon_n = d(z_{n+1}, f(T, z_n))$, for $n \geq 0$. We say the iteration procedure (1.1) is $T-$stable if and only if $\lim_{n\to\infty} \epsilon_n = 0$ implies that $lim_{n\to\infty} z_n = p$.

Remark 1.3. Since the metric is induced by a norm, we have
$\epsilon_n = \|z_{n+1} - f(T, z_n)\|$, for $n \geq 0$ in place of $\epsilon_n = d(z_{n+1}, f(T, z_n))$, for $n \geq 0$ in the definition of stability whenever we are working in a Banach space.

In 2003, Imoru and Olatinwo [9] proved some stability results by employing the following general contractive definition:
for each $x, y \in E$, there exists $\delta \in [0, 1)$ and a monotone increasing function $\varphi : R^+ \to R^+$ with $\varphi(0) = 0$ such that

$$\|Tx - Ty\| \leq \delta\|x - y\| + \varphi(\|x - Tx\|). \tag{1.7}$$

Several other stability results exist in the literature (for details see [1], [4], [5], [8], [9], [19], [20], [21], [22] and [25]).

In 2013, Khan [12], gave a different perspective to iteration procedure, he introduced the following Picard-Mann hybrid iterative scheme for a single nonexpansive mapping T. For any initial point $x_0 \in E$ the sequence $\{x_n\}_{n=0}^{\infty}$ is defined

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by

$$\begin{aligned} x_{n+1} &= Ty_n \\ y_n &= (1-\alpha_n)x_n + \alpha_n T x_n, n \geq 0, \end{aligned} \tag{1.8}$$

where $\{\alpha_n\}_{n=0}^{\infty}$ is a real sequence in [0,1] such that $\sum_{n=0}^{\infty} \alpha_n = \infty$.
He showed that the hybrid scheme (Picard-Mann scheme (1.8)) converges faster than all of Picard (1.1), Mann (1.2) and Ishikawa (1.3) iterative procedures in the sense of Berinde [7] for contractions. He also proved strong convergence and weak convergence theorems with the help of his iterative process (1.8) for the class of nonexpansive mappings in general Banach spaces and applied it to obtain results in uniformly convex Banach spaces.
Motivated by the work of Khan [12], we introduce the following modified Jungck-Mann hybrid iterative procedure and prove common fixed point theorem in [2] for a pair of weakly compatible generalized contractive-like mappings in a Banach space. The author [2] also gave some useful examples to demonstrate that weakly compatibility is the most general of all the compatibility type condtions under consideration.

Definition 1.4 [2]. Let E be a Banach space, Y be an arbitrary set and $S, T : Y \to E$ such that $T(Y) \subseteq S(Y)$. Let $x_0 \in Y$, the Jungck-Mann hybrid iteration scheme $\{Sx_n\}_{n=1}^{\infty}$ is defined by

$$\begin{aligned} Sx_{n+1} &= Ty_n \\ Sy_n &= (1-\alpha_n)Sx_n + \alpha_n T x_n \end{aligned} \tag{1.9}$$

where $\{\alpha_n\}_{n=0}^{\infty}$ is a real sequence in [0,1] such that $\sum_{n=0}^{\infty} \alpha_n = \infty$.

Definition 1.5 [17]. Let E be a Banach space, Y be an arbitrary set. For $S, T : Y \to E$ with $T(Y) \subseteq S(Y)$, where $S(Y)$ is a complete subspace of E. There exists a real number $\delta \in [0, 1)$ and a monotone increasing function $\varphi : R^+ \to R^+$ such that $\varphi(0) = 0$ and for every $x, y \in Y$, we have

$$\|Tx - Ty\| \leq \delta\|Sx - Sy\| + \varphi(\|Sx - Tx\|). \tag{1.10}$$

Definition 1.6 [17]. Let E be a Banach space and $S, T : E \to E$. A point $p \in E$ is called a coincident point of a pair of self maps S, T if there exists a point q (called a point of coincidence) in E such that $q = Sp = Tp$. Two self maps S and T are weakly compatible if they commute at their coincidence points, that is if $Sp = Tp$ for some $p \in E$, then $STp = TSp$.

The author [2], proved convergence theorem of Jungck-Mann hybrid iterative procedure to the common fixed point of a pair of weakly compatible mappings S, T for generalized contractive-like operators, using the following theorem:
Theorem 1.7 [Theorem 3.1 [2]]. Let $(E, \|.\|)$ be a Banach space and $S, T : Y \to E$ be nonself commuting mappings for an arbitrary set Y satisfying the

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qeneralized contractive-like inequality (1.10) with $T(Y) \subseteq S(Y)$. Let w be the coincidence point of S, T, (i.e $Sw = Tw = p$) for each $x_0 \in Y$, the Jungck-(Jungck-Mann) hybrid iterative scheme (1.9) converges strongly to p.
Further, if $Y = E$ and S, T commute at p (that is S and T are weakly compatible), then p is the unique common fixed point of S, T.

Example 1.8.[2] Let $E = \Big([0,2], \|.\|\Big)$. Define T and S by

$$Tx = \begin{cases} \frac{1}{2}, & \text{if } \quad x \in (0,1] \\ 0, & \text{if } \quad x \in \{0\} \cup (1,2] \end{cases} \quad \text{and } Sx = \begin{cases} 0, & \text{if} \quad \text{x=0} \\ x+1, & \text{if} \quad x \in (0,1] \\ x-1, & \text{if} \quad x \in (1,2] \end{cases}$$

$\|Tx - Ty\| \leq \delta\|Sx - Sy\| + \varphi(\|Sx - Tx\|)$, where $\delta = \frac{1}{2}$ and $\varphi(t) = 2\delta t$.
$T(E) = \{0\} \cup \{\frac{1}{2}\}$ and $S(E) = [0,2]$. Then $T(E) \subseteq S(E)$. It is easy to see that $S(0) = T(0) = 0$ and $ST(0) = S(0) = 0$, $TS(0) = T(0) = 0$.
Hence the common fixed point of S and T is 0.

Several other relevant papers of Jungck-type schemes worth studying exist in literature, for detailed survey of some of them see ([15], [16], [17] and [18]).

We only need to prove the stability results for this iteration procedure defined in (1.9) for a pair of weakly compatible maps.

We shall need the following definition and Lemma, to prove our results.
Definition 1.9. Let $(E, \|.\|)$ be a Banach space and $S, T : Y \to E$ nonself commuting mappings for an arbitrary set Y with $T(Y) \subseteq S(Y)$. Suppose S and T have a common fixed point p, that is $(Sp = Tp = p)$ and for each $x_0 \in Y$ let $\{Sx_n\}_{n=0}^{\infty}$ generated by the modified Jungck-Mann hybrid iterative procedure defined by (1.9) converge to p . Let $\{Sz_n\}_{n=0}^{\infty}$ be arbitrary sequence in E, and set $\epsilon_n = \|Sz_{n+1} - f(T, z_n)\|$, for $n \geq 0$. We say the iteration procedure (1.9) is (S,T)-stable if and only if $\lim_{n\to\infty} \epsilon_n = 0$ implies that $lim_{n\to\infty} Sz_n = p$.

Lemma 1.10 [5]. Let δ be a real number satisfying $0 \leq \delta < 1$ and $\{\epsilon_n\}_{n=0}^{\infty}$ a sequence of positive numbers such that $lim_{n\to\infty}\epsilon_n = 0$, then for any sequence of positive numbers $\{u_n\}_{n=0}^{\infty}$ satisfying $u_{n+1} \leq \delta u_n + \epsilon_n$, n=0,1,2,..., we have $lim_{n\to\infty} u_n = 0$.

2 Main Result

Theorem 2.1. Let $(E, \|.\|)$ be a Banach space and $S, T : E \to E$ be nonself weakly compatible mappings satisfying the generalized contrative-like condition

$$\|Tx - Ty\| \quad \leq \quad \delta\|Sx - Sy\| + \varphi(\|Sx - Tx\|). \tag{2.1}$$

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where $\delta \in [0,1)$ and $\varphi : R^+ \to R^+$ a monotone increasing function such that $\varphi(0) = 0$ and for every $x, y \in E$. Suppose S and T have a unique common fixed point p (that is $Sp = Tp = p$). Let $x_0 \in E$ and $\{Sx_n\}_{n=0}^{\infty}$ be the modified Jungck-Mann hybird iterative procedure defined by (1.9) converging to p. Then, the modified Jungck-Mann hybird iterative procedure (1.9) is (S,T)-stable.

Proof:

Let $\{Sx_n\}_{n=0}^{\infty}$ be the theoretical seqeuence and $\{Sz_n\}_{n=0}^{\infty}$ be the approximant sequence.

Let $\{Sz_n\}_{n=0}^{\infty}$ be real a sequence in E.

Let $\epsilon_n = \|Sz_{n+1} - Tu_n\|$, $n = 0, 1, 2, ...,$

where $Su_n = (1-\alpha_n)Sz_n + \alpha_n Tz_n$ and let $lim_{n\to\infty}\epsilon_n = 0$.

Then we shall prove that $lim_{n\to\infty}Sz_n = p$ using the contractive mappings satisfying condition (2.1).

That is,

$$\begin{aligned} \|Sz_{n+1} - p\| &\leq \|Sz_{n+1} - Tu_n\| + \|Tu_n - Tp\| \\ &\leq \epsilon_n + \|Tu_n - Tp\|. \end{aligned} \tag{2.2}$$

Applying condition (2.1) on (2.2), we have

$$\|Sz_{n+1} - p\| \leq \epsilon_n + \delta\|Sp - Su_n\| + \varphi(\|Sp - Tp\|) \tag{2.3}$$

Since $Sp = Tp = p$ and $\varphi(0) = 0$, then (2.3) becomes

$$\|Sz_{n+1} - p\| \leq \epsilon_n + \delta\|p - Su_n\|. \tag{2.4}$$

From (2.4),

$$\begin{aligned} \|p - Su_n\| &= \|(1-\alpha_n+\alpha_n)p - (1-\alpha_n)Sz_n - \alpha_n Tz_n\| \\ &= \|(1-\alpha_n)(p - Sz_n) + \alpha_n(p - Tz_n)\| \\ &\leq (1-\alpha_n)\|p - Sz_n\| + \alpha_n\|Tp - Tz_n\| \\ &\leq (1-\alpha_n)\|p - Sz_n\| + \alpha_n[\delta\|Sp - Sz_n\| + \varphi(\|Sp - Tp\|)] \\ &= (1-\alpha_n)\|p - Sz_n\| + \alpha_n[\delta\|p - Sz_n\| \\ &= (1-\alpha_n+\alpha_n\delta)\|p - Sz_n\| \end{aligned} \tag{2.5}$$

Substituting (2.5) in (2.4), we have

$$\|Sz_{n+1} - p\| \leq \delta[1 - (1-\delta)\alpha_n]\|p - Sz_n\| + \epsilon_n. \tag{2.6}$$

Since $0 \leq \delta < 1$, using Lemma [1.10] in (2.6) yields $lim_{n\to\infty}Sz_n = p$.

Conversely, let $lim_{n\to\infty}Sz_n = p$, we show that $lim_{n\to\infty}\epsilon_n = 0$ as follows:

$$\begin{aligned} \epsilon_n &= \|Sz_{n+1} - Tu_n\| \\ &\leq \|Sz_{n+1} - Sp\| + \|Tp - Tu_n\| \\ &\leq \|Sz_{n+1} - Sp\| + \delta\|Sp - Su_n\| + \varphi(\|Sp - Tp\|) \\ &= \|Sz_{n+1} - Sp\| + \delta\|p - Su_n\| \end{aligned} \tag{2.7}$$

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From (2.7),

$$\begin{aligned} \|p - Su_n\| &\leq (1-\alpha_n)\|p - Sz_n\| + \alpha_n\|Tp - Tz_n\| \\ &\leq (1-\alpha_n)\|p - Sz_n\| + \alpha_n[\delta\|Sp - Sz_n\| + \varphi(\|Sp - Tp\|)] \\ &= (1-\alpha_n)\|p - Sz_n\| + \alpha_n\delta\|p - Sz_n\| \\ &= (1-\alpha_n+\alpha_n\delta)\|p - Sz_n\| = [1-(1-\delta)\alpha_n]\|p - Sz_n\| \qquad (2.8) \end{aligned}$$

Substituting (2.8) in (2.7), we have

$$\epsilon_n \leq \|Sz_{n+1} - p\| + \delta[1-(1-\delta)\alpha_n]\|Sz_n - p\|$$

Since $lim_{n\to\infty}Sz_n = p$ by our assumption, then we have $lim_{n\to\infty}\epsilon_n = 0$. Therefore, the modified Jungck-Mann hybrid iterative procedure (1.9) is (S,T)-stable.This ends the proof.

Theorem 2.1 yields the following corollary:

Corollary 2.2. Let $(E, \|.\|)$ be a Banach space and $T : E \to E$ be a self mapping satisfying the contrative-like condition

$$\|Tx - Ty\| \leq \delta\|x - y\| + \varphi(\|x - Tx\|), \qquad (2.9)$$

where $\delta \in [0,1)$ and $\varphi : R^+ \to R^+$ a monotone increasing function such that $\varphi(0) = 0$ and for every $x, y \in E$. Suppose T has a fixed point p (that is $Tp = p$). Let $x_0 \in E$ and $\{x_n\}_{n=0}^{\infty}$ be the Picard-Mann hybird iterative procedure defined by (1.8) converging to p. Then, the Picard-Mann hybird iterative procedure (1.8) is T-stable.

Theorem 2.3. Let $(E, \|.\|)$ be a Banach space and $S, T : E \to E$ be nonself weakly compatible mappings satisfying the generalized contrative-like condition

$$\|Tx - Ty\| \leq \delta\|Sx - Sy\| + \varphi(\|Sx - Tx\|), \qquad (2.10)$$

where $\delta \in [0,1)$ and $\varphi : R^+ \to R^+$ a monotone increasing function such that $\varphi(0) = 0$ and for every $x, y \in E$. Suppose S and T have a unique common fixed point p (that is $Sp = Tp = p$). Let $x_0 \in E$ and $\{Sx_n\}_{n=0}^{\infty}$ be the Jungck iterative procedure defined by $Sx_{n+1} = Tx_n$, $n \geq 0$ converging to p. Then, the Jungck iterative procedure is (S,T)-stable.

Proof:

The Proof of theorem 2.3 is similar to that of theorem 2.1.

Theorem 2.3 yields the following collolary:

Corollary 2.4. Let $(E, \|.\|)$ be a Banach space and $S, T : E \to E$ be self mapping satisfying the contrative-like condition

$$\|Tx - Ty\| \leq \delta\|x - y\| + \varphi(\|x - Tx\|), \qquad (2.11)$$

where $\delta \in [0,1)$ and $\varphi : R^+ \to R^+$ a monotone increasing function such that $\varphi(0) = 0$ and for every $x, y \in E$. Suppose T has a fixed point p (that is $Tp = p$).

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Let $x_0 \in E$ and $\{x_n\}_{n=0}^{\infty}$ be the Picard iterative procedure defined by (1.1) converging to p. Then, the Picard iterative procedure is T-stable.

Example 2.5: Consider the equation $f(x) = 0$, where f is the real function defined on interval $[0, \frac{\pi}{2}]$ by $f(x) = x^2 - \left(\frac{\pi}{2}\right)^2 \cos(x)$.
f can be decomposed as $f = \frac{\pi}{2}(S - T)$, where the maps S and T are the self mappings in $[0, \frac{\pi}{2}]$ defined by $S(x) := \frac{2}{\pi}x^2$ and $T(x) := \frac{\pi}{2}\cos(x)$. They satisfy inequality (1.10). They coincide at $\omega \approx 1.0792$ and we have $p = S\omega = T\omega \approx 0.7415$. Thus, ω is solution to the equation $f(x) = 0$.
From **Theorem 3.1** [2], the modified Jungck-Mann hybrid iteration scheme $\{Sx_n\}$ given in (1.9) converges to $p = Sw$. Using MATLAB, we have the following table:

n	x_n	Sx_n
0	0.1000	0.1000
1	1.0472	0.6982
2	1.0896	0.7558
3	1.0739	0.7343
$\vdots$	$\vdots$	$\vdots$
21	1.0791	0.7412
$\vdots$	$\vdots$	$\vdots$
45	1.0792	0.7415

Since S is continuous, the fact that $\{Sx_n\}$ converges to Sw implies that the sequence $\{x_n\}$ converges to w, the zero of f.

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