A New Model for Predicting Liquid Loading in Multiphase Gas Wells


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Abstract

Liquid loading problem occurs when there is accumulation of liquid in the wellbore. The ultimate desire of this phenomenon is to reduce gas recovery or completely kill the production well. This challenge can lead to loss in well deliverability and as a result recovery of gas becomes low and cause heavy damage that the remedy would be costly. So many scholars have modeled to predict liquid loading onset but over the years the results gotten have shown one discrepancies or the other and could be difficult to use because of the challenges facing in predicting the bottomhole pressure of a multiphase flow. Numerical integration method was used in this new model while considering the introduction of valve equation along the functional nodes to the fundamental equation. The pressure drop across the functional node has not been considered by previous scholars. The result is very effective when compared analytically with other established work. The results also show the onset of transient flow and how quickly it stabilizes. Furthermore, the numbers of correctly predicted wells when validated with data was higher than the previous results.

INTRODUCTION

The accumulation of water that is co-produced with gas at the bottom of the well during gas production when the transport energy can no longer transport it to the surface is called liquid loading. The liquid loading occurrence causes decline in production rate and in most extreme cases, may cause the well to die. When pressure drop in the wellbore increases, the gas flow velocity will decrease and the bottomhole pressure will increase. At this point, the effective gas permeability near the wellbore is reduced as water saturation increases. This would certainly hamper gas production rate. However, as the transport energy reduces, the water (liquid) at the wellbore cannot be transported to the surface and this would perhaps lead to well blockage and liquid loading.

Many researchers have developed various models on how to predict and control liquid loading but this gives a more accurate result. The most popular one is Turner et al. (1969) Mhunir (2012). Two physical models were presented: the movement of liquid film along the pipe walls and liquid droplets entrained in high velocity gas core. (www.bsee.gov). The study shows that their critical terminal velocity model \(v_{\text{crit}}\) is a function of gravitational force, the drag force and force of buoyancy. Zhou et al.(2009).

Figure 1.0 Single Droplet Force Balance. Zhou et al. (2009)

\[
F_D + F_B > F_G = \text{Upward movement} \\
F_D + F_B < F_G = \text{Downward movement}
\]

DESIGN THEORY

The maximum velocity a free falling particle in fluid medium can attain under the influence of gravity alone is termed terminal velocity i.e. when \(F_D = F_G\) Mohammed Tabatabaei (2008).

This phenomenon, terminal velocity is therefore a function of the particle shape, size, and density and of the viscosity and density of the fluid medium Coleman et al.(1991). By a transformation of the coordinates, a drop of liquid being
transported by a moving gas stream becomes a free falling particle and the same general equations apply. If the gas were moving at a velocity sufficient to suspend a drop without falling, then the slip velocity of the drop would be equal to the velocity of the gas. Robert De Jonge (2007).

\[ F_D = F_G \]

Where \( F_D = C_D A_D P \left( \frac{V_i^2}{2g} \right) \)

And \( F_G = g \left( \rho_i - \rho_g \right) \frac{\pi d^3}{6} \)

Equating equation 3.0 and 4.0 and solve for \( V_i \)

\[ V_i = \frac{2}{\sqrt{\pi}} \left[ \frac{g \left( \rho_i - \rho_g \right) L}{C_D P_g} \right]^{1/2} \]

Turner et al. (1969) after considering Weber number ranges between 20-30 which is the critical value a free falling liquid can attain and upon assumption of a constant droplet in shape and size, equation 6.0 was developed.

\[ V_i = \frac{14.26 \sigma \left( \rho_i - \rho_g \right) \nu}{\rho_g C_D} \]

Assume drag coefficient \( C_D = 0.44 \) then equation 7.0 was developed

\[ V_i = \frac{17.51 \sigma \left( \rho_i - \rho_g \right) \nu}{\rho_g} \]

Turner et al. later compared their models by using field data and it was observed that the movement of liquid film along the pipe model did not give a true representation of the process that occurs in fluid transport and also noted that the liquid droplet entrained model was underestimated and adjusted by 20%.

Numerous authors later modified the existing models. Guo et al was able to establish that with 20% adjustment, Turner et al still under estimate the minimum velocity of the gas capable of transporting liquid out of the well. However, using the established equation, Guo et al. developed gas minimum kinetic energy needed to transport or lift liquid. Guo et al. (2008).

This is expressed in terms of gas per unit volume as

\[ E_K = \frac{\rho_g V_i^2}{2g_c} \] Guo et al. (2008) 8.0

Where \( E_K \) is defined as the kinetic energy in ft-lbf/ft³. Guo et al. (2005).

\( V_i \) is velocity of the gas in ft/s and \( g_c \) is 32.17 lbm-ft/lfbsec².

The particular equation is extensively used in the drilling industry to calculate the minimum gas flow rate needed to efficiently transport drill cuttings from the wellbore during drilling operations. By substituting Eq. (6) into Eq. (8), Eq. (9) was developed. Guo et al. (2005).

\[ E_{km} = 3.157 \frac{\sigma (\rho_i - \rho_g)}{\rho_g} \]

Turner et al. recommended \( c_d = 0.44 \) and also by neglecting the effect of gas density, Eq. (9) becomes

\[ E_{km} = 4.75 \sqrt{\sigma \rho_i} \]

The minimum required gas velocity needed to transport droplets of liquid upward the conduits equal to the minimum required velocity of gas needed to keep it floating (preventing the droplets of liquid from falling) plus the velocity to transport the droplets.

\[ V_{gm} = V_{sl} + V_{tr} \]

While trying to generate an equation for transporting the liquid velocity, this work made use of an empirical constant to give a vivid account of the solid spheres drag coefficients effect, non-stagnation velocity, and the Weber number that was used to calculate the liquid drops falling in air. Guo et al. (2005). However, considering Turner work, the velocity of transport was given as 20% of \( V_{sl} \) in this study. Therefore, using this value gives Eq. (12.0).

\[ V_{gm} = 1.2V_{sl} \]

Putting Eqs. (6) and (12) in to Eq. (8), the equation necessary to calculate the minimum kinetic energy for liquid droplets transport was formulated in Eq. (13).

\[ E_{km} = 6.86 \sqrt{\sigma \rho_i} \]

In this study it is necessary to determine the kinetic energy of gas at a particular flow rate and then compare with Eq.(13), both the gas density \( \rho_g \) and gas velocity \( V_g \) values have to be determined. Guo et al. (2013). Using ideal gas law, the following equations were obtained.

\[ \rho_g = \frac{2.75 \rho_P}{T} \]

and

\[ V_g = 3.27 \times 10^{-2} \frac{Q_g}{AP} \]

\( S_g \) is specific gravity of gas and \( Q_g \) is flow rate of gas in scf/min. By substituting the two Eqs. to Eq.(8), Eqn. 16 was developed.

\[ E_k = 4.49 \times 10^{-5} \frac{S_g^2 Q_g^2}{AP} \]

Equation (16) the kinetic energy is inversely proportional to pressure, that is, as the pressure increases, the gas kinetic energy decreases which suggest that conditions of the bottom hole are the controlling condition for the removal of liquid in
the wellbore where gas has lower kinetic energy and higher pressure Guo et al. (2006). Therefore, it can be concluded that this analysis is in tandem with the process in drilling operations where solid particles do not accumulate top hole but at bottom hole. However, this result is contrary to Turner results which show that only the wellhead conditions are the controlling conditions. Guo et al. (2005)

The New Model: four phase flow model:

To have accurate prediction of the pressure \( p \) in the bottomhole in Eq. (16), 4-phases of gas-oil-water-solid mist flow model was developed where the introduction of valve was considered along the functional nodes. When considering the transport of the 4-phases flowing in a pipe in an upward direction, the differential pressure \( dp \) across a smaller portion of pipe length \( dl \), inclining at an angle \( \theta \) consist of five parameters: the pressure drop as a result friction, the pressure drop across valve along the conduit, pressure drop due to accumulation and kinetic energy and back pressure drop as a result of the fluid weight. For a given fluid flowing across the valve, the pressure drop is calculated based on the fluid velocity and valve opening. Adegboye (2016). This is represented by equation 17.0

\[
\frac{dp}{dt}_{\text{valve}} = \frac{u^2}{c^2 u^2 p} \tag{17.0}
\]

Derivation of the model takes the basis from thermodynamics, the total energy is made up of internal energy, potential energy and kinetic energy. The energy for a steady state system now becomes

\[
(u + e_p + e_k)_{2} - (u + e_p + e_k)_{1} = q_h - w \tag{18.0}
\]

The introduction of kinetic energy theory to determine the minimum energy required

\[
dp = \gamma_{\text{max}}[\cos (\theta) + \frac{\gamma_{\text{max}} v_{\text{max}}}{2 \gamma c_d t} + \frac{u^2}{c^2 u^2 p} - \frac{\gamma_{\text{max}} v_{\text{max}}}{2 \gamma_{c} D}] \tag{19.0}
\]

Numerically, the pressure \( p \) of the fluid flowing upward the pipe according to the new 4-phase model can be solved.

\[
\int_{p_h}^{p} \frac{(dp+1)dp}{p(1+e(\frac{c^2 u^2 p}{p})^2+\frac{w(\frac{c^2 u^2 p}{p})^2+\frac{\gamma_{\text{max}} v_{\text{max}}}{2 \gamma c_d t})}{Kl} = kl \tag{20.0}
\]

where

\[
k = \frac{15.3235Q_{w}+86.0715Q_{o}\theta+86.0715Q_{o}+0.01885Q_{g}Q_{o}}{7700} \tag{21.0}
\]

\[
b = \frac{(0.2456Q_{o}+1.379Q_{w}+1.379Q_{o})}{7700} \tag{22.0}
\]

\[
c = 4.712\times10^{-5} \gamma_{0} \tag{23.0}
\]

\[
d = \frac{5.615(\gamma_{o}+\gamma_{o})+\gamma_{p}}{86400A} \tag{24.0}
\]

\[
e = \frac{f}{2g c D_H \cos \theta} \tag{25.0}
\]

\[
f = \frac{0.174}{2 \log \left(\frac{D}{D_H}\right)} \tag{26.0}
\]

\[
w = \frac{1}{c^2 u^2 P \cos \theta} \tag{27.0}
\]

\[
y = \frac{1}{2 g c \cos \theta} \tag{28.0}
\]

\[
z = \frac{1}{2 g c d t \cos \theta} \tag{29.0}
\]

**COMPARISON AND DISCUSSION**

The 106 test point data from field employed by Turner et al. (1969) was used in this model. The results were compared to their work, Gou and Fadairo models.

**Assumptions**

0.6 gas specific gravity was assumed for the well.

Gas-condensate was 20 dyn/cm.

Gas-water interfacial tension was 60 dyn/cm.

The Wellhead Temperature of 60 °F was used and the roughness pipe wall was approximated to be 0.000015in.

Having considered the pressure drop across the valve along the functional node, a numerical integration approach-the trapezoidal rule, a more critical and accurate rate prediction was given.

**Application**

This new model is an improvement on Turner, Guo and Fadairo models and the applicability gives a more critical and accurate rate prediction of liquid loading.
Figure 2: Calculated Flow Rate Vs Test Flow Rate of Turner.

Figure 3: The Calculated Flow Rate Vs Test Flow Rate of Guo.
Figure 2 summarised Turner et al. (1969) result the controlling condition for liquid load up is the wellhead. Of all the 106 test points from field data used by Turner et al. (1969), 24 wells were incorrectly predicted.

Figure 3 results shows that fluid flow in wellbore is multiphase therefore, bottomhole condition should be the controlling condition and with the introduction of minimum kinetic energy criterion, the result was better than Turner et al.’s result. 20 wells were incorrectly predicted.

Figure 4 was an improvement on Guo et al by considering pressure drop due to both kinetic and accumulation terms that was not considered before and this gave an incorrectly predicted well of 13 wells.

The result of the new model was better than the other three models because it was an improvement on Fadairo et al. (2013). After considering pressure drop across the valve along the functional node, only 10 wells were wrongly predicted using the same data used by Turner et al. (1969) and all other authors.
Influence of Transient Flow Period

The figure 6 below captures the transient property in a gas velocity streams. Both Fadairo et al. and the new model showed that the critical flow rate increases with time. The new model increases from 0 to 150 days and then stabilized while Fadairo’s model increases from 0 to 200 days before it stabilized. This new model attained stability before Fadairo et al. model. This result does not support Guo’s assumption of constant flow rate. This evidently shows that there exists an initial transient flow period at the onset of flow which will elapse and give way to more stable steady state flow. It was observed that the energy needed to carry liquid from the wellbore is more than the one required at the early stage of the production.

Critical Flow Rate Comparisons with Different Pressures

The figure below (Fig 7) showed the comparison of the flow rates of different models against pressure, it was observed that the introduction of valve equation to the new model allows for an accurate estimation of the minimum gas flow rate at different pressures.

The pressure drop along the functional nodes give rise to the highest flow rate as indicated on the graph. This suggested that the valve along the functional node can be used to increase the flow rate and consequently reduce well load up

Figure 6: Transient Curve (Minimum Gas Flow Rate Vs Production Time).

Figure 7: Critical Flow Rates Comparisons with Different Pressures.
CONCLUSION

Based on the velocity of a free falling particle in fluid medium, droplet theory and the introduction of pressure drop across the valve to the fundamental equation, a new improved model was derived and the result shows that it was more accurate than Turner et al. (1969), Guo et al. (2005) and Fadairo et al. (2013).

<table>
<thead>
<tr>
<th>Name of Models</th>
<th>Number of incorrectly predicted well.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turner et al.’s model</td>
<td>24</td>
</tr>
<tr>
<td>Guo et al.’s model</td>
<td>20</td>
</tr>
<tr>
<td>Fadairo et al.’s model</td>
<td>13</td>
</tr>
<tr>
<td>New model</td>
<td>10</td>
</tr>
</tbody>
</table>

The number of incorrectly predicted well is minimal compared to Guo, Turner and fadairo model. It was also observed that the transient period was faster and shorter than the other models. This validates the fact that at a certain time in the production period, the transient period will be over and more steady stable state flow period will come.

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Nomenclature:

- **A**: area of conduit in ft²
- **Ai**: area of conduit in in²
- **Cd**: drag coefficient.
- **Dh**: Hydraulic diameter of the conduit, ft.
- **Ek**: specific kinetic energy of gas ibf-ft /ft³
- **Emk**: minimum kinetic energy required to carry the liquid droplet ibf-ft/ft³
- **Emk**: minimum kinetic energy required to prevent liquid droplet from falling ibf-ft/ft³
- **f**: the moody friction factor, dimensionless. Guo et al. (2003).
- **g**: 32.2 ft/s²
- **L**: pipe (conduit) length
- **Pwf**: wellhead flowing pressure, psia
- **P**: pressure ib/ft³
- **Qg**: insitu gas volumetric flow rate ft³/s
- **Qo**: volumetric flow rate of oil ft³/s
- **Qs**: volumetric flow rate of Solid particle ft³/s
- **Qw**: volumetric flow rate of water ft³/s
- **Sp**: specific gravity of gas
- **So**: produced oil specific gravity, fresh water = 1
- **Sw**: Specific gravity of solid, fresh water = 1
- **Sw**: produced water Specific gravity, fresh water = 1
- **Vmax**: mixture velocity, ft/s
- **Wg**: flow rate of gas weight, lb/s
- **Wo**: flow rate of oil weight, lb/s
- **Ws**: flow rate of solid particle weight, lb/s
- **Ww**: flow rate of water weight, lb/s

APPENDIX

The New Model: four phase flow model:

To have accurate prediction of the pressure p in the bottomhole, 4-phases of gas-oil-water-solid mist flow model was developed where the introduction of valve was considered along the functional nodes. when considering the transport of the 4-phases flowing in a pipe in an upward direction, the differential pressure dp across a smaller portion of pipe length dl, inclining at an angle θ consist of five parameters: the pressure drop as a result friction, the pressure drop across valve along the conduit, pressure drop due to accumulation and kinetic energy and back pressure drop as a result of the fluid weight. For a given fluid flowing across the valve, the pressure drop is calculated based on the fluid velocity and valve opening.

$$\left( \frac{dp}{dl}\right)_{\text{valve}} = \frac{\nu^2}{c^2u^2p}$$  \hspace{1cm} A-1

$$dp = \gamma_{max}\left[\left(\frac{dp}{dl}\right)_{\text{Elev}} + \left(\frac{dp}{dl}\right)_{\text{frict}} + \left(\frac{dp}{dl}\right)_{\text{acc}} + \left(\frac{dp}{dl}\right)_{\text{Kg}}\right]$$  \hspace{1cm} A-2

$$dp = \gamma_{max}\left[\cos (\theta) + \frac{v_{mix}^2}{2g_sD_h} + \frac{v_{mix}^2}{2g_sD_h} + \frac{v^2}{c^2u^2p}\right]$$  \hspace{1cm} A-3

The specific weight of the mixture can be calculated using Eqn A-4. Fadairo et al. (2013)

$$\gamma_{mix} = \frac{W_g + W_w + W_o + W_s}{g_s \sum w_{i} r_{i} t_{i} + \sum r_{i} t_{i} P_{wi}}$$  \hspace{1cm} A-4
$W_S, W_w, W_o$ and $W_g$ are daily Production:

$$W_S = \frac{S_0 q_S 62.4}{86400}$$

$$W_w = \frac{(62.455 q_w 5.615Q_o)}{86400}$$

$$W_o = 62.455 q_o \frac{5.615Q_o}{86400}$$

$$W_g = 0.07655 \frac{q_g}{86400}$$

The total volumetric flow rate of the fluids (solid, water, oil and gas) are all expressed in field units.

$$q_s \text{ ft}^3/\text{sec} = \frac{q_s}{86400}$$

$$q_w \text{ ft}^3/\text{sec} = 5.615 \frac{q_w}{86400}$$

$$q_o \text{ ft}^3/\text{sec} = 5.615 \frac{q_o}{86400}$$

$$q_g \text{ ft}^3/\text{sec} = \frac{14.7(1447Q_o)}{86400(520)}$$

Substituting Eqs. A-5 through A-13 in to Eq.4 and A-14 was obtained

$$\frac{(15.3255q_s+86.0715q_w+86.0715q_o+0.01885q_g)^p}{0.245601+13750q_o+13750q_o} p+1$$

Eqn. A-14 was further simplified to yield Eqn.A-15

$$\gamma_{mx} = \frac{k_p}{b_p+1}$$

Based on total volumetric flow rate, the velocity of the mixture was formulated

$$v_{mx} = \frac{q_o + q_w + q_s}{A}$$

The respective flowrate was substituted into A-16 and A-17 was derived.

$$v_{mx} = \frac{1}{A} \left( \frac{q_s}{86400} + 5.615 \frac{q_w}{86400} + 5.615 \frac{q_o}{86400} + \frac{q_g}{86400} \right) \frac{14.7(1447Q_o)}{86400(520)}$$

$$v_{mx} = \frac{5.615\left(q_o+q_w+q_s\right)}{86400A} + \left( \frac{4.712+10^{-5}T_Q_o}{P_A} \right)$$

Let

$$\frac{4.712+10^{-5}T_Q_o}{P_A} = C$$

and also

$$\frac{5.615\left(q_o+q_w+q_s\right)}{86400A} = d$$

therefore, equation A-18 becomes

$$v_{mx} = \frac{C_p}{d}$$

By substituting for all the parameter in Eqn A-3 and multiplying thru by $\theta$, Eqn.A-22 was developed.

$$dp = \frac{k_p \cos \theta}{b_p + 1} \left[ 1 + \frac{(C_p + d)^2}{2g\rho d^2 c_l \cos \theta} + \frac{(C_p + d)^2}{c_2 \rho^2 d^2 \cos \theta} \right]$$

where

$$e = \sqrt{\frac{f}{2b_p d \rho c_l \cos \theta}}$$

$$z = \frac{1}{2g\rho d^2 c_l \cos \theta}$$

$$w = \frac{1}{c_2 \rho^2 d^2 \cos \theta}$$

$$y = \sqrt{\frac{2g \rho c_l \cos \theta}{b_p}}$$

$$dp = \frac{k_p \cos \theta}{b_p + 1} \left[ 1 + e(C_p + d)^2 + z(C_p + d)^2 \right]$$

let $k_p \cos \theta$ be represented by capital $K$ and Eqn A-28 was established.

$$dp = \frac{k_p}{b_p + 1} \left[ 1 + e(C_p + d)^2 + z(C_p + d)^2 \right]$$

By separation of variables and applying boundary conditions

$$p = \phi \text{ at } L = 0$$

and then integrating over the conduit length gives Eqn. A-29

$$\int_p^{b_p+1} dp = K l$$

REFERENCES


